Canadian Association of Physicists
1994 Prize Exam

This is a three hour exam. National ranking and prizes will be largely based on a student's performance on the three questions in part B of the exam for which written solutions are required. However, performance on the multiple choice questions in part A will be used to determine whose written work will be marked for prize consideration.

The questions in part B of the exam have a range of difficulty. Please be careful to gather as many of the easier marks as possible before venturing into more difficult territory. In some cases an answer to, say, part (a) of a question is needed for part (b). Should you not be able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to finish this exam and part (d) of each question is very challenging and will likely be solved correctly by only a few of the very top Canadian physics students.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions on the answer card/sheet provided to you and most importantly, write your solutions to the three written problems on separate sheets as they will be marked by different people in different locations. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
Radius of Earth	$R_E = 6380$ km
Mass of Earth	$M_E = 5.98 \times 10^{24}$ kg
Mass of Sun	$M_S = 1.99 \times 10^{30}$ kg
Radius of Earth's orbit	$R_{ES} = 1.50 \times 10^{11}$ m
Acceleration due to gravity	$g = 9.81$ m/s^2
Fundamental charge	$e = 1.6 \times 10^{-19}$ C
Mass of electron	$m_e = 9.1 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.67 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ Js
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ Nm^2/C^2
Magnetic constant	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	340 m/s

Part A: Multiple Choice

Question 1

It was once proposed to use a 2 km vertical Sudbury mine shaft for microgravity experiments in a vacuum. How much time would scientists have to do an experiment during one drop?

(a) 20 s
(b) 63 s
(c) 0.64 s
(d) 198 s

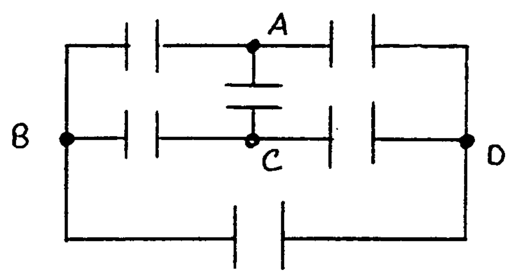
Question 2

If the moon were twice as massive as it is now, and it stayed at the same orbital radius about the earth as it has now, its new orbital period (in terms of its current orbital period T) would be,

(a) T
(b) $T/2$
(c) $T/4$
(d) $2T$

Question 3

In the following circuit composed of identical capacitors, across which terminals would you connect a battery in order for all the capacitors to charge up.



(a) AB
(b) AC
(c) BD
(d) None of the above.

Question 4

Two cylindrical resistors, one of length l and radius r, and the other of length $3l$ and radius $3r$, are made of the same materials. If the resistance of the smaller one is R, what is the resistance of the larger one?

(a) $R/3$
(b) $3R$
(c) $9R$
(d) $27R$

Question 5

A simple pendulum consists of a mass m attached to a light string of length l. If the system is oscillating through small angles, which of the following is true?

(a) The frequency is independent of the acceleration due to gravity, g.
(b) The period depends on the amplitude of the oscillation.
(c) The period is independent of the mass m.
(d) The period is independent of the length l.

Question 6

An astronaut in the space shuttle orbiting the earth performs a trick for a television audience. She inflates a helium filled balloon within the shuttle's controlled atmosphere and lets go of it. To the astonishment of all watching, the balloon

(a) hovers in place where it was released.
(b) rises noticeably away from the earth.
(c) falls noticeably towards the earth.
(d) drifts backwards opposite to the direction of the shuttle's velocity.

Question 7

A boat has a green light (with wavelength $\lambda = 500$ nm) on its mast. What wavelength would be measured and what colour would be observed for this light as seen by a diver submerged in water (index of refraction $n = 1.33$) by the side of the boat.

(a) Green $\lambda = 500$ nm
(b) Red $\lambda = 665$ nm
(c) green $\lambda = 376$ nm
(d) UV $\lambda = 376$ nm

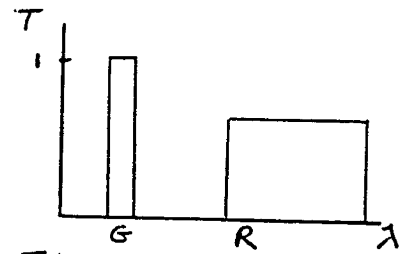
Question 8

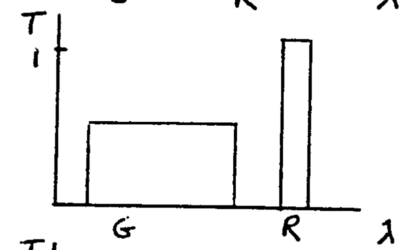
A radio station transmits using a vertical mast. The antenna in your radio is a coil (solenoid) wrapped around a ferrite (iron) rod. What orientation must this antenna have for optimum reception?

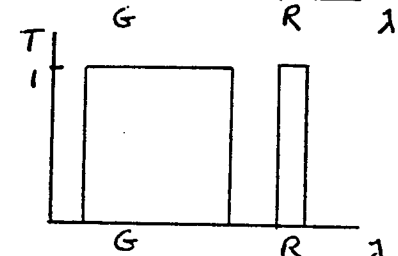
(a) Rod pointing at mast.
(b) Rod parallel to mast.
(c) Horizontal rod pointing 90° from the line of the mast.
(d) Horizontally oriented rod directly above the mast.

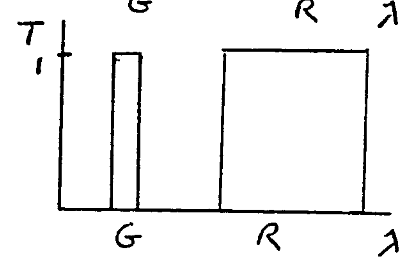
Question 9

You have ten identical filters. One is placed in front of a white light and the light appears red. When all ten are placed in front of the light, it appears to be a dim green colour. Which of the following is a possible transmission characteristic for one of the filters?

(a)


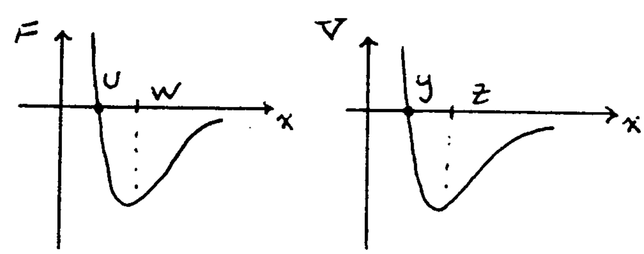
(b)


(c)


(d)


Question 10

Two atoms interact with each other according to the following force F, and potential, V diagrams. What is their equilibrium separation?



(a) The separation u which is equal to y
(b) The separation u which is equal to z
(c) The separation w which is equal to y
(d) The separation w which is equal to z

Question 11

A ball is thrown into the air with an initial speed u. The time interval taken for the ball to rise to its maximum height is t_r. The time interval taken for it to fall back down from this maximum height to its original position is t_f. Under "real life" conditions, which of the following is satisfied by t_r and t_f.

(a) $t_r > t_f$
(b) $t_r < t_f$
(c) $t_r = t_f$
(d) $t_r > t_f$ if u is great enough.

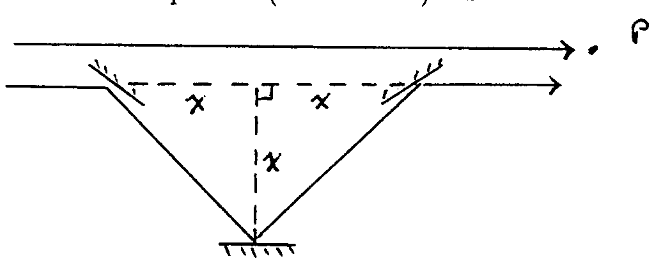
Question 12

An airplane flies a straight path from town A to town B, 500 km away. Town B is due east of town A and a strong wind blows from north to south at 300 km/hr. If the plane's airspeed is 900 km/hr, which of the following statements is true?

(a) Trip time is $\frac{5}{3\sqrt{8}}$ hr.
(b) Plane's ground speed is 600 km/hr.
(c) Plane's heading is 30° North of East.
(d) None of the above.

Question 13

Microwaves of wavelength $\lambda = 5.0$ cm, and intensity I_o, are split and recombined by the metallic mirror system shown. What should x be so that the intensity of the microwaves at the point P (the detector) is zero.



(a) 0.88 cm
(b) 3.54 cm
(c) 6.04 cm
(d) 3.02 cm

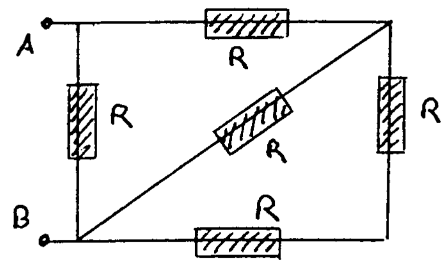
Question 14

Which of the following is a possible expression for the Rydberg, a unit of energy?

(a) $e^4/8m_e\epsilon_o h^2$
(b) $\epsilon_o^2 h^2/8m_e e^4$
(c) $m_e e^4/8\epsilon_o^2 h^2$
(d) $m_e c^2/\epsilon_o he$

Question 15

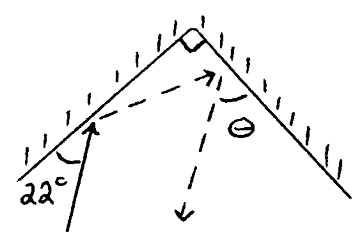
Consider the network of identical resistors shown. The equivalent resistance between the points A and B is,



(a) $5R$
(b) $R/2$
(c) $5R/8$
(d) $2R$

Question 16

A ray of light is directed towards a corner reflector as shown. The incident ray makes an angle of 22° with one of the mirrors. At what angle θ does the ray emerge?



(a) 22°
(b) 68°
(c) 44°
(d) None of the above.

Question 17

Many people's glasses appear to be a blue-green colour when viewed under reflected light. A thin film of index of refraction $n = 1.35$ is applied to the outside surface of the glass so that the film/glass interface does not reflect any red light of wavelength $\lambda = 630$ nm. What thickness must the film layer be in order to achieve this? Take the index of refractions of air and glass to be 1.0 and 1.6 respectively.

(a) 157.5 nm
(b) 315.0 nm
(c) 233.3 nm
(d) 116.7 nm

Question 18

A mass M has the same kinetic energy as a mass m. The ratio of their momenta, p_M/p_m, is,

(a) $\sqrt{M/m}$
(b) $\sqrt{m/M}$
(c) $(m+M)/M$
(d) $(m+M)^2/mM$

Question 19

A stream of water droplets, each of mass $m = 0.001$ kg, are fired horizontally at a velocity of 10 m/s towards a steel plate where they collide. The droplets are spaced equidistantly with a spacing of 1 cm. What is the approximate average force exerted on the plate by the water droplets assuming that they do not rebound after their collision.

(a) 10 N
(b) 100 N
(c) 1 N
(d) 0.1 N

Question 20

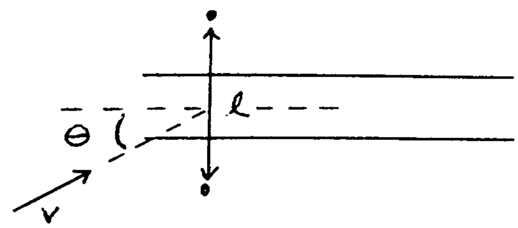
Three identical balls are thrown from the top of a cliff and some time later land at the base of the cliff. Ball A is thrown upwards with speed v, Ball B is thrown downwards with speed v, and Ball C is thrown at speed v and at an angle of 45° above the horizontal. Comparing the speeds, v_A, v_B, and v_C with which the balls hit the ground at the base of the cliff (and ignoring air resistance), you find,

(a) $v_A = v_B > v_C$
(b) $v_A > v_C > v_B$
(c) $v_A = v_B = v_C$
(d) $v_B > v_C > v_A$

Part B

Question 1

In this question we will investigate a possible navigational aid for aircraft approaching an airport. Suppose at one end of a runway there are two radio transmitting towers, one on either side of the runway, separated by a distance $l = 100$ m. The two radio towers are each transmitting a radio signal of frequency $f_o = 12$ MHz in phase with each other. An aircraft with a ground speed of v is flying towards the airport such that its velocity makes an angle θ with the runway as shown. The aircraft, still very far from the airport, has locked onto the signals and is heading directly for the midpoint of the towers.



(a) At the aircraft's current position, the intensity of the signal from each tower separately would be I_o. Find the intensity of the combined signal from both towers received by the aircraft for headings of $\theta = 0$ and $\theta = \pi/2$.

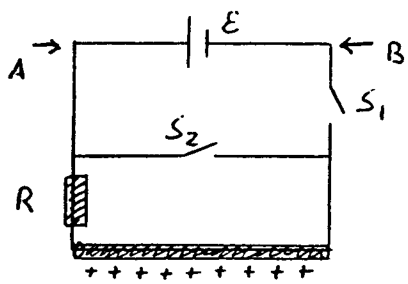
(b) For what headings would the aircraft receive no signal? If the aircraft was commanded to approach the airport with a heading of 30°, what kind of a signal should the captain expect from the radio towers?

(c) The aircraft gets slightly off course and is approaching the airport on a heading of $\theta = 0$ but is heading directly for one of the transmitting towers (as opposed to their midpoint). The aircraft is still very far from the airport but as it draws nearer, the intensity of the signal received from the radio towers starts to drop. How far is the aircraft from the nearest tower when the signal has dropped to a minimum?

(d) The aircraft, back on course now, is approaching the airport with a ground speed of 500 km/hr and on a heading with $\theta = 30°$. The aircraft has a circuit which generates its own 12 Mhz reference signal and compares it to the one being received from the radio towers. A beat is detected in the sum of the two signals. What is the beat frequency? The navigator quickly programs the on-board computer to calculate the aircraft's ground speed in terms of this beat frequency Δf. When the beat frequency is 10 Hz and the signal received from the towers is lower than the 12 Mhz on-board reference signal, what is the plane's ground speed?

Question 2

Consider the circuit shown with the two switches S_1 and S_2. The battery has an emf of $\epsilon = 6$ V and the resistor has a resistance of $R = 1.0\ \Omega$. The bottom wire is encased in an insulated wrapping of mass $M = 0.1$ kg which carries a positive static charge of $Q = 10$ mC. Assume that the masses of all other wires and components are very small as compared to this charge bearing wrapping. The dimensions of the circuit are $l = 5$ cm square. The bridge (or middle connecting) wire, with S_2 in it, is $l/2$ above the bottom wire. The contraption is suspended in such a way that it may pivot about the top wire on the axis AB; the circuit maintains its shape regardless of the switch configuration. A magnetic field, $B = 10.0$ T, and an electric field, $E = 1000$ N/C, both point out of the page.



(a) With both switches open, find the equilibrium angle θ that the plane of the circuit makes with the vertical as it swings upwards due to the presence of the electric field.

(b) With S_1 closed and S_2 open, find the equilibrium angle θ.

(c) Now with S_1 open and S_2 closed, the circuit is swung from hanging straight down, $\theta = 0$, to being horizontal, $\theta = \pi/2$, in a time of $\Delta t = 5$ ms. Estimate the work required to perform this operation.

(d) The circuit is released from the horizontal with both switches open. Describe the motion of the circuit and calculate any relevant parameters necessary to backup your description. What would be the effect of closing S_2 during this motion?

Question 3

Design a spherical spacecraft which could be carried out of the solar system by the pressure of solar radiation alone. Discuss constraints on the materials used. You may assume that the craft can be initially launched from the earth using a conventional rocket so that it may start its journey already having escaped the earth's pull.

The momentum p of a photon is given by $p = E/c$ where E is the energy of the photon and c is the speed of light. The intensity of the solar radiation at the radius of the earth's orbit about the sun is given by the solar constant, $S = 1370$ W/m^2.

Link to solutions ...

Solutions: Part A

1 (a)	5 (c)	9 (a)	13 (c)	17 (d)
2 (a)	6 (a)	10 (b)	14 (c)	18 (a)
3 (d)	7 (c)	11 (b)	15 (c)	19 (a)
4 (a)	8 (c)	12 (a)	16 (b)	20 (c)

Part B

Question 1

(a) The wavelength of the radio signals is $\lambda = c/f_o = 25$ m. Clearly for $\theta = 0$ and $\theta = \pi/2$, the interference is constructive. The amplitude of the signal picked up by the plane is twice the amplitude of the signal from one tower alone. Since intensity is proportional to the square of the amplitude, the intensity received is $I = 4I_o$.

(b) For destructive interference we solve,

$$l\sin(\theta) = (2n+1)\lambda/2 \ ,$$

where $l = 100$ m is the tower separation and n is an integer. This is standard two source interference with the receiver much farther away than the spacing of the sources. The angular headings that result from this are $\theta = 7.2°$, $22.0°$, $38.7°$, and $61.0°$. A heading of $30°$ corresponds to constructive interference and the signal strength from the towers would be a maximum.

(c) The path difference between the signals from the two towers is simply $\Delta r = \sqrt{l^2 + x^2} - x$ where x is the distance of the aircraft from the closest radio tower. For the first occurrence of destructive interference as the aircraft approaches, we set $\Delta r = \lambda/2$. This gives,

$$\sqrt{l^2 + x^2} = \frac{\lambda}{2} + x \ ,$$

which upon squaring both sides yields,

$$x = \frac{1}{\lambda}(l^2 - \frac{\lambda^2}{4}) \ .$$

Solving gives $x = 394$ m. We hope that the plane is well into its approach when it is this close to the runway!

(d) The plane receives a signal of frequency

$$f = f_o\sqrt{\frac{1 + v/c}{1 - v/c}} \ ,$$

where c is the speed of light and we have used the doppler shift formula for electromagnetic waves. Since the problem is nonrelativistic, a student may use the classical doppler formulae without penalty. Solving this is a bit tricky and a calculator is usually not much help. Squaring both sides, however, gives,

$$(1 - v/c)f^2 = (1 + v/c)f_o^2 \ .$$

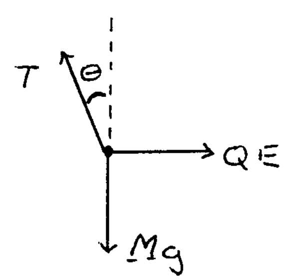
Collecting terms allows us to write,

$$(f - f_o)(f + f_o) = (f^2 + f_o^2)v/c \ .$$

The plane is travelling much slower than the speed of light and we expect that the frequency f will be very similar to f_o. Hence we define $\Delta f = f - f_o$ and $f + f_o \approx 2f_o$ to give, $\Delta f = f_o v/c$. This is the beat frequency in the sum of the signals. Numerically, it is 5.6 Hz. If the beat frequency is 10 Hz, we have $v = 250$ m/s or 900 km/hr. Since the detected frequency is lower than the reference signal, the plane must be heading away from the airport.

Question 2

(a) With both switches open, there is no current in any wire and there is no closed circuit. We consider the following force diagram for the bottom wire.



From this we get the two equations,

$$T\sin(\theta) = QE \ ,$$
$$T\cos(\theta) = Mg \ .$$

Here T is the tension in the supporting side wires. The angle then satisfies, $\tan(\theta) = QE/Mg$. This corresponds to an angle of 84.4°.

(b) With a current of $\epsilon/R = 6.0$ A in the bottom wire, there is a downwards magnetic force on the bottom wire given by $F_m = IlB$ where l is the length of the wire. The angle now satisfies,

$$\tan(\theta) = \frac{QE}{Mg + IlB} \ .$$

This corresponds to an angle of 68.2°.

(c) Here is a situation in which the magnetic flux through the lower closed circuit is changing in time. Initially it is given by $\phi = Bl^2/2$ and it drops to zero in a time of $\Delta t = 5$ ms. The average induced current in the bottom wire is then $I_{av} = \Delta\phi/R\Delta t$. The average current is then $I_{av} = 2.5$ A. In swinging the apparatus upwards, one does work in raising the mass and in driving a current through the resistor which dissipates this energy as heat. After the contraption reaches it new equilibrium, a good estimate for the work done is,

$$W = Mgl + I_{av}^2 R\Delta t \ ,$$

which gives $W = 0.08$ J.

(d) With both switches open, the equilibrium position of the apparatus is at an angle of $\theta_{eq} = 84.4°$ from the vertical. If the apparatus is displaced by a small angle α from this, the net unbalanced force on the bottom wire is directed towards the equilibrium position and is given by,

$$F = Mg\sin(\theta_{eq} + \alpha) - QE\cos(\theta_{eq} + \alpha) \ .$$

Using the trigonometric addition formulae, we rewrite these in the form,

$$F = Mg[\sin(\theta_{eq})\cos(\alpha) + \cos(\theta_{eq})\sin(\alpha)] - QE[\cos(\theta_{eq})\cos(\alpha) - \sin(\theta_{eq})\sin(\alpha)]$$

We know that $Mg\sin(\theta_{eq}) = QE\cos(\theta_{eq})$ so that

$$F = M[g\cos(\theta_{eq}) + \frac{QE}{M}\sin(\theta_{eq})]\sin(\alpha) \ ,$$
$$= Mg_{eff}\sin(\alpha) \ .$$

Compare this to the force equation for a simple pendulum, $F = Mg\sin(\alpha)$ where α is the angular displacement from equilibrium. For a small displacement, the motion of our apparatus will be simply harmonic with a period of $T = 2\pi\sqrt{l/g_{eff}}$. This gives a period of 0.14 s. Closing the switch S_2 will allow magnetically induced currents to form which will damp the pendulum's motion. It will eventually come to rest at θ_{eq}.

Question 3

For a black sphere of radius r, the rate at which solar energy falls upon it at a distance R from the Sun is given by,

$$P = \frac{\Delta E}{\Delta t} = \pi r^2 S(R_{ES}/R)^2 \ .$$

The force F on the sphere is,

$$F\Delta t = \Delta p = \frac{\Delta E}{c},$$

for a black sphere.

The mass m of the (hollow) sphere depends on the thickness of the material d and its density ρ.

$$m = 4\pi r^2 \rho d$$

The force F has to overcome the gravitational force F_g,

$$F_g = \frac{GMm}{r^2},$$

where GM for the Sun can be found to be (using Kepler's 3rd law),

$$GM = 4\pi \frac{R_{ES}^3}{T^2},$$

where T is one year. For acceleration away from Sun,

$$\begin{aligned} d &< \frac{SR_{ES}^2}{4c\rho GM} \\ &= 0.2\mu m \text{ if } \rho = 10^3 \text{ kg/m}^{-3} . \end{aligned}$$

If you have a perfectly reflecting sphere you get a factor of 2 for the change in momentum and $sin^2\theta$ from the angular variation across the sphere. This integrates out to unity, so the answer is the same. A student may avoid calculus by knowing that

$$< \sin^2(\theta) >= \frac{1}{2} .$$

So, $d < \lambda$ for solar radiation. You can make a good 90% reflecting Al thin film of about 50 nm but to make it black might be harder. I have seen self-supporting parylene films of about 1 μm but very floppy. A factor of five less than this is really pushing it!

Canadian Association of Physicists
1995 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee.

The questions in part B of the exam have a range of difficulty. Please be careful to gather as many of the easier marks as possible before venturing into more difficult territory. In some cases an answer to, say, part (a) of a question is needed for part (b). Should you not be able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to completely finish this exam and part (d) of each question is very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions on the answer card/sheet provided to you and most importantly, write your solutions to the three written problems on separate sheets as they will be marked by different people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
Radius of Earth	$R_E = 6380$ km
Mass of Earth	$M_E = 5.98 \times 10^{24}$ kg
Mass of Sun	$M_S = 1.99 \times 10^{30}$ kg
Radius of Earth's orbit	$R_{ES} = 1.50 \times 10^{11}$ m
Acceleration due to gravity	$g = 9.81$ m/s^2
Fundamental charge	$e = 1.6 \times 10^{-19}$ C
Mass of electron	$m_e = 9.1 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.675 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ Js
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ Nm^2/C^2
Magnetic constant	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	340 m/s
Boltzman constant	$k = 1.38 \times 10^{-23}$ J/K
Absolute zero	$T = 0$ K, $T = -273°$C

Part A: Multiple Choice

Question 1

A salvage ship tries to raise a sunken miniature submarine from the bottom of Lake Superior. The submarine and its contents have a mass of 72,000 kg and a volume of 18.9 m^3. What upwards force must be applied to raise the submarine? The density of water is 1000 kg/m^3.

(a) 1.8×10^5 N
(b) 2.0×10^5 N
(c) 4.8×10^5 N
(d) 5.2×10^5 N

Question 2

A little girl is playing with a toy pendulum while riding in an elevator. Being an astute and educated young lass, she notes that the period of the pendulum is $T = 0.5$ s. Suddenly the cables supporting the elevator break and all of the brakes and safety features fail simultaneously. The elevator plunges into free fall. The young girl is astonished to discover that the pendulum has,

(a) continued oscillating with a period of 0.5 s.
(b) stopped oscillating entirely.
(c) decreased its rate of oscillation to have a greater period.
(d) increased its rate of oscillation to have a lesser period.

Question 3

Two identical spring loaded, toy guns shoot projectiles straight upwards. The projectile in gun B has twice the mass as that in gun A. The projectile launched by gun A reaches a height H. Ignoring air resistance, the projectile launched by gun B would reach a height of,

(a) $H/4$
(b) $H/2$
(c) $H/\sqrt{2}$
(d) H

Question 4

An athlete of mass 75 kg runs at 10 m/s. What is his kinetic energy?

(a) 3750 J
(b) 7500 J
(c) 375 J
(d) 750 J

Question 5

The moon is about 60 earth radii from the centre of the earth and completes one orbit in about 28 days. Assuming that the moon executes a circular orbit about the centre of the earth, the acceleration of the moon (in units of g, the acceleration due to gravity at the earth's surface) is closest to,

(a) $g/30$
(b) $g/60$
(c) $g/3600$
(d) g

Question 6

A survey crew sights the top of a radio tower with a small telescope. The angle that the telescope makes with the horizontal is measured to be 53°. The crew moves the telescope 25 m closer to the base of the tower and repeats the process. The new angle that the telescope makes with the horizontal is 60°. What is the height of the tower assuming that the base of the tower is on the same level as the areas where the telescope measurements were made?

(a) 33 m
(b) 43 m
(c) 142 m
(d) 286 m

Question 7

Which of the following is **not** true for electromagnetic waves;

(a) they consist of oscillating electric and magnetic fields.
(b) they travel at different speeds in air depending on their frequency.
(c) they transport energy.
(d) they have no momentum.

Question 8

Two long parallel wires are placed side-by-side on a level table. The two wires carry equal currents in the same direction. Which of the following is true of the magnetic forces exerted on each wire by the other?

(a) One wire experiences a force upwards while the other experiences a downwards force.
(b) Both wires experience an upwards force.
(c) The two wires are attracted to one another.
(d) The two wires repel one another.

Question 9

A piano tuner uses a tuning fork which oscillates with a frequency of 440 Hz (middle A). Middle A on a piano is out of tune, producing a note of frequency f. The piano note and the tuning fork are heard to beat with a frequency of 3 Hz. When the tension in the piano string is decreased, the beat frequency also decreases. The out-of-tune frequency f must have been,

(a) 446 Hz
(b) 443 Hz
(c) 437 Hz
(d) 434 Hz

Question 10

Sudbury's Creighton mine is one of the deepest in the world (2.07 km). In this mine the conditions as compared to those at the surface are,

(a) lower air pressure, higher acceleration due to gravity.
(b) higher air pressure, lower acceleration due to gravity.
(c) higher air pressure, higher acceleration due to gravity.
(d) lower air pressure, lower acceleration due to gravity.

Question 11

A parallel network of resistors consists of a 1 Ω resistor placed in parallel with a 2 Ω resistor. This combination is, in turn, placed in parallel with a 4 Ω resistor. Further 8 Ω, 16 Ω, 32 Ω ... resistors are placed in parallel with the network one by one. To what value does equivalent resistance of the network converge upon?

(a) 0.5 Ω
(b) 2.0 Ω
(c) 0.0 Ω
(d) Does not converge to a finite value.

Question 12

A simple pendulum has a bob of mass 2 kg hanging on a cord of length 1 m. Suppose the pendulum is raised until it is horizontal (an angular displacement of 90°) and then released. What is the speed of the bob at the bottom of its swing?

(a) 9.91 m/s
(b) 19.6 m/s
(c) 3.13 m/s
(d) 4.43 m/s

Question 13

An organ pipe of length L is open at one end and closed at the other. What are the wavelengths of the three lowest frequencies that can be produced with this pipe?

(a) $4L,\ 2L, L$
(b) $2L,\ L,\ L/2$
(c) $2L,\ L,\ 2L/3$
(d) $4L,\ 4L/3,\ 4L/5$

Question 14

A satellite is in orbit at an altitude of one earth radius. What is the orbital speed of this satellite?

(a) 1.56×10^7 m/s
(b) 3.95×10^3 m/s
(c) 7.91×10^3 m/s
(d) 5.59×10^3 m/s

Question 15

A steady non-turbulent stream of water comes out of a tap and falls vertically downward. As it does so, the diameter of the stream appears to get smaller. What is the primary reason?

(a) The water's surface tension constricts the stream.
(b) Air pressure, which decreases with altitude, squeezes the stream.
(c) The water is accelerating under gravity and so the stream must get thinner as the flow rate (velocity times cross-sectional area) must be constant.
(d) The flow does not constrict; it is an optical illusion.

Question 16

A roller coaster car is on a track that forms a circular loop, of radius R, in the vertical plane. If the car is to maintain contact with the track at the top of the loop (generally considered to be a good thing), what is the minimum speed that the car must have at the bottom of the loop. Ignore air resistance and rolling friction.

(a) $\sqrt{2gR}$
(b) $\sqrt{3gR}$
(c) $\sqrt{4gR}$
(d) $\sqrt{5gR}$

Question 17

An object is placed 30 cm in front of a thin, spherical lens. An upright image, twice the size of the object, is formed by the lens. Which of the following attributes best describes the lens.

(a) diverging, focal length 20 cm
(b) converging, focal length 20 cm
(c) diverging, focal length 60 cm
(d) converging, focal length 60 cm

Question 18

A point charge $+Q$ is placed at the centroid of an equilateral triangle. When a second charge $+Q$ is placed at a vertex of the triangle, the magnitude of the electrostatic force on the central charge is 4 N. What is the magnitude of the net force on the central charge when a third charge $+Q$ is placed at another vertex of the triangle?

(a) zero
(b) 4 N
(c) $4\sqrt{2}$ N
(d) 8 N

Question 19

Greenhouse "warming" is caused by an atmosphere which is ,

(a) transparent to visible light but opaque to I.R. light.
(b) transparent to both visible and I.R. light.
(c) opaque to both visible and I.R. light.
(d) warming due to the hot air from the world's increasing population.

Question 20

A particle of mass m and charge q is accelerated through a potential difference V to a velocity $\vec{v}$ towards the south. The particle enters a region with both a magnetic field $\vec{B}$ (pointing eastwards) and electric field $\vec{E}$ (pointing down). The particle travels at constant velocity through this region. The potential difference V must satisfy,

(a) $V = mE^2/2qB^2$
(b) $V = 2mE/qB$
(c) $V = E/qB$
(d) It is not possible for the particle to be undeflected by these fields.

Question 21

A space craft orbiting the moon with an orbital radius of 10^6 m has an orbital period of 45 minutes. When it drops down to a lower orbit of radius 10^5 m, its period has become,

(a) 36 minutes
(b) 85 seconds
(c) 54 minutes
(d) Need to know the mass of the moon for this problem.

Question 22

A simple pendulum is made from a 2 kg block of wood suspended from a light cord of length 1 m. When the pendulum is hanging in such a way that it is stationary and vertical, a bullet is shot horizontally into the block of wood where it sticks. The bullet has a mass of 10^{-2} kg and has a speed of 500 m/s just before its head-on collision with the block. The pendulum will swing to a maximum angle with the vertical of,

(a) 71.3°
(b) 23.4°
(c) 35.7°
(d) 46.8°

Question 23

Electronic components are tested for durability by subjecting them to high accelerations. This is achieved by placing them on a platform attached to a vertically hanging spring. The platform (and the component being tested) oscillate up and down with a frequency of 10 Hz when the spring is stretched by 10 cm and released. What is the maximum acceleration experienced by the electronic component?

(a) $10.0\ \mathrm{m/s^2}$
(b) $2.00\ \mathrm{m/s^2}$
(c) $395\ \mathrm{m/s^2}$
(d) $158\ \mathrm{m/s^2}$

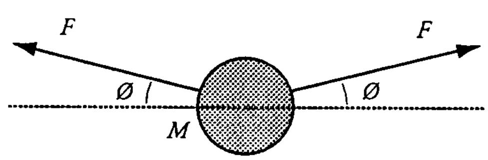
Question 24

A real battery can be considered to be an ideal battery of potential ϵ in series with an internal resistance r. Suppose that the battery is connected to a single "load" resistance R. For what value of R will the power dissipated by the load be a maximum?

(a) r
(b) $r/2$
(c) $2r$
(d) $10r$

Question 25

Two trucks pull on either end of a rope, each with a force of 10,000 N. In the centre of the rope, a ball of mass 20 kg is hanging.



The angle θ that the rope makes with the horizontal is,

(a) 56°
(b) 5.6°
(c) 0.56°
(d) 0°; the rope will obviously be horizontal in this situation.

Part B

Question 1

Physicists at UBC do microgravity experiments on board a NASA KC-135 (Boeing 707). The KC-135 flies parabolic flight paths to follow as closely as possible the trajectory and speed of a free-falling body. Once the plane is on the parabolic flight path (both ascending and descending), its engines are only used to overcome air resistance. The aircraft is known, with some justification, as the "Vomit Comet". The KC-135 cannot fly at a speed of less than 300 km/hr (as it has to be under control at all times) or more than 500 km/hr (or the g-forces in recovery will be too great).

(a) While the plane is executing its parabolic trajectory at an altitude of 10,000 m, a physicist of mass $m = 65$ kg demonstrates her "weightlessness" by floating freely in the large open cargo bay. Draw a force diagram for the physicist showing all forces acting on her body. Find the force of gravity acting on the physicist and find her net acceleration. Explain in your own words, and referring to your preceding calculations, why people call this experience "weightlessness".

(b) At what angle to the horizontal should the KC-135 enter its parabolic path?

(c) How much time do the physicists get under "weightless" conditions?

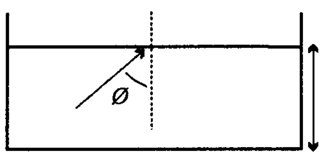
(d) While an aircraft is in level flight and travelling at constant velocity, passengers on board are said to experience a 1-g "force" downwards and can walk around the cabin normally. Sketch the trajectory of the path the aircraft would have to follow so that it could execute a 1-g vertical loop. In this situation, the aircraft must execute a full vertical loop while at all times allowing the passengers to experience only a 1-g "force" towards the floor of the cabin. All on board would be able to walk around normally at all times. What is the acceleration of the aircraft, in units of g, when it is at the top, and bottom of the loop? What is the acceleration of the aircraft, in units of g, when it is headed vertically upwards and downwards?

Question 2

A tank of salt water is allowed to stand undisturbed for a long time. The tank has a square footprint and measures 3 m by 3 m. The depth of the water in the tank is $d = 1$ m. In the absence of any agitation, the concentration of salt in the water is not uniform but increases with depth. Since salt water is more dense that pure water, the density of the water also increases with depth and hence, the index of refraction of the water also increases with depth. Assume that the index of refraction of the water at its surface is $n_o = 1.3$ and the index increases linearly and at the rate of

$\alpha = 0.05\ \mathrm{m}^{-1}$. The walls and floor of the tank are made of clear glass and the air surrounding the tank has an index of refraction $n_a = 1$. This tank can be used to model the optical properties of a variety of situations where the index of refraction varies with altitude.

(a) Suppose, for parts (a) and (b), that we make the simplification that the index of refraction of the water is uniform and has the value $n_o = 1.3$ throughout the tank. A laser beam is shone through the floor of the tank towards the surface of the water as shown. When the beam is pointing straight up, it passes through the salt water and exits into the air above. As the angle between the beam and the normal to the surface is increased, a point is reached where the beam no longer exits the water but is internally reflected back into the tank. Calculate the angle at which this total internal reflection occurs.



(b) A small light bulb is placed directly under the middle of the tank. Looking down from above at the surface of the water, what area on the surface appears to be illuminated by the light? Note that light from a light bulb travels in all directions.

(c) Now let us consider the more complicated case where the water is not of uniform density. Treat the salt water as a large number of layers, each with a distinct index of refraction and sketch the path of the laser beam in part (a). At what angle, from the normal to the bottom of the tank, did the laser beam enter the water so that it would be internally reflected at the top surface? Sketch the path of a ray which entered the tank at an angle greater than this.

(d) Now consider air above a hot roadway. At the surface of the roadway, the temperature of the air is $T_h = 60°\mathrm{C}$ whereas at a height of $l = 1$ m and above, the air is a cool $T_c = 30°\mathrm{C}$. The index of refraction $n(T)$ of the air as a function of temperature is related to the air's density $\rho(T)$ according to the relation,

$$n(T) - 1 \propto \rho(T) \ ,$$

where T is the absolute temperature of the air in degrees Kelvin. You may assume that the density of the air is, in turn, inversely proportional to the air's temperature. The index of refraction of air at 15°C and atmospheric pressure is 1.000276. Sitting in a car with your eyes 1.5 m above the roadway, how far is it in the distance that the roadway appears to shimmer? This is, of course, a mirage.

Question 3

Estimate from your basic bodily dimensions your natural walking speed. Explain clearly, in words and diagrams, the physics you use to make this calculation. If time allows, go beyond your first basic calculation to discuss refinements and improvements to your model for walking. Be creative.

Link to Solutions

Solutions: Part A

1 (d)	2 (b)	3 (b)	4 (a)	5 (c)
6 (c)	7 (d)	8 (c)	9 (b)	10 (b)
11 (a)	12 (d)	13 (d)	14 (d)	15 (c)
16 (d)	17 (d)	18 (b)	19 (a)	20 (a)
21 (b)	22 (d)	23 (c)	24 (a)	25 (c)

Part B

Question 1

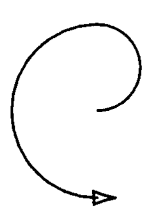
(a) The only force on the physicist is the downwards force of gravity which in the physicist's case is about 638 N. The acceleration is just g downwards. The experience of weightlessness is simply a prolonged period with the absence of any contact forces acting upon the body. This could be achieved in a free-fall situation or in the total absence of a gravitational field. We experience weight in everyday life via the presence of normal forces and such.

(b) The horizontal speed will remain constant throughout the flight. At the apogee, the velocity is at a minimum, there being no vertical speed. So, the horizontal speed is constrained to be 300 km/hr. The KC-135 should enter the parabola as fast as possible to get the longest experimental time; this velocity is 500 km/hr. So, the angle α satisfies $\cos\alpha = 300/500$ and hence, $\alpha = 53°$.

(c) The initial vertical speed is v_h = 400 km/hr (or 111 m/s). The time to reach apogee is the time this takes to go to zero. The total weightless time is then,

$$t_w = \frac{2v_h}{g} , \\ = 22.65 \text{ s} .$$

(d) This is tricky until one realizes that in the absence of gravity, the aircraft would simply fly in a circular path with such a speed and radius so as to have a centripetal acceleration of g. In the presence of gravity, we can imagine the aircraft flying its circular path in a giant room which is in free fall (accelerating downwards at g). Take away the imaginary room and the net trajectory would look something like,



At the top of the loop, the net acceleration would be $2g$ downwards whereas at the bottom of the loop, the acceleration would be zero. At the sides of the loop, the acceleration would be $\sqrt{2}g$ directed at an angle of 45° below the horizontal.

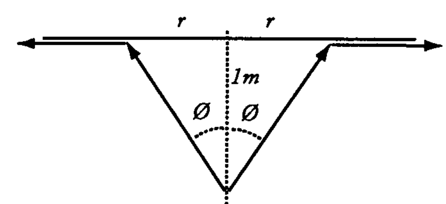
Question 2

(a) Snell's law of refraction gives,

$$1.3 \sin\theta = 1 \sin 90° ,$$

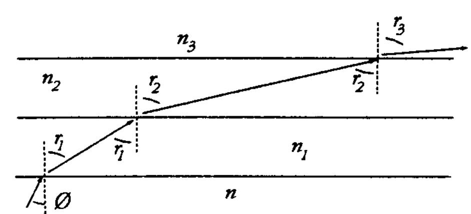
so that the angle for total internal reflection is θ = 50.3°.

(b) The light rays from the bulb which are totally internally reflected at the surface of the water are shown. Light rays incident on the surface at angles less than the critical angle pass out of the water. Hence, there appears to be a circular region illuminated by the bulb when viewed from above.



The illuminated area is $A = \pi r^2$ where r = (1 m) × tan(50.3°). Hence the area is A = 4.55 m^2.

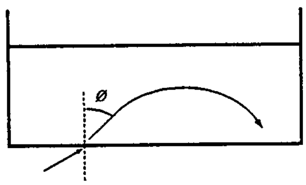
(c) Imagine dividing the water in the tank into many thin layers, each with an index of refraction slightly lower than that of the layer below. The path of a ray through the layers would be as follows.



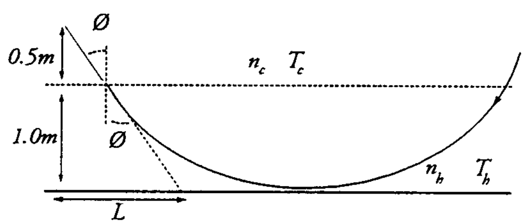
Applying Snell's Law at each interface, we have that

$$n \sin\theta = n_1 \sin r_1 , \\ = n_2 \sin r_2 , \\ = n_3 \sin r_3 ,$$

and so on. Hence, $n \sin\theta = 1.3 \sin(50.3°)$, where n is the index of refraction of the water at the bottom of the tank. Since the index increases at the rate $\alpha = 0.05/\text{m}$, $n = 1.35$. This gives $\theta = 47.8°$. Below is a sketch of a ray that enters the bottom of the tank at an angle greater than this.



(d) The index of refraction, of the air immediately above the roadway will be less than higher up where the air is cooler and more dense. This is a situation similar to our salt-water tank model. A light ray reaching the driver's eye may take the path shown. The apparent source of the ray is shown as a dotted line.



Consider the ideal gas law, $PV = NkT$ where P is the uniform atmospheric pressure of the air, V is the volume occupied by N molecules and T is the air's temperature (in Kelvin). The density of the air is proportional to N/V so that,

$$n(T) - 1 \propto \frac{1}{T} .$$

We are given the index of refraction at $T = 288$ K so that we can find the indices of refraction n_h at $T = 333$ K, and n_c at $T = 303$ K (60°C and 30°C respectively).

$$\frac{n_h - 1}{1.000276 - 1} = \frac{288}{333} ,$$
$$\frac{n_c - 1}{1.000276 - 1} = \frac{288}{303} ,$$

This gives $n_h = 1.000239$ and $n_c = 1.000262$. Using our result from part (c), we can calculate the angle θ for a ray which is totally internally reflected at the roadway.

$$1.000262 \sin\theta = 1.000239 \sin 90° .$$

This gives $\theta = 89.61°$. The geometry of the situation allows us to calculate the distance to the apparent source of the ray from the surface of the roadway.

$$L = (1.5 \text{ m}) \times \tan 89.61° .$$

We find that $L = 221.2$ m. Since the ray is actually originating from the sky, the roadway 221.2 m away appears blue and shimmering.

Question 3

Our natural walking speed is largely determined by the pendulum swing of our legs. My leg has a length of $l = 0.8$ m, and my stride d is very similar; let us assume $l = d$. The moment of inertia of a uniform tube is

$$I = \frac{1}{3} ml^2 .$$

For want of more accurate information let us assume my leg is such a tube. The period of oscillation is then,

$$T = 2\pi\sqrt{\frac{I}{mgh}} ,$$

where $h = l/2$. Hence,

$$T = 2\pi\sqrt{\frac{2l}{3g}} ,$$
$$= 1.47 \text{ s} .$$

Experimentally T is much closer to 1 s due to the uneven mass distribution of my leg (centre of mass is actually higher than the leg's centre) and the fact that we are using the small amplitude formula for a large amplitude oscillation.

A walking velocity v is given by my stride length divided by half the period.

$$v = \frac{d}{T/2} \approx \frac{2l}{T} ,$$
$$= 1.09 \text{ m/s (or about 4 km/h)} .$$

This is a rather leisurely pace; the experimental value of T gives about 6 km/h.

Refinements to the model would include corrections due to the compound pendulum effect of knee, the period of oscillation of the walker's straight arms (runners bend their arms to allow them to oscillate more quickly) and a more reasonable estimate of the centre of oscillation of the leg.

Canadian Association of Physicists
1996 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee. The questions in part B of the exam have a range of difficulty. Please be careful to gather as many of the easier marks as possible before venturing into more difficult territory. In some cases an answer to part (a) of a question is needed for part (b). Should you not be able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to completely finish this exam and parts of each question are very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions on the answer card/sheet provided to you and, most importantly, write your solutions to the three written problems on separate sheets as they will be marked by different people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
Radius of Earth	$R_E = 6.38 \times 10^6$ m
Mass of Earth	$M_E = 5.98 \times 10^{24}$ kg
Mass of Moon	$M_M = 7.35 \times 10^{22}$ kg
Radius of Moon's orbit	$R_{EM} = 3.84 \times 10^8$ m
Acceleration due to gravity	$g = 9.81$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.675 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ Js
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ Nm^2/C^2
Permeability of free space	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	$v_s = 340$ m/s
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K
Absolute zero	$T = 0$ K, $T = -273$°C

Part A: Multiple Choice

Question 1

A toy for firing a ball vertically consists of a vertical spring which is compressed by 0.10 m. A force of 10.0 N is needed to hold the spring at that compression. If a ball of mass 0.050 kg is placed on the compressed spring and the spring is released, the ball will reach a height (above its initial position) of:

(a) 1.0 m
(b) 1.2 m
(c) 1.4 m
(d) 1.6 m

Question 2

A rescue plane is flying horizontally with a speed of 30 m/s and at an altitude of 125 m above the sea when it drops a warning flare. Neglecting air resistance and assuming that the plane does not change its course, speed, or altitude, how far from the plane is the flare when it hits the water?

(a) 146 m
(b) 195 m
(c) 125 m
(d) 150 m

Question 3

A loudspeaker is placed over the open end of a pipe. By changing the frequency of the sound from the speaker, it is found that the pipe has resonances at 700 Hz and 900 Hz but not 800 Hz. This means that:

(a) The pipe is closed at one end and the fundamental is 100 Hz.
(b) The pipe is closed at one end and the fundamental is 200 Hz.
(c) The pipe is open at both ends and the fundamental is 100 Hz.
(d) The pipe is open at both ends and the fundamental is 200 Hz.

Question 4

In a television tube, electrons are accelerated from rest through a potential difference of 1600 V. What is the speed of the electrons after this acceleration?

(a) 16,000 km/s
(b) 20,000 km/s
(c) 24,000 km/s
(d) 28,000 km/s

Question 5

The electron-volt is a measure of:

(a) charge
(b) current
(c) electric field strength
(d) energy

Question 6

A current of 5 A passes along a wire of length 1.0 m. The wire is at right angles to a uniform magnetic field of strength 0.15 T. The force acting on the wire is:

(a) 0
(b) 0.75 N
(c) 33 N
(d) 0.03 N

Question 7

An 8 μF capacitor is charged to a potential of 120 V. How much work was required to do this?

(a) 2.7×10^{-12} J
(b) 1.2×10^{-1} J
(c) 9.6×10^{-4} J
(d) 5.8×10^{-2} J

Question 8

A flashlight is operated on four batteries placed in series. Each battery has an internal resistance r. If one of the batteries is accidentally placed the wrong way around, the total internal resistance of the four cells will now be:

(a) $r/4$
(b) $3r$
(c) $r/5$
(d) $4r$

Question 9

A ray of light is passing from air into glass. If the angle of incidence, with respect to the normal to the interface, is increased:

(a) Total internal reflection will occur when the angle of incidence equals the critical angle.
(b) Total internal reflection will occur when the angle of incidence is less than the critical angle.
(c) Total internal reflection will occur when the angle of incidence is greater than the critical angle.
(d) The refractive angle will increase but there will be no total internal reflection.

Question 10

A mass of 20 g is hung from the end of a light vertical spring and is set oscillating with an amplitude of 10 cm. Its total energy is found to be 4 J. If the mass is now replaced with a mass of 40 g and it is again set oscillating with an amplitude of 10 cm, its total energy is now:

(a) 2 J
(b) 4 J
(c) 5.6 J
(d) 8 J

Question 11

A 65 kg girl, riding in an elevator, weighs herself by standing on a scale. What is the reading of the scale if the elevator is accelerating downwards at 2 m/s^2?

(a) 65.0 kg
(b) 0 kg
(c) 51.7 kg
(d) 78.3 kg

Question 12

A highway curve of radius 30 m is banked so that a car travelling at 40 km/hr can travel around it without slipping even if there is no friction between the car's tires and the road surface. Without friction, a car travelling faster than this will slide up the curve, while a car travelling slower will slide down the curve. Find the angle of elevation of the banked highway curve.

(a) 67°.
(b) 23°.
(c) 45°.
(d) 90°.

Question 13

A ball is thrown upwards into the air rising to a height h. Accounting for the "real life" condition of air resistance, the time t_1 that the ball takes to rise to its maximum height and the time t_2 that it takes to fall back down to its initial position obey:

(a) $t_1 = t_2$
(b) $t_1 < t_2$
(c) $t_1 > t_2$
(d) $t_1 + t_2 = \sqrt{8h/g}$

Question 14

An airplane flies northwards from town A to town B and then back again. There is a steady wind blowing towards the north so that for the first stage of the trip, the airplane is flying in the same direction as the wind and for the return half of the journey, the airplane is flying directly into the wind. The total trip time T_w, as compared to the total trip time in the absence of any wind T_o, obeys:

(a) $T_w = T_o$
(b) $T_w > T_o$
(c) $T_w < T_o$
(d) $T_w = 2T_o$

Question 15

The space shuttle moves in a circular orbit about the earth at a constant speed. To change the orbit radius, the crew temporarily activates the main engine which accelerates the space shuttle in the direction of its motion. After the main engine is switched off again, the shuttle will be in an elliptical orbit with:

(a) a larger average radius and a lower average speed.
(b) a larger average radius and a higher average speed.
(c) a smaller average radius and a lower average speed.
(d) a smaller average radius and a higher average speed.

Question 16

Which of the following expressions has the correct units to represent the radius of a hydrogen atom in its ground state? (only one expression is correct)

(a) $\epsilon_o h^2/\pi m_e e^2$
(b) $h^2 m_e e^2/4\pi\epsilon_o$
(c) $m_e c^2/h^2 e^4$
(d) $e^4 m_e/8\epsilon_o^2 h^2$

Question 17

If the electric field is zero within some region of space, the electric potential within that region:

(a) must be zero.
(b) must be positive.
(c) must be negative.
(d) must be a constant value.

Question 18

In comparing the properties of visible light waves to microwaves, which of the following statements is FALSE?

(a) Visible light waves travel at the same speed in glass as do microwaves.
(b) Visible light waves have a higher frequency in glass than do microwaves.
(c) Visible light waves travel at the same speed in vacuum as do microwaves.
(d) Both visible light waves and microwaves can be refracted by glass.

Question 19

A boy sits at the top of a hemispherical mound of ice of radius r. He begins to slide (from rest) downwards without friction. At what height above the ground does he leave contact with the ice?

(a) The boy does not leave contact with the ice.
(b) $r/3$
(c) $r/2$
(d) $2r/3$

Question 20

A rocket for mining the asteroid belt is designed like a large scoop. It is approaching asteroids at a velocity of 10^4 m/s. The asteroids are much smaller than the rocket. If the rocket scoops asteroids at the rate of 100 kg/s, what thrust (force) must the rocket's engines provide in order for the rocket to maintain a constant velocity? Ignore any variation in the rocket's mass due to the burning of fuel.

(a) 10^3 N
(b) 10^6 N
(c) 10^9 N
(d) 10^{12} N

Question 21

The sensitivity of the human ear is greatest near 3000 Hz. This result is best explained by the fact that:

(a) the ear canal is a resonant cavity with a fundamental frequency near 3000 Hz.
(b) at a frequency of 3000 Hz the wavelength of sound is approximately the distance between the ears.
(c) 3000 Hz is the frequency at which the skull resonates.
(d) none of the above statements explains the result.

Question 22

A long piece of rubber is wider than it is thick. When it is stretched in length by some amount:

(a) its thickness decreases but its width increases.
(b) its thickness decreases but its width remains constant.
(c) its thickness increases but its width decreases.
(d) both its thickness and width decrease.

Question 23

A new medical procedure to cure near-sightedness involves changing the shape of the eye's lens using a precisely controlled laser beam. A near-sighted eye cannot focus clearly on distant objects. To improve the vision of the patient, we must:

(a) uniformly decrease the thickness of the lens.
(b) uniformly increase the thickness of the lens.
(c) make the middle of the lens thinner.
(d) make the outer rim of the lens thinner.

Question 24

A proton travels in a circular orbit of radius 1 cm in a magnetic field of strength 0.5 T. The kinetic energy of the proton is:

(a) 3.35×10^{-27} J
(b) 1.67×10^{-27} J
(c) 3.83×10^{-16} J
(d) 1.91×10^{-16} J

Question 25

A metal rod 30 cm long moves at 8 m/s in a plane perpendicular to a magnetic field of 0.05 T. The velocity of the rod is in a direction perpendicular to its length. The potential difference induced between the ends of the rod is:

(a) 48 V
(b) 6.40×10^{-20} V
(c) 120 mV
(d) A potential difference will not be induced in this case.

Part B

Question 1

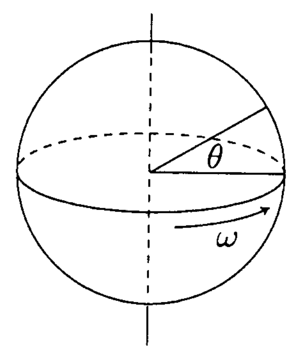
The study of the acceleration due to gravity of the earth can be very interesting. A variety of factors, such as the earth's spin and altitude, result in variations of "g". In this question, we shall investigate these variations.

(a) First let's assume that the earth is a non-spinning sphere. Find an expression for "g" at the earth's surface in terms of the earth's mass M_E, radius R_E and G, the gravitational constant.

(b) Modify your expression from part (a) to give the acceleration due to gravity at an altitude of h above the earth's surface. Simplify your expression in the limit that $h \ll R_E$ (the altitude is much less than the radius of the earth). Compare the weights of a 90 kg mountaineer at sea level and at the top of Everest at an altitude of about 8000 m. Hint: you may use the approximation that $1/(1+x)^2 \approx 1 - 2x$ if $|x| \ll 1$.

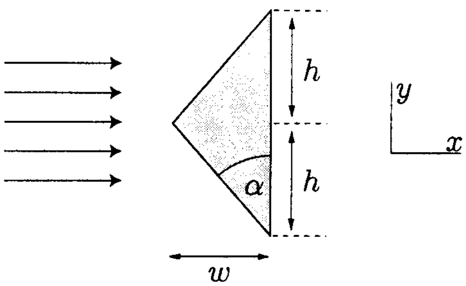
(c) The acceleration due to gravity depends on latitude because the earth is spinning. Find the percentage difference between g at the poles and at the equator due to the earth's spin. Hint: consider the weight of an object at the two locations.

(d) Find an expression for the magnitude of the freefall acceleration of an object near the surface of the earth as a function of the latitude θ. Assume that the earth is spherical and spinning.



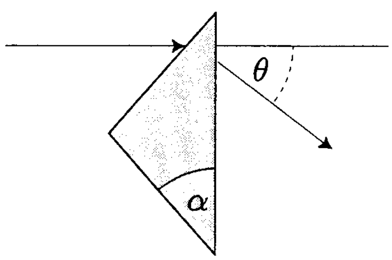
Question 2

It is possible to exert forces on small objects in a controlled fashion using a laser. Consider a laser beam incident upon a small glass prism as shown. The prism has an index of refraction $n = 1.6$ and the laser light is of frequency f. The prism is isosceles with a base of $2h$, a height w and a base angle of α.



(a) Sketch the possible paths for a ray of laser light entering the top half of the prism.

(b) Find the range of values for the angle α so that the light will exit from the long side of the prism as shown and find an expression for the deflection angle θ (you might want to do this first!). Only an approximate numerical value for α is required. Henceforth, you may assume that the angle α satisfies this condition.



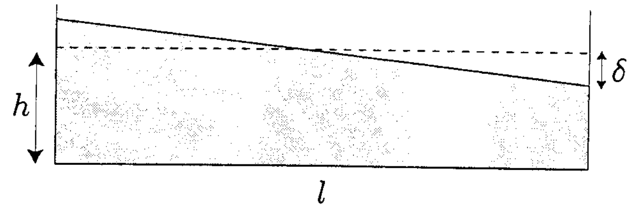
(c) The momentum p of a photon is related to its energy E by $E = pc$ where c is the speed of light. Find the vertical force exerted on the prism by a stream of r photons per second incident upon its top half. You may express your answer in terms of the deflection angle θ.

Question 3

In some large lakes, typically ones that are long and narrow, a strange phenomenon occurs. The water will oscillate up and down slowly first being high at one end and low at the other and then later reversing itself. It is much like the water in a bathtub sloshing back and forth and is not to be confused with regular waves on the water's surface. The phenomenon is called "seiching".

(a) How can seiching be initiated and how might it decay?

(a) Make a physical model for seiching and find an expression for a lake's period of oscillation. Possible factors include the lake's length l, width w, depth h, and amplitude of oscillation δ. Most simple models might begin by considering the lake to be a rectangular tank of water with the following geometry.



An exact solution is not expected for this question. Rather, a simpler system that you can imagine and that has characteristics in common with this problem can be used to model seiching and to estimate a period of oscillation.

You may compare your model with data from Lake Geneva in Switzerland and Lake Vättern in Sweden. Lake Geneva has a length of 60 km and an average depth of 150 m. Its oscillation period is 76 minutes with amplitudes of up to 2 m. Seiching on Lake Geneva can last for up to a week. Lake Vättern is 123 km long, has an average depth of 50 m and seiches with a period of about 3 hours.

Link to Solutions

Solutions: Part A

1 (a)	2 (c)	3 (a)	4 (c)	5 (d)
6 (b)	7 (d)	8 (d)	9 (d)	10 (b)
11 (c)	12 (b)	13 (b)	14 (b)	15 (a)
16 (a)	17 (d)	18 (a)	19 (d)	20 (b)
21 (a)	22 (d)	23 (c)	24 (d)	25 (c)

Part B

Question 1

(a) The acceleration due to gravity at the earth's surface is,

$$g = \frac{GM_E}{R_E^2} .$$

(b) At an altitude of h above the earth's surface, the acceleration due to gravity would be,

$$\begin{aligned} a_g &= \frac{GM_E}{(R_E + h)^2} , \\ &= \frac{GM_E}{R_E^2(1 + h/R_E)^2} , \\ &\approx g(1 - 2h/R_E) . \end{aligned}$$

The weight of a 90 kg person at sea level would be,

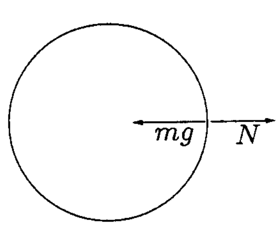
$$\begin{aligned} W &= mg \\ &= 90 \times 9.81 = 882.9 \text{ N} . \end{aligned}$$

At 8000 m altitude,

$$\begin{aligned} W &= 882.9 \times (1 - 0.00251) \\ &= 880.7 \text{ N} . \end{aligned}$$

This is only about 0.25% lighter.

(c) The force diagram for an object on the surface of the earth at the equator is shown.



This gives us,

$$mg - N = m\frac{v^2}{R_E} .$$

If the angular frequency of the earth's spin is $\omega = 2\pi/T$ where $T = 24$ hr is the period of rotation then,

$$\begin{aligned} N &= m(g - v^2/R_E) , \\ &= m(g - R_E\omega^2) , \\ &= m(g - \frac{4\pi^2 R_E}{T^2}) . \end{aligned}$$

Hence the effective acceleration due to gravity is,

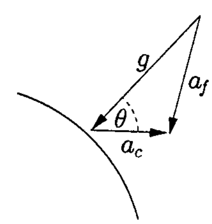
$$\begin{aligned} g_{eff} &= g - \frac{4\pi^2 R_E}{T^2} , \\ &= 9.78 \text{ m/s}^2 . \end{aligned}$$

The percentage difference between g at the poles and the effective value at the equator is 0.3%. Not a big deal.

(d) If the angular frequency of the earth's spin is $\omega = 2\pi/T$ where $T = 24$ hr is the period of rotation then,

$$\begin{aligned} a_c &= v^2/R = R\omega^2 , \\ &= (R_E \cos\theta)\omega^2 , \end{aligned}$$

The vector diagram shown allows us to find the freefall acceleration knowing the acceleration due to gravity and the centripetal acceleration due to the earth's spin.

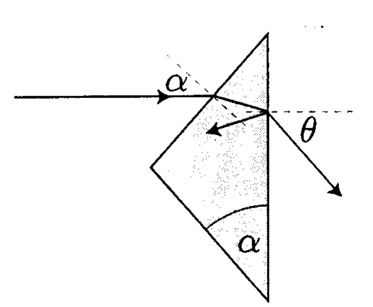


Hence,

$$a_f^2 = g^2 + a_c^2 - 2ga_c\cos\theta .$$

Question 2

(a) The ray may either exit the prism as shown or totally internally reflect from the interior glass/air interface.



(b) Applying Snell's law at the two interfaces we find,

$$\sin\alpha = n\sin\beta \ ,$$
$$n\sin(\alpha-\beta) = \sin\theta \ .$$

From this we have,

$$\sin\theta = n\sin(\alpha - \sin^{-1}(\frac{\sin\alpha}{n})) \ .$$

A solution for θ exists if,

$$\alpha - \sin^{-1}(\frac{\sin\alpha}{1.6}) \leq 0.675\text{rad} \ .$$

This is difficult to solve analytically but it is pretty easy to play with a calculator (be sure that it is in radian mode) to find that α must be less than about 1.33 rad (about 76°).

(c) The vertical change in momentum of a photon incident upon the to half of the prism is,

$$\Delta p_y = -\frac{E}{c}\sin\theta \ .$$

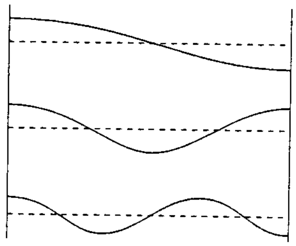
The prism's change in momentum is equal and opposite and the force exerted on it is given by $F = \Delta p/\Delta t$. Since there are r photons per second incident upon the prism,

$$F_y = \frac{rE}{c}\sin\theta \ .$$

The quantity rE is the power (in W) incident upon the prism.

Question 3

There are many possible models that may be considered including studies of the horizontal motion of the water's centre of mass and modelling the water as an oscillating (rectangular) rigid body. One could model the oscillation as water sloshing back and forth in a u-shaped garden hose. One method would be to consider the wavelength of a standing wave on a closed body of water.



The wavelength would be,

$$\lambda = \frac{2l}{n+1} \ ,$$

where n is a mode-number. In terms of the speed of a water wave v, the period of oscillation would be $T = \lambda/v$. The speed of a water wave can only depend on the depth of the water and the acceleration due to gravity. Dimensional analysis gives $v \approx \sqrt{gh}$. Hence,

$$T = \frac{2l}{(n+1)\sqrt{gh}} \ .$$

For $n = 0$, we get periods of 52 minutes for the Lake Geneva data and 3 hours for Lake Vättern.

Seiching could be initiated by winds, tidal action or localized cells of high or low air pressure. Seiching would decay by the drag of the various layers of water against each other (boundary layer drag).

Canadian Association of Physicists
1997 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee. The questions in part B of the exam have a range of difficulty. Please be careful to gather as many of the easier marks as possible before venturing into more difficult territory. In some cases an answer to part (a) of a question is needed for part (b). Should you not be able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to completely finish this exam and parts of each question are very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions on the answer card/sheet provided to you and, most importantly, write your solutions to the three written problems on separate sheets as they will be marked by different people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
Radius of Earth	$R_E = 6.38 \times 10^6$ m
Radius of Moon	$R_M = 1.70 \times 10^6$ m
Mass of Earth	$M_E = 5.98 \times 10^{24}$ kg
Mass of Moon	$M_M = 7.35 \times 10^{22}$ kg
Mass of Sun	$M_S = 1.99 \times 10^{30}$ kg
Radius of Moon's orbit	$R_{EM} = 3.84 \times 10^8$ m
Radius of Earth's orbit	$R_{ES} = 1.50 \times 10^{11}$ m
Acceleration due to gravity	$g = 9.81$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.675 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ Js
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ Nm^2/C^2
Permeability of free space	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	$v_s = 340$ m/s
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K
Absolute zero	$T = 0$ K, $T = -273$°C

Part A: Multiple Choice

Question 1

Two astronauts, each of mass 75 kg, are floating next to each other in space, outside the space shuttle. One of them pushes the other through a distance of 1 m (an arm's length) with a force of 300 N. What is the final relative velocity of the two?

(a) 2.0 m/s
(b) 2.83 m/s
(c) 4.0 m/s
(d) 16.0 m/s

Question 2

Twelve identical resistors (each of resistance R) are placed along the 12 edges of a cube. What is the effective resistance between opposite corners of the cube?

(a) $2R/3$
(b) $5R/6$
(c) R
(d) $12R$

Question 3

A heavy wooden beam of length L lies on the ground. You must carry it with a friend who happens to be twice as strong as you are (she can carry twice as much weight with the same effort). If you pick up the beam at one end, at what distance from your end should your friend carry it in order to be fair?

(a) $2L/3$
(b) $3L/5$
(c) $3L/4$
(d) L

Question 4

A car travelling at 80 km/h needs a braking distance of 50 m to come to a complete stop. What braking distance should the same car need if its initial speed were 160 km/h? You may assume that the same braking force can be applied in both cases.

(a) 70.7 m
(b) 100 m
(c) 141 m
(d) 200 m

Question 5

What would the length of the earth's day be if a pendulum suspended at some point along the equator did not swing (i.e., when it is pulled from its equilibrium point, it does not swing down again)?

(a) 24 hr
(b) 1.2 hr
(c) 1.4 hr
(d) 2.4 hr

Question 6

Two electrons are a certain distance apart from one another. What is the order of magnitude of the ratio of the electric force between them to the gravitational force between them?

(a) 10^8:1
(b) 10^{28}:1
(c) 10^{31}:1
(d) 10^{42}:1

Question 7

A balloon rising vertically with a speed of 5 m/s releases a sand bag at an instant when it is 20 m above the ground. How long after its release does it take for the sand bag to hit the ground? You may ignore air resistance.

(a) 2.01 s
(b) 2.59 s
(c) 2.81 s
(d) 3.01 s

Question 8

A cord is tied to a pail of water and the pail is swung in a vertical circle of radius 1 m. What must the minimum velocity of the pail be at its highest point so that no water spills out?

(a) 3.1 m/s
(b) 5.6 m/s
(c) 20.7 m/s
(d) 100.5 m/s

Question 9

An object of mass 5 kg hangs from a spring and oscillates with a period of 0.5 s. By how much will the equilibrium length of the spring be shortened when the object is removed.

(a) 0.75 cm
(b) 1.50 cm
(c) 3.13 cm
(d) 6.20 cm

Question 10

A certain organ pipe produces a sound of frequency 440 Hz in air. If the pipe is filled with helium at the same temperature and pressure, what frequency will be produced? Sound travels three times faster in helium than it does in air.

(a) 147 Hz
(b) 440 Hz
(c) 880 Hz
(d) 1320 Hz

Question 11

A ladder of length 10 m and mass 20 kg (and with a uniform mass distribution) leans against a slippery vertical wall. The ladder makes an angle of 30° with respect to the vertical. Friction between the ladder and the ground prevents it from sliding downwards. What is the magnitude of the force exerted on the ladder by the wall?

(a) 0 N
(b) 0.57 N
(c) 5.7 N
(d) 57 N

Question 12

A ball of mass 1.0 kg is whirled on the end of a string in a horizontal circle of radius 1.5 m and with a constant speed of 2.0 m/s. The work done on the ball by the tension in the string in one complete revolution is,

(a) 0 J
(b) 2.7 J
(c) 8.0 J
(d) 25.1 J

Question 13

A source of sound emitting a note of constant frequency moves in a horizontal circle at constant speed. A distant observer hears a note which fluctuates in pitch over a range of 20 Hz once every second. If the rate of rotation of the source is doubled, the observer would hear a fluctuation in pitch,

(a) greater than 20 Hz once every second.
(b) of 20 Hz twice every second.
(c) less than 20 Hz once every two seconds.
(d) greater than 20 Hz twice every second.

Question 14

Four identical charges are located at the corners of a square. The electric field is zero at,

(a) any of the four corners of the square.
(b) any of the mid-points of the sides of the square.
(c) the centre point of the square.
(d) at no point within or on the square.

Question 15

The unit of electric charge may be expressed as,

(a) ampere·newton·meter / watt
(b) ampere·volt
(c) ampere / second
(d) ampere·ohm

Question 16

A rectangular coil of wire rotates about an axis which is perpendicular to a uniform magnetic field at a steady rate. Consider the instant when the plane of the coil is parallel to the magnetic field lines. At that instant the induced electromotive force is,

(a) minimum.
(b) maximum.
(c) zero.
(d) constant, the same at all times.

Question 17

An airplane flies from a town A to a town B when there is no wind and takes a time T_o for a return trip. When there is a wind blowing in a direction from town A to town B, the plane's time for a similar return trip, T_w, would satisfy,

(a) $T_o > T_w$
(b) $T_o < T_w$
(c) $T_o = T_w$
(d) the result depends on the distance between the towns.

Question 18

You have an ample supply of 3 μf capacitors. What is the minimum number of these capacitors required to make a circuit with an equivalent capacitance of 2.25 μf.

(a) 3
(b) 4
(c) 5
(d) 6

Question 19

Three 100 W light bulbs are connected in series to a 120 volt power source. If two of the light bulbs are replaced by 60 W bulbs, the brightness of the remaining 100 W bulb is,

(a) brighter than it was before.
(b) dimmer than it was before.
(c) the same brightness as it was before.
(d) will not illuminate.

Question 20

The earth orbits the sun in an elliptical path. Of the following statements, which are true?

I The earth's orbital speed is constant.
II The angular momentum of the earth with respect to the sun is a constant.
III The force acting on the earth due to the sun is a constant.
IV The earth's orbital speed is faster in the spring and fall than it is in the summer and winter.

(a) I only.
(b) II only.
(c) IV only.
(d) II and IV only.

Question 21

A ball is thrown upwards into the air. Taking into account air resistance, the forces acting on the ball while in upwards flight are,

(a) a decreasing upwards force and a constant downwards force.
(b) a decreasing upwards force and a decreasing downwards force.
(c) a decreasing downwards force.
(d) an increasing downwards force.

Question 22

A bullet travelling at 500 m/s and with a mass of 100 g collides and remains embedded in a small lead weight of mass 5 kg. The weight is suspended from a fixed support by a cord of length 5 m and may swing freely. If the weight was initially at rest, what is the period of oscillation of the system immediately after the collision and how does it change as the amplitude of the oscillation decays due to air resistance?

(a) 4.5 s and constant in time.
(b) 4.5 s and increasing in time.
(c) 0.7 s and constant in time.
(d) none of the above.

Question 23

A peculiar force acting upon an object has the equation $F = -cx^3$ where x is the displacement of the object from some origin and $c = 1$ N/m^3. What is the magnitude of the work done by this force on the object as the object moves from $x = -2$ m to $x = +2$ m?

(a) 16 J
(b) 8 J
(c) 4 J
(d) 0 J

Question 24

With what minimum speed, with respect to the earth, must an object be launched from the earth so as to escape the gravitational pull of the solar system? You may Ignore the effect of the earth, moon and other planets.

(a) 5.6×10^3 m/s
(b) 11.2×10^3 m/s
(c) 12.2×10^3 m/s
(d) 42.1×10^3 m/s

Question 25

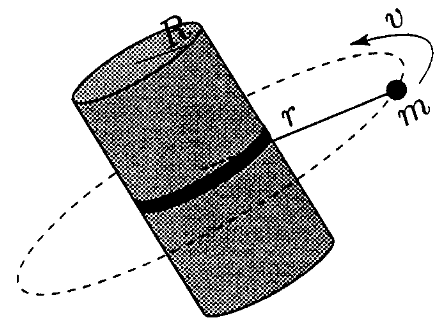
Sound waves from deep within a room and of frequency 440 Hz are incident upon a tall, narrow, and open window. An observer outside is far from the window (at least much farther from the window than the window is wide). The line from the observer to the window makes an angle of 30° with respect to the normal to the plane of the window. Shutters begin to close so that the width of the window is decreasing at 6 m/s. The observer hears,

(a) a 440 Hz note beating at 4.29 Hz.
(b) a 440 Hz note beating at 3.88 Hz.
(c) a 440 Hz note steadily decreasing in intensity.
(d) initially a 440 Hz note with a steadily increasing frequency.

Part B

Question 1

A massive satellite in orbit (*i.e.* in a weightless environment) has a rotating cuff that allows a small mass m on the end of a cord to circle about the satellite at a radius r with a speed v.



The initial speed of the mass is v_o and the initial radius of its circular path is r_o. A mechanism within the cuff allows the cord to be drawn in so that the radius of revolution for the mass decreases. The centre of revolution for the mass remains constant and you may assume that the satellite is much, much more massive than m.

(a) Find the initial tension T_o in the string.

(b) Suppose that the cord begins to be drawn in at a constant rate (r decreases uniformly from its initial value r_o). Express the speed v of the mass as a function of r, r_o and of its initial speed v_o.

(c) If the cord snaps beyond a maximum tension $T_{\max}$ and the radius of the satellite is R, what constraint must be satisfied in order for the mass to be brought within the satellite?

(d) What amount of work is necessary to bring the mass from radius r_o to radius R?

Question 2

A solid metal sphere of radius 2 cm carries a net negative charge $-Q$. The air around the sphere will hence be subjected to an electric field. If an electric field in excess of 3.0×10^6 V/m is applied across an "air molecule", the molecule will ionize. This will appear as sparking and is called "break down". You may assume spherical symmetry in this problem (although in actuality, sparking will break this symmetry).

(a) Sketch the electric field lines within and around the charged sphere and show on your diagram how the excess charge on the sphere is distributed.

(b) What is the minimum charge that the sphere must have to cause the break down of air near its surface?

(c) It requires about 1.6×10^{-18} J of energy to ionize an air molecule. Consider a free electron near the surface of the sphere carrying this minimum charge as in part (b). What radial distance must the electron travel before attaining a kinetic energy equal to the ionization energy for an air molecule? An electron typically travels only 1.0×10^{-6} m before colliding with an air molecule. Is it possible for our accelerated electron to cause ionization of air molecules away from the surface of the sphere?

(d) What is the potential (in units of V) on the surface of the sphere when break down just occurs?

Question 3

A spherical balloon of radius $R = 5$ cm has within it a small, solid lead sphere of mass m and radius r. The mass of the balloon is negligible as compared to the mass of the lead sphere. The system is buoyantly neutral, floating just below the surface in a pool of water.

The balloon is given a light downwards push. Describe the motion of the system qualitatively, and as quantitatively as possible.

Hint: A spherical object of radius r moving without turbulence and with a velocity v through a fluid of viscosity η experiences a retarding force given by,

$$F_v = 6\pi r \eta v \ .$$

Water has a viscosity of $\eta = 1.0 \times 10^{-3}$ Pa S and a density of $\rho_w = 1000$ kg/m^3. Lead has a density of $\rho_l = 11.3 \times 10^3$ kg/m^3.

Link to Solutions

Solutions: Part A

1 (c)	2 (b)	3 (c)	4 (d)	5 (c)
6 (d)	7 (b)	8 (a)	9 (d)	10 (d)
11 (d)	12 (a)	13 (d)	14 (c)	15 (a)
16 (b)	17 (b)	18 (b)	19 (b)	20 (b)
21 (c)	22 (d)	23 (d)	24 (c)	25 (b)

Part B

Question 1

(a) The tension in the cord for any radius of revolution and speed is,

$$T = \frac{mv^2}{r} .$$

Hence the initial tension is just $T_o = mv_o^2/r_o$.

(b) Conservation of angular momentum gives,

$$L = mv_o r_o = mvr ,$$

so that $v = v_o r_o/r$.

(c) The angular momentum $L = mvr$ is constant and the tension is

$$T = \frac{mv^2}{r} = \frac{L^2}{mr^3} .$$

The condition for the mass to reach $r = R$ is then

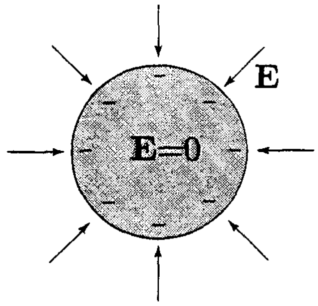
$$T_{\text{max}} > \frac{L^2}{mR^3} = \frac{mv_o^2 r_o^2}{R^3} .$$

(d) The work W is given by the integral of the tension over radius:

$$\begin{aligned} W &= \int_{r_o}^{R} \frac{L^2}{mr^3} dr \\ &= \frac{L^2}{2m}\left(\frac{1}{R^2} - \frac{1}{r_o^2}\right) \end{aligned}$$

Question 2

(a) There is no electric field within a solid conductor and the excess charge is uniformly distributed on the sphere's surface. The electric field outside of the sphere is the same as one that would be formed by a point charge $-Q$ located at the centre of the sphere.



(b) The electric field at the surface of the sphere is just,

$$E = \frac{1}{4\pi\epsilon_o}\frac{-Q}{r^2} ,$$

where r is the radius of the sphere. For $E = 3.0 \times 10^6$ V/m, we find $Q = 1.33 \times 10^{-7}$ C.

(c) The potential energy of an electron at the surface of the sphere is,

$$U_r = \frac{1}{4\pi\epsilon_o}\frac{eQ}{r} ,$$

where e is the charge on an electron. The potential at some radius R is then,

$$U_R = \frac{1}{4\pi\epsilon_o}\frac{eQ}{R} .$$

The difference must satisfy $U_r - U_R = 1.6 \times 10^{-18}$ J. Hence we find,

$$8.35 \times 10^{-3} = \frac{1}{2.0 \times 10^{-2}} - \frac{1}{R} .$$

Solving for R gives,

$$R = \frac{2 \text{ cm}}{1 - 1.67 \times 10^{-4}} .$$

The electron must travel $d = R - 2$ cm. We find $d = 3.34 \times 10^{-6}$ m. It is unlikely that an accelerated electron will cause breakdown away from the surface as it will likely collide with air molecules before attaining enough energy.

(d) The potential at the surface of the sphere is,

$$V = \frac{1}{4\pi\epsilon_o}\frac{-Q}{r} ,$$

so that $V = -60,000$ volts. This is pretty high.

Question 3

In equilibrium, the buoyant force on the system must equal its weight. Hence,

$$\rho_w \cdot \frac{4}{3}\pi R^3 g = mg ,$$

From this we find the mass of the lead sphere to be,

$$m = \frac{4}{3}\pi\rho_w R^3 .$$

The mass is 0.52 kg and since we know the density of lead, its radius can be found from,

$$m = \frac{4}{3}\pi\rho_l r^3 .$$

We find $r = 2$ cm.

If the system is given a light downwards push, it will descend slightly. As it descends, its radius will continually decrease due to water pressure reducing both the buoyant force and the retarding force. The system will accelerate downwards and may reach a high speed. At some depth, the balloon will have collapsed around the lead sphere and the radius of the system will no longer decrease. As it continues to accelerate downwards, the retarding force will increase again due to the increasing velocity. A terminal velocity will be reached.

The retarding force on the collapsed system is,

$$F_v = 6\pi\eta r v ,$$

where v is the speed of the system. The equation of motion for the system is,

$$mg - 6\pi\eta r v - \frac{4}{3}\pi\rho_w g r^3 = ma .$$

The sphere will eventually reach a terminal velocity (when $a = 0$) which satisfies,

$$6\pi\eta r v_t + \frac{4}{3}\pi\rho_w g r^3 = mg .$$

Substitution of the mass of the lead sphere gives,

$$6\pi\eta r v_t = \frac{4}{3}\pi\rho_w g(R^3 - r^3) .$$

The terminal velocity is then,

$$v_t = \frac{2\rho_w g}{9\eta r}(R^3 - r^3) .$$

The terminal velocity is $v_t = 13,000$ m/s. This is really fast and likely invalidates out turbulent free assumption.

Canadian Association of Physicists
1998 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee. The questions in part B of the exam have a range of difficulty. Please be careful to gather as many of the easier marks as possible before venturing into more difficult territory. In some cases an answer to part (a) of a question is needed for part (b). Should you not be able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to completely finish this exam and parts of each question are very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions on the answer card/sheet provided to you and, most importantly, write your solutions to the three written problems on separate sheets as they will be marked by different people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
Radius of Earth	$R_E = 6.38 \times 10^6$ m
Radius of Moon	$R_M = 1.70 \times 10^6$ m
Mass of Earth	$M_E = 5.98 \times 10^{24}$ kg
Mass of Moon	$M_M = 7.35 \times 10^{22}$ kg
Mass of Sun	$M_S = 1.99 \times 10^{30}$ kg
Radius of Moon's orbit	$R_{EM} = 3.84 \times 10^8$ m
Radius of Earth's orbit	$R_{ES} = 1.50 \times 10^{11}$ m
Acceleration due to gravity	$g = 9.81$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.675 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ Js
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ Nm^2/C^2
Permeability of free space	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	$v_s = 340$ m/s
Density of air	$\rho = 1.2$ kg/m^3
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K
Absolute zero	$T = 0$ K, $T = -273°$C
Energy Conversion	1 cal=4.184 J
Avogadro's number	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Combustion Energy of Sugar	15 MJ/kg

Useful Relation:

For $x \ll 1$, $(1+x)^{-n} \approx (1 - nx)$.

Part A: Multiple Choice

Question 1

What mechanical power P is required to move an object of mass m against an opposing force F at a constant velocity v?

(a) $P = Fv$
(b) $P = mv^2/2$
(c) $P = mv/F$
(d) $P = F^2/mv$

Question 2

A rocket generates a thrust force by ejecting hot gases from an engine. If it takes 1 ms to combust 1 kg of fuel, ejecting it at a speed of 1000 m/s, what thrust is generated?

(a) 1000 N
(b) 10,000 N
(c) 100,000 N
(d) 1,000,000 N

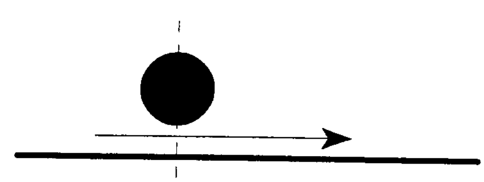
Question 3

A rubber band (two parallel strands of elastic material) has a spring constant of 10 N/m. If the band is cut in one place such that it now forms a single long strand of elastic material, what is its new spring constant?

(a) 20 N/m
(b) 40 N/m
(c) 5 N/m
(d) 2.5 N/m

Question 4

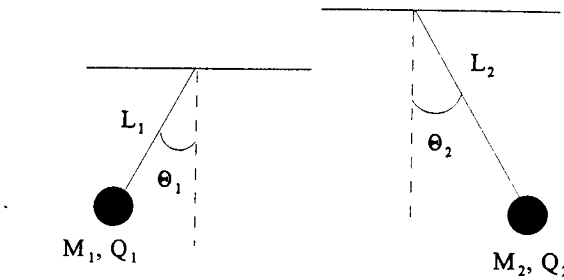
A ball rests upon a flat piece of paper on a table top. The paper is pulled quickly and horizontally to the right as shown. Relative to the its initial position with respect to the table, the ball initially;



(a) moves to the right.
(b) moves to the left.
(c) remains stationary.
(d) its motion depends on the speed at which the paper was removed.

Question 5

Two small spheres with masses m_1 and m_2 hang on weightless, insulating threads with lenghts l_1 and l_2. The two spheres carry a charge of q_1 and q_2 respectively. The spheres hang such that they are level with one another and the threads are inclined to the vertical at angles θ_1 and θ_2. Which of the following conditions is required if $\theta_1 = \theta_2$.



(a) $m_1 = m_2$

(b) $|q_1| = |q_2|$

(c) $l_1 = l_2$

(d) none of the above

Question 6

An astronaut aboard the space shuttle, which is in orbit about the earth, is said to be weightless because;

(a) there is no force of gravity on her.

(b) a radially outwards force acts on her which cancels the force of gravity.

(c) she is falling in the earth's gravitational field.

(d) she is at such a position between the earth and the moon that their forces of gravity cancel each other.

Question 7

Beats are heard when the A strings of two violins are played. The beat frequency decreases as the tension in the A string of violin 1 is slowly increased. Which of the following statements is correct?

(a) The fundamental frequency of the A string in violin 1 is less than that for violin 2.

(b) The fundamental frequency of the A string in violin 1 is greater than that for violin 2.

(c) The fundamental frequency of the A string in violin 1 may be greater or less than that for violin 2 depending on the linear mass densities of the two strings.

(d) The fundamental frequency of the A string in violin 1 may be greater or less than that for violin 2 depending on the temperature.

Question 8

A proton (mass m_p, charge $+e$) and a deuteron (mass $2m_p$, charge $+e$) are initially at rest and are accelerated through the same potential difference ΔV. Which of the following is true concerning the final properties of the two particles?

(a) They have the same speed.

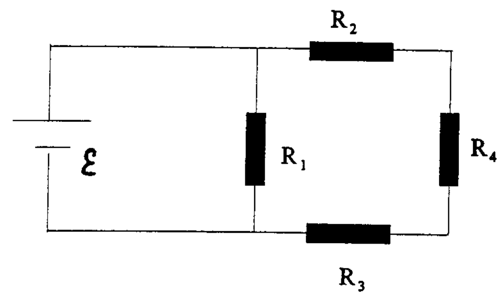
(b) They have the same momentum.

(c) They have the same kinetic energy.

(d) They have been subjected to the same impulse.

Question 9

An ideal battery is connected to a circuit of four resistors as shown. If the resistance of R_4 is increased, then:



(a) The current through R_1 decreases.

(b) The potential difference across R_1 increases.

(c) The potential difference across R_4 increases.

(d) The potential difference across R_2 decreases.

Question 10

A ray of light is incident upon an air/water interface (it passes from air into water) at an angle of 45°. Which of

(1) wavelength

(2) frequency

(3) speed of propagation

(4) direction of propagation

change as the light enters the water?

(a) 1,3 only

(b) 3,4 only

(c) 1,2,4 only

(d) 1,3,4 only

Question 11

Three freight cars, each of mass m, are being pulled by an engine exerting a force F on the lead car. What is the force exerted on the trailing car by the middle one?

(a) F

(b) $F/2$

(c) $F/3$

(d) $2F/3$

Question 12

The lowest frequency to resonate in a pipe, closed at one end and open at the other, is 300 Hz. Which of the following frequencies will not resonate in this particular pipe?

(a) 900 Hz
(b) 2,100 Hz
(c) 3,300 Hz
(d) 3,600 Hz

Question 13

A long metal bar, 30.0 cm in length , is aligned along a north-south line and moves eastwards at a speed of 10.0 m/s. A uniform magnetic field of 4.00 T points downwards. If the south end of the bar has a potential of 0 V, the induced potential at the north end of the bar is,

(a) +12 V
(b) -12 V
(c) 0 V
(d) cannot be determined since there is no closed circuit.

Question 14

A proton is moving in a uniform electric field, E, of unknown magnitude and direction. There is a uniform magnetic field of 0.01 T pointing in the y-direction. The proton moves in the x- direction at a constant velocity of 10 km/s. All directions are with respect to the usual right handed x-y-z coordinate system. Which of the following is correct?

(a) $E = 100$ V/m, in the z-direction.
(b) $E = 100$ V/m, in the negative z-direction.
(c) $E = 100$ V/m, in the x-direction.
(d) $E = 100$ V/m, in the negative y-direction.

Question 15

A aluminum block has a cavity within it which is completely closed. The block is placed in a region permeated by a uniform electric field which is directed upwards. Which of the following is a correct statement describing conditions in the interior of the block's cavity?

(a) The electric field in the cavity is directed upwards.
(b) The electric field in the cavity is directed downwards.
(c) There is no electric field in the cavity.
(d) The electric field in the cavity is of varying magnitude and is zero at the exact center.

Question 16

Two copper spheres, A and B, are identical in all respects but A carries a charge of -3 μC whereas B is charged to $+1$ μC. The two are brought together until they touch and then separated so that they are nearby but otherwise insulated. Which of the following is true concerning the electrostatic force F between the two spheres?

(a) $F = 0$ as one of the spheres is uncharged.
(b) $F = 0$ as both of the spheres are uncharged.
(c) F is attractive.
(d) F is repulsive.

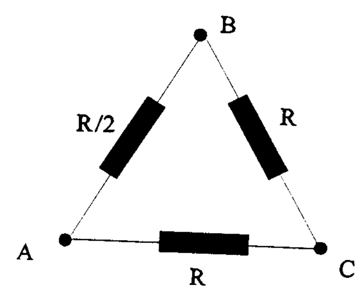
Question 17

A real battery has an emf ϵ and an internal resistance r. A variable resistor R is connected across the terminals of the battery. A current I is drawn from the battery and the potential difference across the terminals of the battery is V. If R is slowly decreased to zero, which of the following best describes I, and V?

(a) I decreases to zero; V approaches ϵ.
(b) I approaches an infinite value; V decreases to zero.
(c) I approaches ϵ/r; V approaches ϵ.
(d) I approaches ϵ/r; V decreases to zero.

Question 18

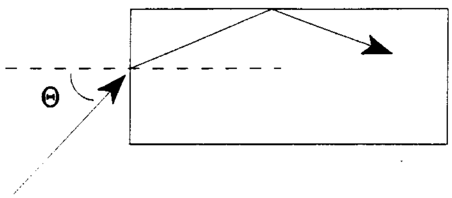
Three resistors are connected together as shown. A current I flows into A and out of B. What is the power dissipated by the resistor $R/2$?



(a) $8I^2R/25$
(b) I^2R
(c) $16I^2R/5$
(d) $25I^2R/32$

Question 19

Light enters one end of an optical fibre as shown. The uniform index of refraction of the cylindrical fibre is $n = 1.4$. What is the maximum angle θ that the incident light may have so that the light ray is totally internally reflected within the fibre. Take the index of refraction of air to be unity; all results shown to two significant digits.



(a) 78°
(b) 46°
(c) 44°
(d) 12°

Question 20

Two balls of the same mass are dropped from the same height onto the floor. The first ball bounces upwards from the floor elastically. The second ball sticks to the floor. The first applies an impulse to the floor of I_1 and the second applies an impulse I_2. The two impulses obey,

(a) $I_2 = 2I_1$.
(b) $I_2 = I_1/2$.
(c) $I_2 = 4I_1$.
(d) $I_2 = I_1/4$.

Question 21

A glass of water has an ice cube floating in it. The level of the water is marked on the glass with a grease-pencil. Once the ice cube has completely melted, and assuming that there has been no evaporation, the water level, relative to the initial marking, has;

(a) gone up.
(b) gone down.
(c) remained the same.
(d) cannot say based on the above information.

Question 22

A bit of mud stuck to a bicycle's front wheel detaches and is flung horizontally forward when it is at the top of the wheel's rotation (directly above the wheel's hub). The bicycle is moving forward at a speed v and the bicycle is rolling along without slipping. At the instant that the bit of mud hits the ground, how far in front of the bicycle's front wheel hub is it? (Ignore air resistance)

(a) $\sqrt{2rv^2/g}$
(b) $\sqrt{4rv^2/g}$
(c) $\sqrt{8rv^2/g}$
(d) $\sqrt{16rv^2/g}$

Question 23

An exploratory rocket of mass m is in orbit about the sun at a radius of $R_{ES}/10$ (one tenth of the radius of the earth's orbit about the sun). To exit this orbit, it fires its engine over a short period of time. This quickly doubles the velocity of the rocket while halving its mass (due to fuel consumption). Immediately after the burn, what is the kinetic energy of the rocket?

(a) $10GM_Sm/R_{ES}$
(b) $5GM_Sm/R_{ES}$
(c) $20GM_Sm/R_{ES}$
(d) $GM_Sm/2R_{ES}$

Question 24

A light bulb shines light along the x-axis and through two parallel ideal polarizing filters, one with a fixed polarizing axis, and one with an axis that rotates about the x-axis and in the yz-plane. Looking towards the light bulb through the combined filtering system, you see;

(a) A bulb almost disappearing twice per revolution, and reaching a maximum intensity twice per revolution. The maximum brightness is not as bright as looking at an unfiltered bulb.
(b) As in (a) but once per revolution.
(c) As in (a) but with a maximum brightness as bright as an unfiltered bulb.
(d) As in (a) but with a bulb that disappears completely.

Question 25

Two identical metallic spheres are both charged to $+Q$. The spheres are fixed in place, with their centers $2l$ apart from one another. They are joined by a light, in-flexible rod. Sliding back and forth on the rod (ignore friction) is a small disk of mass m charged to q. The disk is in equilibrium when it half way between the spheres. If the disk is displaced by a small amount from its equilibrium point, it will oscillate with a period of:

(a) $2\pi\sqrt{\pi\epsilon_o ml^3/Qq}$
(b) $2\pi\sqrt{Qq/\pi\epsilon_o ml^3}$
(c) $2\pi\sqrt{4\pi\epsilon_o ml^3/Qq}$
(d) $2\pi\sqrt{4\pi\epsilon_o ml^2/Qq}$

Part B

Question 1

A ball of mass $m = 1.0$ kg is dropped from a height $h_1 = 1.0$ m above a hard table top. It rebounds to a height $h_1' < h_1$. The coefficient of restitution between the ball and the table is $\epsilon = \sqrt{h_1'/h_1} = 0.7$. The radius of the ball is $r = 0.5$ cm.

(a) What is the ratio of the ball's kinetic energy just after it has bounced to that just before it bounces?

(b) If the duration of the collision between the ball and the table top is $\Delta t = 0.01$ s, what is the average force exerted on the ball by the table?

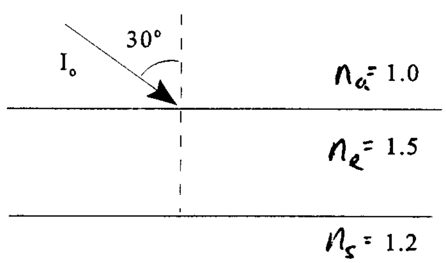
(c) From what height, h_2, above the table top and directly above the first ball, must a second ball be dropped so that it collides with the first ball immediately after the first ball has finished its bounce from the table top. The second ball is identical to the first but with a mass of 100 g.

(d) If the coefficient of restitution between the two balls is 1.0, find the rebound height, h_2', of the second ball above the table top.

Question 2

Laminar optical devices are important in the semi-conducting electronics industry. It is important to know how light interacts with such an interface and so let's consider the interaction of light with a simple laminar structure.

Suppose that a beam of light of intensity I_o shines upon a laminar interface with a small angle of incidence $\theta_i = 30°$. The interface is a laminar *sandwich* made of two layers. There is a thick substrate layer , with an index of refraction $n_s = 1.2$, and a thin mid-layer, with an index of refraction $n_l = 1.5$. The index of refraction of air, n_a may be taken to be unity. The thickness of the mid-layer is large enough so that interference effects are not an issue.



(a) Find the angle θ_l that light refracted through the air/mid-layer interface makes with the normal. Hence, find the angle of incidence for light impinging upon the substrate. Finally, find the refracted angle, θ_s, for light passing through the mid-layer/substrate interface.

Note: For unpolarized light incident upon an n_1/n_2 interface with an incident angle θ_i, there will be a reflected ray and a refracted ray (at an angle of θ_r to the normal). The fraction of the incident light reflected is given by,

$$R = \frac{1}{2}\left(\frac{\tan^2(\theta_r - \theta_i)}{\tan^2(\theta_r + \theta_i)} + \frac{\sin^2(\theta_r - \theta_i)}{\sin^2(\theta_r + \theta_i)}\right) .$$

The fraction of light transmitted is $T = 1 - R$.

(b) Using your angles from part (a), calculate values for R and T for reflection and transmission at the air/mid-layer, mid-layer/substrate, and mid-layer/air interfaces.

(c) Find an expression for the intensity of the light which is reflected from the laminar interface. Evaluate your expression to as many significant figures as possible.

Question 3

A hummingbird has a mass of $m = 10$ g. It hovers in much the same manner as a helicopter except that instead of rotating, its wings sweep back and forth, developing lift on both parts of the stroke. The vertical, cross sectional area that the wings sweep through is $A = 0.01$ m^2.

(a) Estimate the mechanical power output P the hummingbird needs to generate for hovering.

(b) Estimate the mass of sugar the hummingbird needs to ingest (as nectar) in order to hover for one hour.

Note: This question requires you to make assumptions and to estimate physical quantities. Marks are awarded for the quality of these assumptions and estimations so please explain them clearly. You should, however, attempt the question using your assumptions whatever the quality of your estimations.

Link to Solutions

Solutions: Part A

1 (a)	2 (d)	3 (d)	4 (a)	5 (a)
6 (c)	7 (a)	8 (c)	9 (c)	10 (d)
11 (c)	12 (d)	13 (a)	14 (b)	15 (c)
16 (d)	17 (d)	18 (a)	19 (a)	20 (b)
21 (c)	22 (b)	23 (a)	24 (d)	25 (a)

Part B

Question 1

(a) We can take all heights to be measured between the table top and the bottom of the ball (although it makes no difference if the top or middle of the ball is chosen as a reference point). Since the kinetic energies before and after the bounce satisfy,

$$mgh_1 = E_{Ki} \quad mgh_1' = E_{Kf} \ ,$$

then $E_{Kf}/E_{Ki} = h_1'/h_1 = 0.49$.

(b) Using $F_{avg} = \Delta p/\Delta t$ we write,

$$\begin{aligned} F_{avg} &= \frac{m(v_i + v_f)}{\Delta t} \ , \\ &= \frac{m(\sqrt{2gh_1} + \sqrt{2gh_1'})}{\Delta t} \ , \\ &= \frac{m\sqrt{2gh_1}}{\Delta t}(1 + 0.7) \ , \\ &= 474 \text{ N} \ . \end{aligned}$$

(c) If $h_1 = gt^2/2$, then it takes $t = 0.452$ s for the ball to fall to the table. The collison time is 0.01 s so that the total time that second ball may fall for is $t = 0.462$ s. The ball must fall from a height of $h_2 = gt^2/2 = 1.05$ m above the first ball. Adding the 1.0 cm height of the first ball gives 1.06 m.

(d) The velocity of the balls just before colliding is,

$$\begin{aligned} v_1 &= 0.7 \times \sqrt{2gh_1} = 3.10 \text{ m/s} \ , \\ v_2 &= \sqrt{2gh_2} = 4.54 \text{ m/s} \ . \end{aligned}$$

Since the mass of the second ball is one tenth the mass of the first, m, our momentum and energy conservation equations are,

$$\begin{aligned} 10{v_1'}^2 + {v_2'}^2 &= 10v_1^2 + v_2^2 = 117 \text{ J} \ , \\ 10v_1' + v_2' &= 10v_1 - v_2 = 26.5 \text{ m/s} \ , \end{aligned}$$

Solving for $v_2' = 9.36$ m/s gives, $h_2' = {v_2'}^2/2g = 4.47$ m. Adding the height of a ball gives $h_2' = 4.48$ m above the table top.

Question 2

(a) The reflected light is directed at 30° to the normal. Snell's law gives us the refraction angle for the $n_a - n_l$ interface,

$$n_a \sin(\theta_i) = n_l \sin(\theta_l) \ .$$

Hence, $\theta_l = 19.5°$. The angle of light incident upon, and reflected from, the substrate layer is also 19.5°. Applying Snell's law again gives the refraction angle for the light passing into the substrate.

$$n_l \sin(\theta_l) = n_s \sin(\theta_s) \ .$$

Hence $\theta_s = 24.6°$. Light reflecting from the substrate is incident upon the air at an angle of 19.5° and the refraction angle is 30°.

(b) Evaluating the expressions given for the reflection and transmission coefficients gives,

$$\begin{aligned} &R_{al} = 0.04 \quad T_{a1} = 0.96 \\ &R_{ls} = 0.0124 \\ &R_{la} = 0.04 \quad T_{la} = 0.96 \end{aligned}$$

(c) The intensity of the ray which reflects from the first interface is just $R_{al}I_o$ or $0.04I_o$. The refracted or transmitted ray may reflect from the second interface and in turn, be transmitted through the first. Its intensity is $T_{al}R_{ls}T_{la}I_o$ or $0.23I_o$. The total for these first two possible rays is $0.27I_o$.

Now the problem is to sum the intensities of all possible reflected rays. Simplifying our notation to $R_o = 0.04$, $R = 0.25$ and $T = 0.96$ we write,

$$\begin{aligned} I &= I_o(R_o + TRT + TRR_oRT + TRR_oRR_oRT + \dots) \ , \\ &= I_oR_o + I_oTRT(1 + R_oR + (R_oR)^2 + \dots) \ ; \end{aligned}$$

Here one can use a calculator to estimate a limit for this series or one can recognise the geometric series.

$$\frac{I}{I_o} = R_o + \frac{TRT}{1 - R_oR} \ .$$

Hence $I/I_o = 0.2727272727\dots$.

Question 3

(a) As the bird's wings sweep back and forth, they will deflect a mass of air downwards with velocity $\Delta v = v$. The mass of air involved is approximately $m_a = \rho A \Delta x$ where Δx is the length of the wings sweep. The acceleration of the air downwards is approximately $a = \Delta v/\Delta t$. The lift is then,

$$\begin{aligned} L &= m_a a \ , \\ &= \rho v^2 A = mg \ . \end{aligned}$$

The power is just $P = Lv$ which gives,

$$P = \rho v^3 A \ ,$$
$$= \sqrt{(mg)^3/\rho A} \ ,$$
$$1/3 \text{ W} \ .$$

(b) Hydrocarbons burn yielding about 40 MJ/kg. One can get this from reading food labeling (300 ml of Coke has about 100 k cal or about 4 MJ), binding energy information (a hydrogen atom has 13.6 eV which gives about 1 MJ/g), human caloric intake *vs.* output considerations or any clever estimate. For a typical animal efficiency of 25%, about 100 mg (or 1% of the bird's mass) is needed for an hours hover. Huge variations from this answer will be accepted. In actual fact, the bird needs about 4 times this amount (due to drag effects and since nectar is CHO, not CH). Any student commenting this will be richly rewarded.

Canadian Association of Physicists
1999 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee. The questions in part B of the exam have a range of difficulty. Please be careful to gather as many of the easier marks as possible before venturing into more difficult territory. In some cases an answer to part (a) of a question is needed for part (b). Should you not be able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to completely finish this exam and parts of each question are very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions on the answer card/sheet provided to you and, most importantly, write your solutions to the three written problems on separate sheets as they will be marked by different people in different parts of Canada. Good luck.

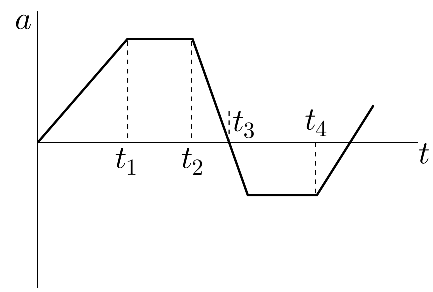
Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
Radius of Earth	$R_E = 6.38 \times 10^6$ m
Radius of Moon	$R_M = 1.70 \times 10^6$ m
Mass of Earth	$M_E = 5.98 \times 10^{24}$ kg
Mass of Moon	$M_M = 7.35 \times 10^{22}$ kg
Mass of Sun	$M_S = 1.99 \times 10^{30}$ kg
Radius of Moon's orbit	$R_{EM} = 3.84 \times 10^8$ m
Radius of Earth's orbit	$R_{ES} = 1.50 \times 10^{11}$ m
Acceleration due to gravity	$g = 9.81$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.675 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ Js
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ Nm^2/C^2
Permeability of free space	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	$v_s = 340$ m/s
Density of air	$\rho = 1.2$ kg/m^3
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K
Absolute zero	$T = 0$ K, $T = -273^\circ$C
Energy Conversion	1 cal=4.184 J
Avogadro's number	$N_A = 6.02 \times 10^{23}$ mol^{-1}

Part A: Multiple Choice

Question 1

The time dependence of the acceleration of a car is as shown. Which time corresponds to the maximum speed of the car?



(a) t_1
(b) t_2
(c) t_3
(d) t_4

Question 2

Two identical loudspeakers are supplied with identical sinusoidal signals. They are placed close to one another in a room and it is noted that there are regions of increased and decreased sound intensity in the vicinity of the speakers. Which of the following actions will not change this pattern?

(a) Moving one of the speakers.
(b) Changing the amplitude of the signal.
(c) Changing the frequency of the signal.
(d) Replacing the air in the room with Helium gas.

Question 3

Two black objects of the same diameter, a sphere and a disc, are placed in front of a uniform beam of light. The plane of the disc is perpendicular to the light rays. The force acting upon them by the light is:

(a) zero
(b) bigger on the disc.
(c) bigger on the sphere.
(d) the same on both.

Question 4

A 1 kg mass rests on the ground, at sea-level and at the equator. What would its weight be if the earth were spinning ten times faster than is presently the case?

(a) 9.80 N
(b) 3.37 N
(c) 6.43 N
(d) 9.76 N

Question 5

An ideal Ammeter—a device to measure current in an electrical circuit—should have an internal resistance which is,

(a) zero.
(b) infinite.
(c) equal to the resistance of the circuit's load.
(d) equal to the internal resistance of the circuit's power source.

Question 6

A 100,000 kg aircraft drops a 1000 kg package of supplies over an arctic research station. What approximate force is felt by the 100 kg pilot at the instant of the release?

(a) 1 N
(b) 10 N
(c) 100 N
(d) zero

Question 7

A converging lens forms an image of an object on a screen. The image is real and has twice the size of the object. If the positions of the screen and the object are interchanged, leaving the lens in its original position, what is the new image size on the screen?

(a) Twice the object size.
(b) Same as the object size.
(c) Half the object size.
(d) Can't say as it depends on the focal length of the lens.

Question 8

A bucket full of water is attached to the end of a rope and allowed to swing back and forth as a pendulum from a fixed support. The bucket has a hole in its bottom that allows water to leak out. How does the period of motion change with the loss of water?

(a) The period does not change.
(b) The period continuously decreases.
(c) The period continuously increases.
(d) The period increases to some maximum and then decreases again.

Question 9

A pencil is placed vertically on a table top with its pointy end up and its sticky eraser end down. As it falls over from this unstable position, its point of contact with the table remains stationary. During its fall, the acceleration of its tip,

(a) remains less than g at all times.
(b) exceeds g at some point.
(c) becomes g just before hitting the table.
(d) is constant.

Question 10

Two 20 g worms climb over a 10 cm high, very thin, wall. One worm is thin and 20 cm long. The other is fat (but still thin compared to its length) and only 10 cm long. What is the ratio of the potential energy of the thin worm as compared to that of the fat worm when each is half way over the top of the wall?

(a) 1:1
(b) 2:1
(c) 2:3
(d) 1:2

Question 11

Suppose that the "Man Who Skied Down Everest" went straight down an incline of 40° to the horizontal and subject to a coefficient of kinetic friction of 0.10. Starting from rest, how long did it take him to reach a speed of 50 km/hr (Ignore air resistance)?

(a) 2.5 s
(b) 9.0 s
(c) 1.5×10^2 s
(d) 2.0 s

Question 12

A stone is thrown vertically downward from the edge of a cliff with an initial speed of 10 m/s. Just before hitting the ground, it has a final speed of 30 m/s. If instead, the stone were thrown horizontally outwards from the top of the cliff with the same initial speed as before, what final speed would it have immediately before hitting the ground?

(a) 10 m/s
(b) 20 m/s
(c) 30 m/s
(d) 40 m/s

Question 13

Two objects of equal mass, and heading towards each other with equal speeds, undergo a head-on collision.Which of the following statements is correct?

(a) Their final velocities must be zero.
(b) Their final velocities may be zero.
(c) Each must have a final velocity equal to the other's initial velocity.
(d) Their velocities must be reduced in magnitude.

Question 14

A positive point charge $+Q$ is placed at $x = 0$ and a negative point charge $-Q$ is placed at $x = a$. The magnitude of the electrostatic force between the two is F. If another point charge $+Q$ is paced at $x = -a$, the net force on the charge at the origin ($x = 0$) is,

(a) $2F$ in the negative x-direction.
(b) F in the positive x-direction.
(c) $5F/4$ in the positive x-direction.
(d) $2F$ in the positive x-direction.

Question 15

A simple circuit contains an ideal battery and a resistance R. If a second resistor is placed in parallel with the first,

(a) the potential across R will decrease.
(b) the current through R will decrease.
(c) the current delivered by the battery will increase.
(d) the power dissipated by R will increase.

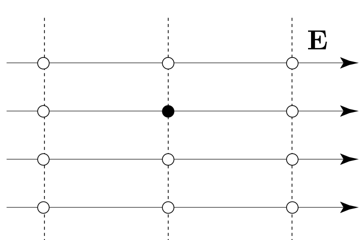
Question 16

A positively charged insulator is brought near (but does not touch) two metallic spheres that are in contact. The metallic spheres are then separated. The sphere which was initially farthest from the insulator will have,

(a) no net charge.
(b) a negative charge.
(c) a positive charge.
(d) either a positive or negative charge.

Question 17

A uniform electric field is as shown. How many of the labelled points have the same electric potential as the shaded point?



(a) 2
(b) 3
(c) 4
(d) 8

Question 18

A spherical capacitor is formed by two metallic and concentric spherical shells. The capacitor is then charged so that the outer shell carries a positive charge and the inner shell carries an equal but negative charge. Even if the capacitor is not connected to any circuit, the charge will eventually leak away due to the small conductivity of the material between the shells. What is the character of the magnetic field induced by this leakage current?

(a) Radially outwards from the inner shell to the outer shell.
(b) Radially inwards from the outer shell to the inner shell.
(c) Circular field lines between the shells and perpendicular to the radial direction.
(d) No magnetic field is induced.

Question 19

Which of the following has units of magnetic field?

(a) kg C^{-1} s^{-1}
(b) kg A^{-1} s^{-1}
(c) N C^{-1} m^{-1}
(d) J A^{-1} m^{-1}

Question 20

Near the geographic north pole is the magnetic north pole which is, in fact, a south magnetic pole. At this point, the earth's magnetic field points straight down. An electron is projected southwards and horizontally from the magnetic north pole. The magnetic force on the electron will cause it to be deflected to the,

(a) Up
(b) Down
(c) East
(d) West

Question 21

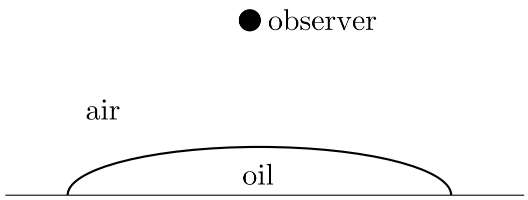
A diffraction grating has 300 "lines" per mm etched upon it. When light of wavelength 550 nm is normally incident upon the grating, how many bright spots appear on a screen a short distance away?

(a) 1
(b) 6
(c) 12
(d) 13

Question 22

A drop of oil ($n = 1.4$) is on a glass ($n = 1.5$) sheet and is observed from directly above by reflected white light.

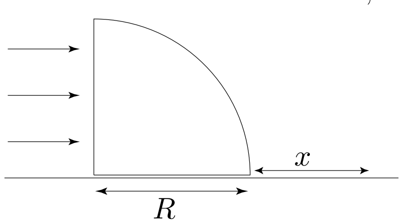
A number of circular constructive interference bands are observed for each colour in the visible spectrum. Approximately how thick is the oil film where one observes the third blue band from the outside edge of the drop. The wavelength of blue light may be taken to be $\lambda_b = 450$ nm.



(a) 482 nm
(b) 675 nm
(c) 1125 nm
(d) 1350 nm

Question 23

A uniform, horizontal beam of light is incident upon a prism as shown. The prism is in the shape of a quarter-cylinder, of radius $R = 5$ cm, and has the index of refraction $n = 1.5$. A patch on the table top for a distance x from the cylinder is unilluminated. The value of x is,



(a) 1.71 cm
(b) 2.24 cm
(c) 2.50 cm
(d) 5.00 cm

Question 24

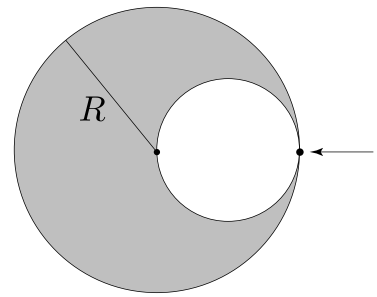
Consider the following types of electromagnetic radiation: radio waves, infra-red, visible light. Which of the following statements are true?

(i) only radio waves can be used to transmit audio information.
(ii) only infra-red radiation is emitted by very hot objects.
(iii) only visible light can be detected by humans.

(a) only i is true.
(b) only ii is true.
(c) only iii is true.
(d) None of the statements are true.

Question 25

In the film Armageddon, astronauts land on an asteroid and are able to walk around "normally" due to the gravitation of the large rock. Suppose that a spherical asteroid has a mass M and a radius R. A spherical hole of radius $R/2$ is excavated from the asteriod as shown. What is the gravitational acceleration at a point on the surface of the asteroid at a point just above the excavation?

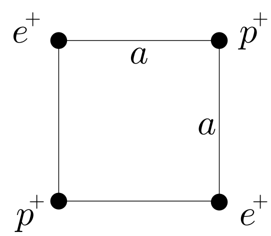


(a) $\frac{GM}{R^2}$
(b) $\frac{GM}{2R^2}$
(c) $\frac{GM}{8R^2}$
(d) $\frac{7GM}{8R^2}$

Part B

Question 1

Four charges, each of charge $+e$, are at the four corners of a square of side $a = 1$ cm. The four charges are fixed in place and are considered to be of point-size. Gravity plays no role in this question.



(a) Sketch a diagram of the electric field due to this charge system.

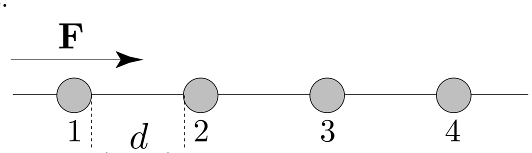
(b) What is the net force acting upon one of the charges?

(c) What is the electrostatic potential energy of this charge system?

(d) Now suppose that two of the charges are protons and the other two are positrons. A positron is the electron's anti-particle. It has the same mass as an electron but a charge of $+e$. The four charges are suddenly released. What are their speeds when they are a significant distance apart from one another?

Question 2

Suppose that we have a string of equally spaced beads of mass m, their surfaces being separated by a distance d, that are free to slide without friction on a thin wire. Suppose that a constant force F acts on the first bead, initially at rest, causing it to accelerate along the wire as shown. This force acts only on the first bead and might be created by a well directed, steady stream of air. The first bead will collide with the second, which will in turn collide with the third, and so on. Suppose that all collisions are elastic.



(a) What is the speed of the first bead immediately before and immediately after its collision with the second bead?

(b) What is the speed of the second bead immediately before and immediately after its collision with the third bead?

(c) Remember that the constant force is always acting upon the first bead. What is the time interval between subsequent collisions between the first and second beads? What then is the average speed of the first bead? What is the speed of the "shock wave" that travels down the wire?

(d) If the whole process is repeated, but with collisions which are perfectly inelastic, what is the terminal speed of the shock wave formed?

Question 3

From everyday experience and observations, estimate the brightness of a full moon as compared to the noon day sun. Brightness is defined as the luminous intensity as measured in units of power per unit area. To arrive at a reasonable numerical estimate, you will need to estimate some physical parameters. It is essential that you explain the reasoning behind your estimates and include detailed drawings and explanations to support your arguments.

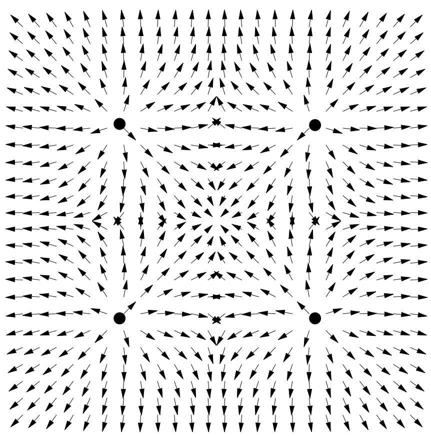
Solutions: Part A

1 (c)	2 (b)	3 (d)	4 (c)	5 (a)
6 (b)	7 (c)	8 (d)	9 (b)	10 (c)
11 (a)	12 (c)	13 (b)	14 (d)	15 (c)
16 (c)	17 (b)	18 (d)	19 (a)	20 (d)
21 (d)	22 (a)	23 (a)	24 (d)	25 (b)

Part B

Question 1

(a)



(b) The force on, say, the top left positron is given by,

$$F = 2 \times \frac{1}{4\pi\epsilon_o} \cdot \frac{e^2}{a^2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{4\pi\epsilon_o} \cdot \frac{e^2}{(\sqrt{2}a)^2} \ ,$$
$$= \frac{1}{8\pi\epsilon_o} \cdot \frac{e^2}{a^2} \cdot (1 + 2\sqrt{2}) \ ,$$
$$= 4.41 \times 10^{-24} \text{ N} \ ,$$

and acts in a direction away from the square and along the square's diagonal.

(c) The electrostatic potential energy of the system is the sum of the pairwise potential energies.

$$U = 4 \times \frac{1}{4\pi\epsilon_o} \cdot \frac{e^2}{a} + 2 \times \frac{1}{4\pi\epsilon_o} \cdot \frac{e^2}{(\sqrt{2}a)} \ ,$$
$$= \frac{1}{4\pi\epsilon_o} \cdot \frac{e^2}{a} \cdot (4 + \sqrt{2}) \ ,$$
$$= 1.25 \times 10^{-25} \text{ J} \ .$$

(d) The positrons are many orders of magnitude less massive than the protons so that very shortly after being released, they will have moved very far away and reached a high velocity. Conservation of energy gives,

$$U = \frac{1}{4\pi\epsilon_o} \cdot \frac{e^2}{(\sqrt{2}a)} + 2 \times \frac{1}{2} m_e v_e^2 \ .$$

Substituting numerical data gives $v_e = 345$ m/s. In the remaining time,the speed of the positron does not change significnalty and the protons are eventually accelerated to a speed v_p. Again, conservation of energy gives,

$$\frac{1}{4\pi\epsilon_o} \cdot \frac{e^2}{(\sqrt{2}a)} = 2 \times \frac{1}{2} m_p v_p^2 \ .$$

This gives $v_p = 3.1$ m/s.

Question 2

(a) Immedaitely before its collision with the second bead, the first bead has a speed of $v_1 = \sqrt{2Fd/m}$. Immediately afterwards, it has a speed of zero.

(b) Immedaitely before its collision with the third bead, the second bead has a speed of $v_1 = \sqrt{2Fd/m}$. Immediately afterwards, it has a speed of zero.

(c) The time between subsequent collisions of the first bead with the second is governed by

$$d = \frac{1}{2} a t^2 \ ,$$

where $a = F/m$ so that the average speed of the first bead is $v_o = d/t = v_1/2$. The first bead moves at an average speed which is half the speed v_1 of the shock wave that travels down the wire.

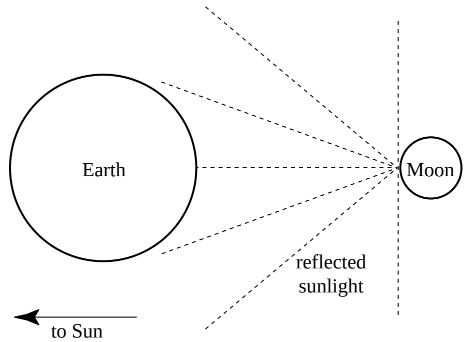
(d) Suppose that after a time, the clump of beads that have collided (and are all stuck together) is travelling with a constant (terminal) velocity v. In a time interval Δt there are $v\Delta t/d$ further collisions increasing the mass of the clump by $mv\Delta t/d$ and increasing the momentum of the clump by $mv^2\Delta t/d$. Since $F = \Delta p/\Delta t$, $v = \sqrt{Fd/m}$. The shock wave moves at a speed of $v_1/\sqrt{2}$.

Question 3

A reasoned argument may come from many directions. A geometric estimate might argue that as much sunlight is incident upon a unit area on the moon as upon the earth (the moon's orbital radius about the earth is not significant as compared to the earth's orbital radius about the sun). The intensity of light per unit area is about,

$$I_o = \frac{P}{4\pi R_{ES}^2} \ ,$$

where P is the sun's power output. We can estimate that the moon reflects about $r = 0.3$ (30%) of this light. We need to know how much light, reflected by the moon, is incident upon a unit area on the earth. The geometry for a full moon is,



Hence,

$$I = rI_o \cdot \pi R_M^2 \cdot \frac{1}{2\pi R_{EM}^2} \ ,$$
$$\frac{I}{I_o} = \frac{rR_M^2}{2R_{EM}^2} \ .$$

This gives a relative intensity of about 3.0×10^{-6}.

Canadian Association of Physicists
2000 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee. The questions in part B of the exam have a range of difficulty. Please be careful to gather as many of the easier marks as possible before venturing into more difficult territory. In some cases an answer to part (a) of a question is needed for part (b). Should you not be able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to completely finish this exam and parts of each question are very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions on the answer card/sheet provided to you and, most importantly, write your solutions to the three written problems on separate sheets as they will be marked by different people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
Radius of Earth	$R_E = 6.38 \times 10^6$ m
Radius of Moon	$R_M = 1.70 \times 10^6$ m
Mass of Earth	$M_E = 5.98 \times 10^{24}$ kg
Mass of Moon	$M_M = 7.35 \times 10^{22}$ kg
Mass of Sun	$M_S = 1.99 \times 10^{30}$ kg
Radius of Moon's orbit	$R_{EM} = 3.84 \times 10^8$ m
Radius of Earth's orbit	$R_{ES} = 1.50 \times 10^{11}$ m
Acceleration due to gravity	$g = 9.81$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.675 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ Js
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ Nm^2/C^2
Permeability of free space	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	$v_s = 340$ m/s
Density of air	$\rho = 1.2$ kg/m^3
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K
Absolute zero	$T = 0$ K, $T = -273$°C
Energy Conversion	1 eV=1.6×10^{-19} J
Avogadro's number	$N_A = 6.02 \times 10^{23}$ mol^{-1}

Part A: Multiple Choice

Question 1

Two high performance bicycle wheels have the same mass but one is a solid disk and the other is a hoop with very light (as compared to the mass of the rim) spokes. Which wheel takes the most energy to accelerate to a speed v? Ignore any air and rolling resistances.

(a) The disk wheel requires more energy.
(b) The spoked wheel requires more energy.
(c) Both take the same amount of energy.
(d) It depends on the radius of each wheel.

Question 2

An astronaut on a strange planet finds that the acceleration due to gravity is two times greater than that on earth. Which of the following could explain this?

(a) The mass of the planet is half that of the earth's but its radius is the same.
(b) The radius of the planet is half that of the earth's but its mass is the same.
(c) Both the mass and the radius are twice the earth's values.
(d) Both the mass and the radius are half the earth's values.

Question 3

A ball of mass of 10 g is hung from the end of a light vertical spring and set oscillating with an amplitude of 10 cm. The period of the oscillation is 1 s. If the ball is now replaced by another with a mass of 40 g and again set oscillating with the same amplitude, its new period of oscillation will be,

(a) 1 s.
(b) 2 s.
(c) 4 s.
(d) 8 s.

Question 4

The human ear canal, which is open to the atmosphere and ends at the ear drum, is about 3 cm long. Which of the following is the best approximation to the fundemental resonant frequency of the ear canal?

(a) 1,500 Hz
(b) 3,000 Hz
(c) 6,000 Hz
(d) 11,000 Hz

Question 5

A metal ball is connected to the ground with a wire via a switch. The switch is initally closed (*i.e.*the ball is connected to ground) while a charge $+Q$ is brought close to the ball (but not touching). While the charge is near the ball, the switch is opened and then the charge is taken away. The charge on the ball is now,

(a) zero.
(b) positive.
(c) negative.
(d) unchanged from its initial charge.

Question 6

In the light from a distant star, a particular spectral line is observed. The wavelength of this line varies between being about 2% shorter than the same line as observed in an earth based laboratory, and about 2% longer. Which of the following best describes the star.

(a) The star is moving away from the earth.
(b) The star is moving towards the earth.
(c) The star is very, very massive as compared to the sun.
(d) The star is orbiting about some massive hidden object.

Question 7

A metal bar rotates about a vertical axis which passes through the center of the bar. The axis of rotation is perpendicular to the length of the bar. There is a uniform magnetic field in the vertical direction. The emf (or potential difference) induced between the two ends of the bar is,

(a) zero.
(b) a sinusoidally oscillating value.
(c) a non-zero positive value.
(d) a non-zero negative value.

Question 8

A projector shows an image which is in focus but too large for the screen on which it is shown. Since the projector and the screen are fixed in place, the projectionist must,

(a) adjust the projector's lens by moving it closer to the screen.
(b) adjust the projector's lens by moving it farther from the screen.
(c) replace the projector's lens with one having a shorter focal length.
(d) replace the projector's lens with one having a longer focal length.

Question 9

In photographic darkrooms, the only source of light is usually a red light bulb. After spending time in a darkroom, a person's eye becomes adjusted to the light but everything appears to be in black and white. Since a red surface reflects all red light, it will appear "white". Which of the following colours will appear to be the brightest?

(a) Green
(b) Blue
(c) Purple
(d) Black

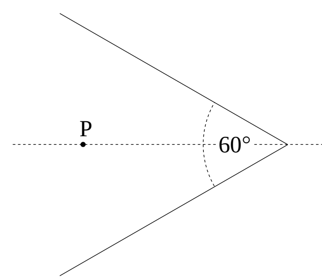
Question 10

Millikan's oil drop experiment attempts to measure the charge on a single electron, e, by measuring the charge on tiny oil drops suspended in an electrostatic field. It is assumed that the charge on the oil drops is due to just a small number of excess electrons. The charges 3.90×10^{-19} C, 6.50×10^{-19} C, and 9.10×10^{-19} C are measured on three drops of oil. The charge of an electron is deduced to be,

(a) 1.3×10^{-19} C.
(b) 1.6×10^{-19} C.
(c) 2.6×10^{-19} C.
(d) 3.9×10^{-19} C.

Question 11

Two mirrors are joined together so that they make an angle of 60° as shown.



A person stands at a point P which is on the line which bisects the 60° angle. How many images of herself does the person see?

(a) 1
(b) 3
(c) 6
(d) infinite number

Question 12

Consider a roller coaster with a loop-the-loop. The loop is not circular (that would be too dangerous) but has a radius of curvature which decreases with height. A roller coaster car starts from rest from the top of a hill which is 5 m higher than the top of the loop. It rolls down the hill and through the loop. What must the radius of curvature at the top of the loop be so that the passengers of the car will feel, at that point, as if they have their normal weight?

(a) 5 m
(b) 10 m
(c) 15 m
(d) 20 m

Question 13

Two identical conducting balls have positive charges $q_1 \neq q_2$ respectively. The balls are brought together so that they touch and then put back in their original positions. The force between the balls is,

(a) The same as it was before the balls touched.
(b) Greater than before the balls touched.
(c) Less than before the balls touched.
(d) Zero.

Question 14

Helium $^4_2$He becomes a superfluid at temperatures $T <$ 2.18 K. A superfluid flows with no viscosity. This behaviour can only be explained using quantum physics and it can only happen if the de Broglie wavelength of a helium atom, of mass m, is comparable to the inter-atomic spacing of the fluid. Which of the following could be an expression for λ, the de Broglie wavelength?

(a) $\lambda = h/\sqrt{3mkT}$
(b) $\lambda = \sqrt{3mkT}/rh$
(c) $\lambda = \sqrt{h}/3mkT$
(d) $\lambda = 3mkT/\sqrt{h}$

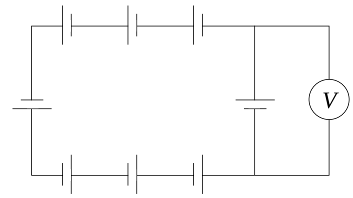
Question 15

In a game of egg-toss, two players toss an egg back and forth between them while moving farther and farther apart. The loser is the person who breaks the egg when catching it. The force required to break an egg's shell is about 5 N and a typical egg has a mass of about 50 g. If the players are 10 m apart, and the eggs are thrown with initial velocities directed at 45° above the horizontal, what is the shortest time that a player can take to arrest the motion of the egg so as not to break it?

(a) 0.01 s
(b) 0.10 s
(c) 0.50 s
(d) 1.00 s

Question 16

A circuit is comprised of 8 identical batteries connected in series as shown.

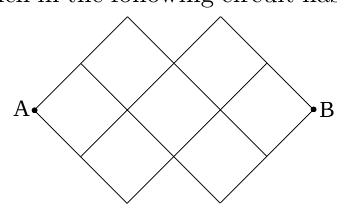


Each battery has an emf of 1.5 V and an internal resistance of 0.2 Ω. What is the reading on a voltmeter connected across any one of the batteries?.

(a) 0.0
(b) 1.3 V
(c) 1.5 V
(d) 12 V

Question 17

Each branch in the following circuit has a resistance R.



The equivalent resistance of the the circuit between the points A and B is,

(a) R
(b) $2R$
(c) $4R$
(d) $8R$

Question 18

Which of the following statements is true concerning the elastic collision of two objects?

(a) No work is done on any of the two objects, since there is no external force.
(b) The work done by the first object on the second is equal to the work done on the second by the first.
(c) The work done by the first object on the second is exactly the opposite of the work done on the second by the first.
(d) The work done on the system depends on the angle of collision.

Question 19

Which of the following statements is false?

(a) The momentum of a heavy object is greater than that of a light object moving at the same speed.

(b) In a perfectly inelastic collision, all the initial kinetic energy of the colliding bodies is dissipated.

(c) The momentum of a system of colliding bodies may be conserved even though the total mechanical energy may not.

(d) The velocity of the center of mass of a system is the system's total momentum divided by its total mass.

Question 20

Which of the following statements is true?

(a) The observed doppler shift for sound waves depends only on the relative motion of the source and the receiver.

(b) Only transverse waves can diffract.

(c) Two wave sources that are out of phase by 180° are incoherent.

(d) Interference patterns are only observed for coherent sources.

Question 21

An airplane circumnavigates the globe by flying above the equator and at a constant altitude above sea level. If the plane repeats the journey but flies at an altitude which is 1000 m higher than before, how much larger a distance through the atmosphere does the plane travel as compared to the first time?

(a) 6.283 km

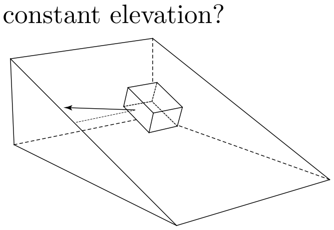
(b) 62.83 km

(c) 628.3 km

(d) 6283 km

Question 22

A block of mass m is at rest on an inclined plane. The coefficients of static and kinetic friction between the block and the plane are μ_s and $\mu_k < \mu_s$. The angle that the plane makes with the horizontal satisfies $\tan\theta = \mu_s$. A string is attached to the block. Is it possible to pull on the string in such a way that the block will slide with constant velocity along a horizontal line? That is to say, can the block slide along a line of constant elevation?



(a) Yes

(b) Yes, but only if $\mu_k mg\cos\theta < mg\sin\theta$

(c) Yes, but only if $\mu_k mg\cos\theta > mg\sin\theta$

(d) No

Question 23

A proton is moving with a speed of u directly towards an alpha-particle (Helium nucleus) which, when the two particles were very far apart from one another, was initially at rest. What is the separation of the particles at their point of closest approach? You may assume that $m_\alpha = 4m_p$.

(a) $r = e^2/\pi\epsilon_o m_p u^2$

(b) $r = e^2/4\pi\epsilon_o m_p u^2$

(c) $r = 5e^2/4\pi\epsilon_o m_p u^2$

(d) $r = 5e^2/8\pi\epsilon_o m_p u^2$

Question 24

Red light from a HeNe laser has a wavelength of 630 nm. If it is normally incident upon an optical diffraction grating with 2000 lines per centimeter, how many maxima (including the central maximum) may be observed on a screen which is far from the grating?

(a) 14

(b) 15

(c) 16

(d) 17

Question 25

Which of the following statements is true?

(a) The capacitance of a capacitor is the total amount of charge it can store.

(b) The equivalent capacitance of two capacitors in series is the sum of the two capacitances.

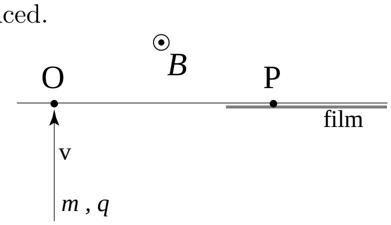
(c) A dielectric material inserted between the conductors of a capacitor increases its capacitance.

(d) The electrostatic energy stored by a capacitor is equal to the ratio of the charge on either conductor to the potential difference between the conductors.

Part B

Problem 1

A mass spectrometer measures the mass of ions. An ion is accelerated to a high speed and injected into a region with a uniform magnetic field which is perpendicular to the ion's velocity. The ion will follow a curved trajectory and impact a photographic plate some distance away from the injection point. By measuring this distance, the ion's mass can be deduced.



Suppose singly charged ($q = +e$) ions of ${}^{39}_{19}$K and ${}^{41}_{19}$K are accelerated to an energy of 500 eV and injected into a magnetic field $B = 0.7$ T. The velocity of the ions upon entering the magnetic field is perpendicular to the line OP.

(a) What speed do each of the ions have when they impact the photographic plate at a point P?

(b) What is the shape of the trajectory for the ions? Find the distance OP for each of the two ions.

(c) In reality, things are not quite so simple. Variations in the ions' velocities can cause problems with the resolution of the machine. If the initial energy of the ions cannot be held precisely at 500 eV, there will be a small variation in the distances OP. Suppose that the energy of the ions is 500 ± 5 eV. Can the two ions be distinguished from one another when they hit the screen?

(d) Suppose that the velocity of the ions when they enter the magnetic field cannot be held perpendicular to the line OP. If there is a small variation of $\pm\alpha$ about a perpendicular injection angle, find the variation in the length of OP. Can the two ions be distinguished from one another if $\alpha = 3°$?

Problem 2

A town B is directly east of a town A. The two towns are a distance L apart. In the absence of any wind, a plane can fly from town A to B in a time T. The speed of the airplane, $v_o = L/T$ is called the plane's air-speed. Assume in all that follows that the plane always has the same air-speed. If the plane flys with a tail wind of speed v_w, then it will have a speed with respect to the ground, v_g given by $v_g = v_o + v_w$. This is called the plane's ground-speed.

(a) Explain how the plane could have a ground-speed of zero.

(b) If a wind v_w blows from the west (*i.e.*to the east), find the time that the plane would take for a round trip from town A to B and back again. Show that it is greater than T.

(c) If a wind v_w blows from the south (*i.e.*to the north), sketch a diagram showing the heading that the plane must fly so as to fly in a straight line from town A to town B. Again, find the round trip time for flying in such a cross wind.

(d) Show that regardless of the wind direction, the round trip time for a flight from town A to B and back again is longer than if there was no wind whatsoever. Find an expression for the round trip time if a wind v_w blows in an arbitrary direction.

Problem 3

Leonardo da Vinci was commissioned by the Duke of Milan to construct a Bronze statue as a tribute to his late father. The proposed size was to be twenty three feet tall and weigh 80 tons (73,000 kg). For this task Leonardo had to develop new casting methods to construct this massive statue. By 1499 the French invaded Milan and he had only completed a 22-foot clay model which was used by the French soldiers for target practice. The project was never completed.

Heavy statues need to be placed upon very strong concrete pedestals. The compressive strength of high strength concrete ranges from 10 – 70 N/mm^2. Lets take a typical high-strength concrete with a compressive strength of $s = 50$ N/mm^2 and a density of $\rho = 1250$ kg/m^3. This concrete will crumble if a pressure of greater than this is applied.

(a) Suppose Leonardo's statue (a horse with four feet touching the ground) is to be placed on such a concrete pedestal. What must be the minimum area of each of the horse's feet so as not to damage the pedestal?

(b) Now, more generally, what is the maximum height of a conical mountain made of high strength concrete?

(c) Finally, outline the design constraints that go into making a free standing concrete structure with the maximum possible height.

Solutions: Part A

1 (b)	2 (d)	3 (b)	4 (b)	5 (c)
6 (d)	7 (a)	8 (d)	9 (c)	10 (a)
11 (c)	12 (a)	13 (b)	14 (a)	15 (b)
16 (a)	17 (b)	18 (c)	19 (b)	20 (d)
21 (a)	22 (a)	23 (c)	24 (b)	25 (c)

Part B

Question 1

(a) The magnetic field does no work on the ion. The impact speed is the same as the ion's initial speed. Using,

$$E = \frac{1}{2}mv^2 \ ,$$

we find that

$$v = \sqrt{\frac{2E}{Am_p}} \ .$$

We have approximated the mass of the ion to be the atomic number times the mass of a proton. We ignore the mass of the electrons and the minor differnces between proton and neutron masses. We do not need to worry about binding energies. Hence,

$$v_{39} = 4.96 \times 10^4 \text{ m/s} \ ,$$
$$v_{41} = 4.83 \times 10^4 \text{ m/s} \ .$$

(b) The trajectory is circular with a radius given by,

$$r = \frac{Am_p v}{eB} \ , \quad = \frac{\sqrt{2Am_pE}}{eB} \ .$$

The distance OP is just twice the radius. Hence,

$$OP_{39} = 5.76 \text{ cm} \ ,$$
$$OP_{41} = 5.91 \text{ cm} \ .$$

(c) One could work out the variations in OP by putting in maximum and minimum values for the energy, but it is faster to look at the variation of radius with energy.

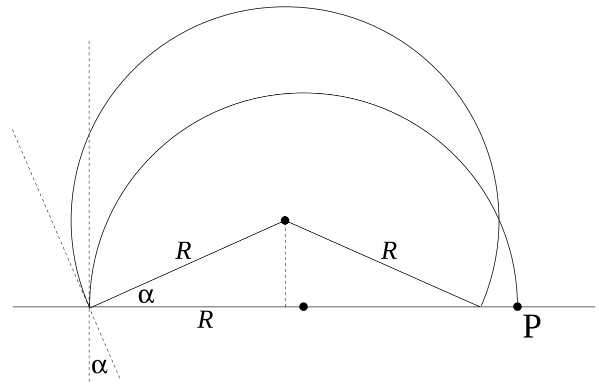
$$\Delta AB = 2\Delta r \ , \quad = \frac{\sqrt{2Am_pE}}{eB} \times \frac{\Delta E}{E} \ , \quad = r \times \frac{\Delta E}{E} \ .$$

Since $\Delta E/E = 0.01$, we find,

$$\Delta OP_{39} \approx \Delta OP_{41} \approx 0.06 \text{ cm} \ .$$

The two positions are more than 0.12 cm apart so that the two ions are distinguishable.

(d) Consider the same ion entering the field normally and with an angle α as shown.



Both trajectories are circular with the same radius. The final positions of the ions are $2r$ and $2r\cos\alpha$ respectively. It is easy to see that we get the same result for an ion injected at an angle of $-\alpha$. Hence the variation in the position of the ion is,

$$\Delta OP = \pm 2r(1 - \cos\alpha) \ , \quad = 4r\sin^2\frac{\alpha}{2} \ , \quad \approx r\alpha^2 \ .$$

For the small angle of $\alpha = \pi/60$, we have

$$\Delta OP_{39} \approx \Delta OP_{41} \approx 0.02 \text{ cm} \ .$$

The ions are still distinguishable.

Question 2

(a) The plane may have no ground-speed if it flys into a wind equal to its air-speed.

(b) The time for the first leg is,

$$t_1 = \frac{L}{v_o + v_w} \ ,$$

whereas the time for the second leg is,

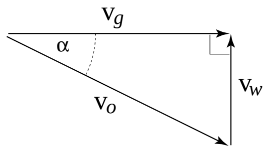
$$t_2 = \frac{L}{v_o - v_w} \ ,$$

We must assume that $v_o > v_w$ or else the plane will never return. The total time is,

$$t = \frac{2L}{v_o} \times \frac{1}{1 - (v_w/v_o)^2} \ .$$

Since in the absence of wind, the time would be $2L/v_o$, the time with the wind is larger.

(c) The heading of the plane must obey,



This gives,

$$v_g = v_o \cos\alpha \ ,$$
$$v_w = v_o \sin\alpha \ .$$

Hence,

$$v_g = v_o\sqrt{1-(v_w/v_o)^2} \ .$$

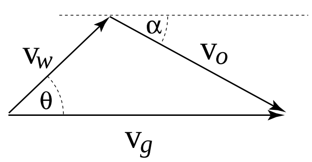
The time forthe first leg of the trip is just $t_1 = L/v_g$ which gives,

$$t_1 = \frac{L}{v_o} \times \frac{1}{\sqrt{1-(v_w/v_o)^2}} \ .$$

Symmetry dictates that the times for both legs of the trip are equal so that the total time is just,

$$t = \frac{2L}{v_o} \times \frac{1}{\sqrt{1-(v_w/v_o)^2}} \ .$$

(d) If the wind blows at an angle of θ north of east then,



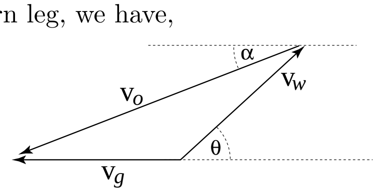
This requires that,

$$v_g = v_w \cos\theta + v_o \cos\alpha$$
$$0 = v_w \sin\theta - v_o \sin\alpha$$

Hence,

$$v_g = v_w \cos\theta + v_o\sqrt{1-(v_w/v_o)^2 \sin^2\theta} \ .$$

For the return leg, we have,



This requires that,

$$v_g = v_o \cos\alpha - v_w \cos\theta$$
$$0 = v_o \sin\alpha - v_w \sin\theta$$

Hence,

$$v_g = v_o\sqrt{1-(v_w/v_o)^2 \sin^2\theta} - v_w \cos\theta \ .$$

The total tip time is then,

$$\begin{aligned} t &= \frac{2L}{v_o} \times \frac{\sqrt{1-(v_w/v_o)^2\sin^2\theta}}{1-(v_w/v_o)^2} \ , \\ &> \frac{2L}{v_o} \ . \end{aligned}$$

Question 3

(a) Each hoof must have an area of:

$$\begin{aligned} A &= \frac{1}{4} \times \frac{mg}{s} \\ &= 35.8 \text{ cm}^2 \end{aligned}$$

Each hoof must be at last 6 cm square.

(b) For a conical mountain of radius r and height h, the pressure on the base surface area is,

$$\begin{aligned} P &= \frac{\frac{1}{3}\pi r^2 h\rho g}{\pi r^2} \ , \\ &= \frac{\rho g h}{3} \ . \end{aligned}$$

Setting this equal to the compressive strength of concrete gives,

$$h = \frac{3s}{\rho g} \ .$$

The maximum height, before the base layer of concrete begins to crumble, is 12.2 km. Consider that the highest mountains in the world (which are not as strong as high strength concrete) are about 8 km high.

(c) Any particular layer of concrete within the tower only needs to be strong enough to hold up the weight of the concrete above it. The cross sectional area $A(x)$ of the tower must decrease with the height x. The defining constraint for the cross sectional area of a tower is then,

$$sA(x) = \rho g V(x) \ ,$$

where $V(x)$ is the volume of the tower above the height x. A number of models could be examined. The conical tower will be quickly found to have too much mass above any cross-sectional area to be the most efficient shape. The tower must have curved sides.

The best students may be able to express this volume as,

$$V(x) = \int_x^H A(x)dx \ ,$$

where H is the height of the tower. The trick now is to differentiate both sides of the resulting equation to get,

$$\frac{dA(x)}{dx} = -\frac{\rho g}{s} A(x) \ .$$

Clearly, the cross sectional area of the tower decreases exponentially. The rate of curvature is least for a low density, high strength material! The tower need not be round in cross section although this would have the greatest cross sectional area (and hence strength) for a given perimeter.

A solution based on calculus is not required. The basic constraint applied to a sensible model will be judged accordingly. Students might examine a series of blocks, each smaller than the one below, stacked upon one another. The defining relationship can also be found from noting that the weight of a small segment of tower is given by,

$$\Delta W = \rho g A(x) \Delta x \ .$$

The difference between the weight that would need to be supported by the top surface of this segment as opposed to that for the bottom surface is $-s\Delta A$ so that,

$$s\Delta A = -\rho g A(x) \Delta x \ .$$

This gives,

$$\frac{\Delta A(x)}{\Delta x} = -\frac{\rho g}{s} A(x) \ .$$

Canadian Association of Physicists
2001 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee. The questions in part B of the exam have a range of difficulty. Please be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three written problems on **separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Acceleration due to gravity	$g = 9.80$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ J·m/C^2
Permeability of free space	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	$v_s = 343$ m/s
Density of air	$\rho = 1.2$ kg/m^3
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K
Energy Conversion	1 eV=1.6×10^{-19} J
Avogadro's number	$N_A = 6.02 \times 10^{23}$ mol^{-1}

Part A: Multiple Choice

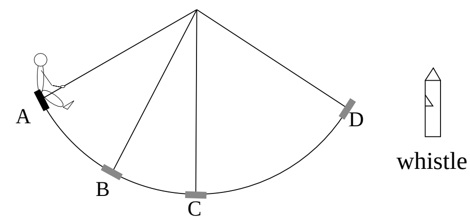
Question 1

Three identical resistors are connected across a voltage source V so that one of them is in parallel with two others which are connected in series. The power dissipated through the first one, compared to the power dissipated by each of the other two, is approximately

(a) the same.
(b) half as much.
(c) twice as much.
(d) four times as much.

Question 2

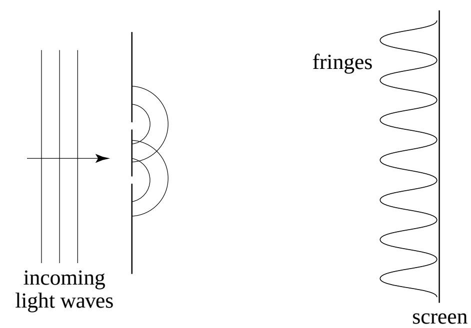
The diagram below shows various positions of a child in motion on a swing. Somewhere in front of the child a stationary whistle is blowing. At which position will the child hear the highest frequency for the sound of the whistle?



(a) At B when moving toward A.
(b) At B when moving toward C.
(c) At C when moving toward B.
(d) At C when moving toward D.

Question 3

In a Young double slit experiment, green light is incident on the two slits. The interference pattern is observed on a screen. Which one of the following changes would cause the observed fringes to be more closely spaced?



(a) Reducing the separation between the slits.
(b) Using blue light instead of green light.
(c) Using red light instead of green light.
(d) Moving the light source further away from the slits.

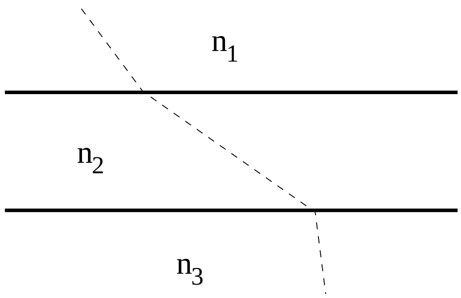
Question 4

A thin lens with focal length f is to be used as a magnifying glass. Which of the following statements regarding this situation is true?

(a) A converging lens must be used, and the object be placed at a distance greater than $2f$ from the lens.
(b) A diverging lens must be used, and the object be placed between f and $2f$ from the lens.
(c) A converging lens must be used, and the object be placed at a distance less than f from the lens.
(d) A diverging lens must be used, and the object be placed at any point other than the focal point.

Question 5

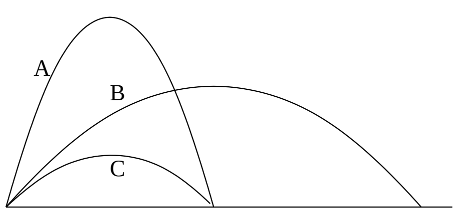
The figure shows the path of a ray of light as it passes through three different materials with refractive indices n_1, n_2 and n_3. The figure is drawn to scale. What can we conclude concerning the indices of these three materials?



(a) $n_3 < n_2 < n_1$.
(b) $n_3 < n_1 < n_2$.
(c) $n_2 < n_1 < n_3$.
(d) $n_1 < n_3 < n_2$.

Question 6

The diagram shows the trajectory of three artillery shells. Each was fired with the same initial speed. Which shell was in the air for the longest time? (Ignore air friction.)



(a) Shell A.
(b) Shell B.
(c) Shell C.
(d) Shells A and C were in the air for equal time, which was longer than for shell B.

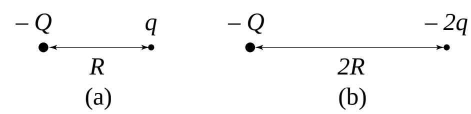
Question 7

Let V be electric potential and E the magnitude of the electric field. At a given position, which of the following statements is true?

(a) E is always zero where V is zero.
(b) V is always zero where E is zero.
(c) E can be zero where V is nonzero.
(d) E is always nonzero where V is nonzero.

Question 8

Two test charges are brought successively to a position some distance away from a negative charge $-Q$. First, test-charge q is placed at a distance R from $-Q$. Then it is removed and a test-charge $-2q$ is placed at a distance $2R$ from $-Q$. The sign of q is unknown. Compared with the electric potential at the position of the test-charge in the first case, the electric potential at the position of the second test-charge is



(a) the same.
(b) twice as large.
(c) half as large.
(d) impossible to determine unless the sign of q is known.

Question 9

A simple pendulum, consisting of a mass m at the end of an unstretchable string of length L, swings upward, making an angle θ with the vertical. The work done *by* the tension in the string is

(a) Zero.
(b) mgL
(c) $mgL\cos\theta$
(d) $-mgL\sin\theta$

Question 10

In problems involving electromagnetism it is often convenient and informative to express answers in terms of a constant, α, which is a combination of the Coulomb constant, $k_e = 1/4\pi\epsilon_0$, the charge of the electron, e, and $\hbar = h/2\pi$, h being Planck's constant. For instance, the lowest energy that a hydrogen atom can have is given by $E = -\frac{1}{2}\alpha^2 mc^2$, where m is the mass of the electron and c is the speed of light. Which of the following is the correct expression for α? (HINT: non-relativistic kinetic energy is $\frac{1}{2}mv^2$, where v is speed.)

(a) $\dfrac{k_e e^2}{\hbar c}$.
(b) $\dfrac{\hbar}{k_e e^2 c}$.
(c) $\dfrac{k_e e^2 \hbar}{c}$.
(d) $\dfrac{k_e e^2 c}{\hbar}$

Question 11

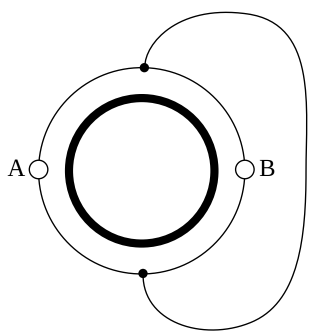
A mass suspended from a spring is oscillating up and down. Consider the following two statements:

(1) At some point during the oscillation, the mass has zero velocity but it is accelerating.
(2) At some point during the oscillation, the mass has zero velocity and zero acceleration.

(a) Both occur at some time during the oscillation.
(b) Neither occurs during the oscillation.
(c) Only (1) occurs.
(d) Only (2) occurs.

Question 12

A very long solenoid perpendicular to the page generates a downward magnetic field whose magnitude increases with time. This induces an emf in a conducting wire loop around the solenoid which lights two identical bulbs connected in series along the wire. Now two points diametrically opposed on the wire loop are shorted with another wire lying to the right of bulb B in the plane of the page. After the shorting wire is inserted,



(a) bulb A goes out, and bulb B dims.
(b) bulb A goes out, and bulb B gets brighter.
(c) bulb B goes out, and bulb A dims.
(d) bulb B goes out, and bulb A gets brighter.

Question 13

A quantity of charge, Q, is distributed uniformly through a sphere of radius R. A smaller sphere, of radius d and concentric with the large sphere, is now removed from it, leaving a spherical cavity with no charge in it. The charge density of the remaining shell has not changed. The electrostatic potential at a distance $r > R$, outside the shell, is

(a) $\frac{1}{4\pi\epsilon_0}\frac{Q}{r}\frac{r^3}{d^3}$
(b) $\frac{1}{4\pi\epsilon_0}\frac{Q}{r}\frac{R^3-d^3}{R^3}$
(c) $\frac{1}{4\pi\epsilon_0}\frac{Q}{r}\frac{R^3}{d^3}$
(d) $\frac{1}{4\pi\epsilon_0}\frac{Q}{r}\frac{d^3}{R^3}$

Question 14

A vertical spring with constant 250 N/m and negligible mass is attached to a horizontal plate. A block of mass 0.255 kg is dropped onto the spring which compresses by 0.12 m before coming momentarily to rest. The following two numbers are the work done by the force of gravity, W_g, and the work done by the spring, W_s, (both over the total distance that the spring is compressed).

(a) $W_\mathrm{g} = 0.30$ J, $W_\mathrm{s} = -1.8$ J.
(b) $W_\mathrm{g} = 0.30$ J, $W_\mathrm{s} = 1.8$ J.
(c) $W_\mathrm{g} = -0.30$ J, $W_\mathrm{s} = 1.8$ J.
(d) $W_\mathrm{g} = -0.30$ J, $W_\mathrm{s} = -1.8$ J.

Question 15

A stone of mass m is attached to a light strong string and whirled in a *vertical* circle of radius r. At the exact bottom of the path, the tension in the string is three times the weight of the stone. The stone's speed at that point is given by

(a) $2\sqrt{gr}$
(b) $\sqrt{2gr}$
(c) $\sqrt{3gr}$
(d) $4gr$

Question 16

You are *rapidly* pumping air into a tire with a bicycle pump. At the same time, helium gas is *rapidly* escaping out of a balloon. Which of the following statements is true?

(a) The pump cools down while the helium gas warms up as it is expelled.
(b) The pump warms up while the helium gas cools down as it is expelled.
(c) The pump cools down while the helium gas also cools down as it is expelled.
(d) The pump warms ups while the helium has also warms up as it is expelled.

Question 17

A particle, moving through a certain region of space, experiences a non-zero magnetic force. Which of the following is possible?

(a) A magnetic field exists in that region and changes the speed of the particle.
(b) A magnetic field exists in this region and the particle's velocity vector is parallel to the magnetic field vector.
(c) A magnetic field exists in this region and the particle is moving at right angle to the magnetic field.
(d) A magnetic field exists in this region and the particle is moving in the direction opposite to the magnetic field vector.

Question 18

The astronauts aboard a Space Shuttle in circular orbit around the Earth wish to transfer the shuttle to a new circular orbit at higher altitude. In this new orbit, the shuttle will have

(a) a smaller orbital speed and a larger total energy.
(b) a smaller orbital speed and a smaller total energy.
(c) a larger orbital speed and a larger total energy.
(d) a larger orbital speed and a smaller total energy.

Question 19

A car pushes a stalled truck up a hill. The car exerts a force of magnitude F_1 on the truck. The truck exerts a force of magnitude F_2 on the car. Then

(a) $F_1 = F_2$ since these forces form an action-reaction pair.
(b) $F_1 = F_2$ only if the car and truck do not accelerate.
(c) $F_1 > F_2$, or else the car and the truck would not move.
(d) $F_1 < F_2$ because the truck is heavier.

Question 20

A convex mirror has its centre of curvature located behind the mirror. The image formed by such a mirror is

(a) upside down, enlarged and virtual.
(b) upside down, reduced and virtual.
(c) right side up. reduced and real.
(d) right side up, reduced and virtual.

.

Question 21

A toy airplane is travelling in a horizontal circle at a constant speed at the end of a tether wire. The magnitude of the tension in the wire is F. Then the wire is played out to twice its original length and the plane is made to fly at twice its original speed. The tension in the wire is now

(a) $\frac{1}{4}F$
(b) $\frac{1}{2}F$
(c) F
(d) $2F$

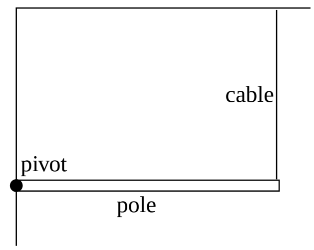
Question 22

The plates of a parallel-plate capacitor are separated by a solid dielectric. This capacitor and a resistor are connected in series across the terminals of a battery. Now the plates of the capacitor are pulled slightly farther apart. When equilibrium is restored in the circuit,

(a) the potential difference across the plates has increased.
(b) the energy stored on the capacitor has increased.
(c) the capacitance of the capacitor has increased.
(d) the charge on the plates of the capacitor has decreased.

Question 23

A uniform pole is attached to a vertical wall by a frictionless pivot. The pole is held horizontal by a vertical cable attached to the ceiling as shown. Considering torques on the pole about the axis of the pivot, which one of the statements below the figure is correct?



(a) The magnitude of the torque due to the tension in the cable is equal to the magnitude of the torque due to the weight of the pole.
(b) The magnitude of the torque due to the tension in the cable is greater than the magnitude of the torque due to the weight of the pole.
(c) The magnitude of the torque due to the tension in the cable is less than the magnitude of the torque due to the weight of the pole.
(d) The tension in the cable is equal to the weight of the pole.

Question 24

The minimum speed with respect to air that a particular jet aircraft must have in order to keep aloft is 300 km/hr. Suppose that as its pilot prepares to take off, the wind blows eastward at a ground speed that can vary between 0 and 30 km/hr. Ignoring any other fact, a safe procedure to follow, consistent with using up as little fuel as possible, is to

(a) take off eastward at a *ground* speed of 320 km/hr.
(b) take off westward at a *ground* speed of 320 km/hr.
(c) take off westward at a *ground* speed of 300 km/hr.
(d) take off westward at a *ground* speed of 280 km/hr.

Question 25

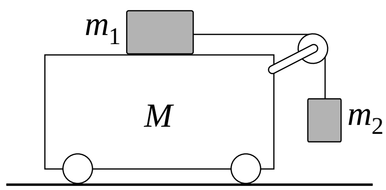
A mass m is moving at speed v perpendicular to a rod of length d and mass $M = 6m$ which pivots around a frictionless axle running through its centre. It strikes and sticks to the end of the rod. The moment of inertia of the rod about its centre is $Md^2/12$. Then the angular speed of the system right after the collision is

(a) $2v/3d$.
(b) $2v/d$.
(c) v/d.
(d) $3v/2d$.

Part B

Problem 1

Suppose that initially the system of masses shown below is held motionless. All surfaces are frictionless, and the pulley is fixed to mass M. The string is completely inextensible: its length is constant. We are interested in the acceleration of each mass at the instant right after they are released. Note that at this instant the string holding m_2 is still *vertical*.



(a) Draw free-body diagrams showing and labelling all forces that act on the three masses at the instant right after they are released.

(b) Write Newton's second law as it applies to each mass. Explain briefly why M accelerates.

(c) Let A be the acceleration of mass M, a_1 that of m_1, and a_2 that of m_2. Find a relation between the accelerations of the three masses.

(d) Determine the tension T in the string and the three accelerations in terms of the masses and of the gravitational acceleration, $\boldsymbol{g}$.

(e) Take the limits of your results in part (d) as $M \to \infty$. What do they become if instead you take $m_1 \to 0$, *or* if you take $m_2 \to 0$ (not both at the same time)? Do your results make sense in all those limits?

Problem 2

A 100 kg (when empty) cylindrical bottle of gas, containing 100 l of air at a pressure of 100 atmospheres, falls over on the lab floor, breaking the valve. This leaves a circular hole of radius 2.0 cm at the end of the bottle, through which the air escapes in a direction parallel to the long axis of the bottle.

(a) Let u be the exhaust speed of the air relative to the lab, *assumed to be constant*, and M_i, M_f the initial and final mass of the bottle, respectively. The bottle slides across a floor with a kinetic coefficient of friction $\mu = 0.15$.

Calculate $\Delta M/\Delta t$, the maximum rate at which the mass of the bottle changes if u is limited by the speed of sound, approximately equal to 343 m/s. HINT: under our assumptions, $\Delta M/\Delta t$ is constant, and the escaping air (as opposed to the air still in the bottle) has a density of 1.2 kg/m^3.

Write down the fundamental conservation law which governs the motion of the bottle. Calculate its acceleration as the air starts escaping.

(b) Suppose that the acceleration imparted to the bottle by the escaping gas and that you calculated in part (a) remains the same until the air has finished escaping at the constant rate $\Delta M/\Delta t$. What would be the *total* distance covered by the bottle when it finally comes to a stop?

(c) NOTE: BONUS marks will be awarded if you can derive the following expression for the maximum speed the bottle can ever attain once all the air has escaped, and which does not assume that the acceleration of the bottle is constant while the air is escaping:

$$v_f = \frac{m}{M_f}u \; - \; \frac{\mu g/2M_f}{(\Delta M/\Delta t)}(M_f^2 - M_i^2)$$

where m is the mass of the escaped air, neglecting the air left in the bottle at the end.

Problem 3

Consider a long horizontal cylindrical shell, of length l, radius R which can rotate freely about its longitudinal axis with a moment of inertia I. The material that composes it is electrically insulating and non-magnetic. A massless string attached to a vertically hanging mass m is then wound around the cylinder drum. The mass is released from rest at time $t = 0$.

(a) Determine the angular acceleration and kinetic energy of the system after the hanging mass has fallen a distance h.

(b) A net amount of positive charge Q, of negligible mass, is deposited uniformly on the outside drum of the shell before the mass is released. Redo part (a) under these conditions. Calculate the difference in kinetic energy between the two cases $Q = 0$ and $Q \neq 0$. Do you have any idea as to where the "missing" kinetic energy went?

HINT: The magnetic field due to a very long solenoid of length l and N turns carrying a current I produces no net magnetic field outside the solenoid, but a net uniform magnetic field of strength $\mu_0 NI/l$ inside the solenoid. The field is directed along the solenoid axis.

Solutions to CAP Exam 2001

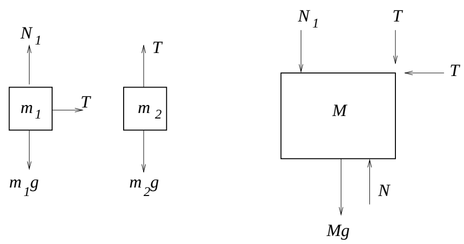
Part A

1 (d)	2 (d)	3 (b)	4 (c)	5 (c)
6 (a)	7 (c)	8 (c)	9 (a)	10 (a)
11 (c)	12 (d)	13 (b)	14 (a)	15 (b)
16 (b)	17 (c)	18 (a)	19 (a)	20 (d)
21 (d)	22 (d)	23 (a)	24 (c)	25 (a)

Part B

Problem 1

a) Free-body diagrams:



mg represents the weight of an object. N is the reaction force exerted by the ground on the cart, and N_1 is the reaction force between m_1 and M. T is the tension in the string.

b) The three equations of motion are

$$\begin{aligned} T &= m_1 a_1 & (1) \\ m_2 g - T &= m_2 a_2 & (2) \\ T &= MA & (3) \end{aligned}$$

where a_1, a_2 and A are the accelerations of m_1, m_2 and M, respectively. The vertical forces on M cancel each other, and so do those acting on m_1. M accelerates (to the left) because the string exerts a force T to the left on the pulley.

c) Imagine yourself accelerating with respect to the ground, so that M is stationary with respect to you. Then the acceleration of m_1 *with respect to you* (and therefore with respect to M) is a_2 to the right, since the length of the string does not change. With respect to the ground, we must have

$$a_1 = a_2 - A \qquad (4)$$

d) In (b) and (c), we have four equations with four unknowns. (1) and (3) give $a_1 = (M/m_1)A$. Equation (4) then gives $a_2 = (1 + M/m_1)A$. Inserting this in (2) and eliminating T with (3) leads to

$$\begin{aligned} A &= \frac{m_1 m_2\, g}{M(m_1+m_2)+m_1 m_2} \\ T &= \frac{M\, g\, m_1 m_2}{M(m_1+m_2)+m_1 m_2} \end{aligned}$$

Then

$$\begin{aligned} a_1 &= (M/m_1)A &= \frac{Mg\, m_2}{M(m_1+m_2)+m_1 m_2} \\ a_2 &= \left(1+\frac{M}{m_1}\right)A &= \frac{(M+m_1)m_2\, g}{M(m_1+m_2)+m_1 m_2} \end{aligned}$$

e) It is understood that the limit $M \to \infty$ is a mathematical short-hand for making M large enough that the other masses in the problem become small with respect to it: $M \gg m_1$, $M \gg m_2$.

$$\begin{aligned} M \to \infty \implies A \to 0, \qquad T \to \frac{m_1 m_2}{m_1+m_2}\, g, \\ a_1 = a_2 \to \frac{m_2}{m_1+m_2}\, g \end{aligned}$$

This is the standard result for the massless pulley problem.

$$m_1 \to 0 \implies A \to 0,\ T \to 0,\ a_1 \to g,\ a_2 \to g$$

T goes to zero (because $m_1 = 0$, no net horizontal force acts on M). $a_1 = g$ may look strange, but it just means that the body must have the same acceleration as m_2 since the string that connects them has a fixed length.

$$m_2 \to 0 \implies A,\ a_1,\ a_2,\ T \to 0$$

If $m_2 = 0$, the tension in the string is also zero, and the rest of the system cannot start moving.

Problem 2

a) $d_t M = -\rho_{\rm air} A\, v_{\rm sound} = -0.52$ kg/s, with A the area of the hole, V the volume of gas in the bottle, and $\rho_{\rm air} = 1.2$ kg/m^3 at standard pressure, so 120 kg/m^3 at 100 atmospheres. The rate at which the momentum Mv of the bottle changes equals minus the rate at which the momentum of the escaping air changes, minus the magnitude of the friction force, $F_{\rm fr} = \mu M g$. Thus, **conservation of total momentum** demands that

$$F = d_t(Mv) = -(d_t M)\, u - \mu M g$$

The initial mass of air in the bottle is $m = (0.1\ \text{m}^3)(120\ \text{kg/m}^3)$. Then we find, with $M = M_i = 112$ kg, that the initial acceleration is

$$a_i = F/M_i = -\frac{(d_t M)}{M_i}\, u - \mu g = 0.11\ \text{m/s}^2$$

b) At 0.52 kg/s, it takes 23 s for 12 kg of air to escape. At a constant acceleration of 0.11 m/s^2, the bottle reaches a speed of $v_f = at = 2.6$ m/s over a distance of $\frac{1}{2}at^2 = 30$ m. Note that during this segment of the bottle's motion, we must neglect the friction force, otherwise the acceleration would not be constant. To find the distance d covered after the gas has escaped, we note that $F_{\rm fr} d = K$, where $K = \frac{1}{2}M_f v_f^2$ is the maximum kinetic energy reached by the bottle, which energy is going to be dissipated by the friction force $F_{\rm fr} = \mu M_f g$ as it deccelerates the bottle. This yields $d = 2.3$ m, so that the bottle travels a total of 32 m.

c) BONUS part: integrating

$$d(Mv) = -u\,dM - \frac{\mu M g}{d_t M} dM$$

gives the final momentum and speed (m is the total mass of the escaped air)

$$p_f = -u\,(M_f - M_i) - \frac{\mu g/2}{d_t M}(M_f^2 - M_i^2), \quad \text{or}$$

$$v_f = mu/M_f - \frac{\mu g/2M_f}{d_t M}(M_f^2 - M_i^2)$$

Problem 3

a) Let α be the angular acceleration of the cylinder. Its equation of motion is $I\alpha = TR$, where I is its moment of inertia around its rotation axis, and T is the tension in the string. The equation of motion of the mass is $-T + mg = ma = m\alpha R$, where a is its linear acceleration. Solving for α yields

$$\alpha = \frac{mgR}{I + mR^2}$$

The total kinetic energy, K, acquired by the system must be equal to the gravitational potential energy lost by the mass. So

$$K = mgh$$

b) From Ampère's law, $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I$, applied to a circular loop outside the cylinder and perpendicular to it, one finds, due to the symmetry of the problem, that the magnitude of the magnetic field $\boldsymbol{B}$ induced by the current I created by the rotating charge is $B = \mu_0 I/l$, directed along the axis of the cylindrical shell (into the page if the cylinder is perpendicular to it and accelerates clockwise). This also results from the hint given in the question. Since the current is $I = \Delta Q/\Delta t = Q\omega/2\pi$, we arrive at $B = \mu_0 \omega Q/2\pi l$.

Now since B is increasing, the increase of the magnetic flux $\Phi_B = B\pi R^2$ inside the shell generates an electric field $\boldsymbol{E}$ tangent to the shell and perpendicular to its axis, according to Faraday's law. This field retards the acceleration of the charge, thereby also slowing down the increase of Φ_B (Lenz's law). Faraday's law applied along a circular path of radius R around the surface of the cylinder gives

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = E(2\pi R) = -d_t \Phi_B = -\pi R^2 \mu_0 d_t I$$

where $d_t I = (Q/2\pi) d_t \omega = Q\alpha/2\pi$. Then

$$E = -\frac{\mu_0 Q R}{4\pi l}\,\alpha$$

This electric field produces a retarding torque, $(QE)R$ on the cylinder, as expected.

The equations of motion for the cylinder and the mass are now, respectively,

$$I\alpha = TR + QER, \qquad -T + mg = m\alpha R$$

where, again, T is the tension in the string. Eliminating this and solving for α yields

$$\alpha = \frac{mgR}{I + mR^2 + \mu_0 Q^2 R^2/4\pi l}$$

This acceleration is indeed smaller than that predicted in the zero-charge situation, due to the increase of the inertial factor in the denominator.

The kinetic energy is still $K = \frac{1}{2}(I + mR^2)\omega^2$, but we cannot equate this to the potential energy lost by the falling mass. Instead, we note that the angular acceleration is constant, so that it is related to the angular speed by $\omega^2 = 2\alpha\Delta\phi$, where $\Delta\phi$ is the total angular displacement, equal to h/R, of the cylinder when the mass has fallen a distance h. Then $K = (I + mR^2)\alpha h/R$, or

$$K = mgh\,\frac{I + mR^2}{I + mR^2 + \mu_0 Q^2 R^2/4\pi l}$$

The total kinetic energy after a drop h is reduced in the same proportion as the acceleration and the "missing" energy is

$$\Delta K = -mgh\,\frac{\mu_0 Q^2 R^2/4\pi l}{I + mR^2 + \mu_0 Q^2 R^2/4\pi l}$$

[NOTE: The missing kinetic energy can readily be seen to be

$$\Delta K = -\frac{B^2}{2\mu_0}\pi R^2 l$$

which is equal to minus the magnetic-field energy stored inside the shell, as predicted by Maxwell's theory of electromagnetism. **The student is not asked to find this last expression, however**.]

Canadian Association of Physicists
2002 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. However, performance on the multiple choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the National Committee. The questions in part B of the exam have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three written problems on **separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Acceleration due to gravity	$g = 9.80$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ J·m/C^2
Permeability of free space	$\mu_o = 4\pi \times 10^{-7}$ N/A^2
Speed of sound in air	$v_s = 343$ m/s
Index of refraction of water	$n_{\mathrm{w}} = 1.33$
Energy conversion	1 eV=1.6×10^{-19} J

Part A: Multiple Choice

Question 1

A little girl is holding a helium-filled balloon on a string while riding in a closed elevator going down a very tall building at constant speed. The elevator shaft is maintained under vacuum. Suddenly the elevator cable snaps, sending the elevator into free fall. In her surprise the girl lets go of the string. She is even more surprised to see

(a) the balloon rising.

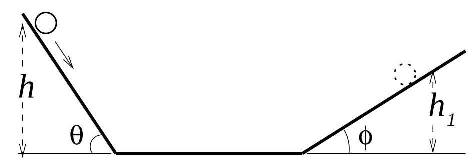
(b) the balloon floating downward.

(c) the balloon remaining stationary.

(d) the balloon bouncing slowly between the floor and the ceiling.

Question 2

A demonstration apparatus sits on a table in the lab. It consists of a metal track (shown as a thick solid line in the figure below) along which a perfectly spherical marble can roll without slipping. In one run, the marble is released from rest at a height h above the table on the left section, rolls down one side and then up the other side without slipping, briefly stopping when it has reached h_1. Assuming the table to be horizontal and neglecting air drag as well as any energy loss due to rolling,



(a) $h_1 < h$ always, because the friction that keeps the ball rolling must dissipate energy.

(b) $h_1 = h$, since total mechanical energy is conserved.

(c) $h_1 = h$ only if tilt angles ϕ and θ are equal.

(d) $h_1 = h$ only if tilt angle ϕ is larger than θ.

Question 3

In the movie *Jurassic Park*, as a jeep is being pursued by a dinosaur, one can see the animal in the jeep's side rear-view mirror on which is printed: "Objects in mirror are closer than they appear." Then,

(a) the mirror is flat.

(b) the mirror is concave.

(c) the mirror is convex.

(d) the mirror could be either convex or concave.

Question 4

A lump of sticky chewing-gum with mass m moves at speed v toward another lump of mass M at rest. They collide and stick together, both moving at speed V. Nothing else is known about the conditions under which the collision takes place. Which of the following statements is the most correct?

(a) Neither total kinetic energy nor total linear momentum can be conserved.

(b) This is an elastic collision in which both total kinetic energy and total linear momentum are conserved; the final speed is $V = v/2$.

(c) This is an inelastic collision, and in such collisions, total linear momentum is always conserved; the final speed is $V = v/4$.

(d) This is an inelastic collision in which total linear momentum is conserved, provided no external force can deliver an impulse to the system during the collision.

Question 5

According to the Archimedes principle,

(a) a body immersed in a fluid experiences an upward force equal to the weight of the fluid it displaces.
(b) a body immersed in a fluid experiences an upward force equal to the weight of the object.
(c) a body immersed in a fluid displaces an amount of fluid equal to the volume of the body.
(d) a body immersed in a liquid floats only if its density is smaller than that of the liquid.

Question 6

A guitar string of length L, with both of its ends fixed, is vibrating, producing a sound composed of a fundamental frequency and its integer multiples (harmonics). To emphasize all the even-numbered harmonics at the expense of the odd-numbered ones, the furthest distance away from either end where one should hold down the string is

(a) $L/2$. (b) $L/3$.
(c) $L/4$. (d) $L/5$.

Question 7

A sinusoidal wave is moving along a string of uniform density. If you double the frequency of the wave,

(a) the wavelength of the wave doubles.
(b) the speed of the wave doubles.
(c) the speed of the wave remains about the same.
(d) the period of the wave doubles.

Question 8

Two point-charges, each with a charge of +1 μC, lie some finite distance apart. On which of the segments of an infinite line going through the charges is there a point, a finite distance away from the charges, where the electric potential is zero, assuming that it vanishes at infinity?

(a) Betweeen the charges only.
(b) On either side outside the system.
(c) Impossible to tell without knowing the distance between the charges.
(d) Nowhere.

Question 9

Standing on the ground, you are holding with your hand a string from which is suspended a stone. The reaction force, associated—in the sense of Newton's 3$^{\text{rd}}$ law—with the force of Earth's gravity on the stone, is the force exerted by

(a) the string on your hand. (b) the stone on the string.
(c) your body on the Earth. (d) the stone on the Earth.

Question 10

At the shallow end of a swimming pool, the water is 70.0 cm deep. The diameter of the cone of light emerging from the water into the air above, emitted by a light source 10.0 cm in diameter at the bottom of the pool and measured by an observer on the edge of the pool 2.50 m away, is'

(a) 1.60 m. (b) 1.70 m.
(c) 1.75 m. (d) 1.80 m.

Question 11

Of two quantities of water and steam, both at 100°C and having the same mass, in immediate contact with the same area of your skin, which is likeliest to cause a more severe burn?

(a) The steam. (b) There is no difference.
(c) The water. (d) It depends on the steam's pressure.

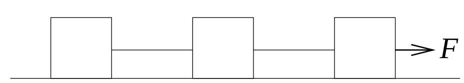
Question 12

A varying electric current running through a coiled wire induces an emf across the coil. The voltage measured by a voltmeter across the terminals of the coil

(a) is smaller or larger than the emf, depending on how fast the current changes.
(b) is larger than the emf.
(c) is equal to the emf.
(d) is smaller than the emf.

Question 13

Three blocks of identical mass are connected by strings as shown below. The whole system is accelerated to the right along a frictionless, horizontal surface by a force F. The net force acting on the middle block is



(a) zero. (b) F.
(c) $2F/3$. (d) $F/3$.

Question 14

A spaceship is moving straight toward Earth with its engine turned off. Let M and m be the masses of the Earth and of the spaceship, respectively, with R the distance from the spaceship to the centre of the Earth. As a result of the gravitational force of the Earth, in moving from position 1 to position 2, the kinetic energy of the spaceship increases by

(a) $\dfrac{GMm(R_2 - R_1)}{R_1 R_2}$. (b) $\dfrac{GMm(R_1 - R_2)}{R_1 R_2}$.
(c) $\dfrac{GMm(R_2 - R_1)}{R_1^2}$. (d) $\dfrac{GMm(R_1 - R_2)}{R_1^2 R_2^2}$.

Question 15

Two masses, M_a and M_b, attached to the end of springs a and b, respectively, undergo simple harmonic motion on a horizontal, frictionless surface. If the periods of motion are identical, and if $M_\mathrm{b} = 2M_\mathrm{a}$, the amplitudes A of oscillation of the two masses are related by

(a) $A_\mathrm{b} = A_\mathrm{a}/\sqrt{2}$. (b) $A_\mathrm{a} = A_\mathrm{b}/4$.

(c) $A_\mathrm{a} = A_\mathrm{b}$. (d) none of the above.

Question 16

Consider a circuit which contains a battery and a single resistance R. If a second resistance is added in parallel with R,

(a) the voltage across R will decrease.

(b) the current through R will increase.

(c) the total current drawn from the battery will increase.

(d) the power dissipated in R will decrease.

Question 17

Starship *Enterprise* is in a circular orbit of period 300 minutes around planet Thera at an altitude of 1000 km. A Klingon spaceship then enters another circular orbit around Thera at an altitude of 19000 km. Thera has a radius of 5000 km. The Klingon spaceship circles Thera in

(a) 600 minutes. (b) 1200 minutes.

(c) 1800 minutes. (d) 2400 minutes.

Question 18

A simplified bola consists of two (instead of the usual three) small but heavy balls connected to a common point by identical lengths of string. To launch it, you hold one of the balls overhead and swing the other ball in a horizontal circle about your wrist. You then let go of the bola which very soon starts rotating around its centre of mass, as shown below. Which one of the following statements holds true when the rotation axis changes from (1) to (2)?

(a) Angular momentum is the same, but angular speed increases.

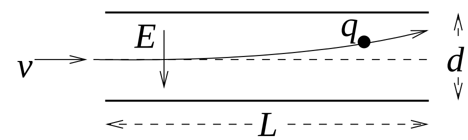
(b) Angular momentum and angular speed remain the same.

(c) Angular momentum is the same, but angular speed decreases.

(d) Angular momentum and angular speed both increase.

Question 19

In an ink-jet printer, an ink droplet of mass m is given a negative charge q by a computer-controlled charging unit, and then enters at speed v the region between two deflecting parallel plates of length L separated by distance d (see figure below). All over this region exists a downward electric field which you can assume to be uniform. Neglecting the gravitational force on the droplet, the maximum charge that it can be given so that it will not hit a plate is most closely approximated by



(a) $\dfrac{mv^2E}{dL^2}$. (b) $\dfrac{mv^2d}{EL^2}$.

(c) $\dfrac{md}{E(vL)^2}$. (d) $\dfrac{m(vL)^2}{Ed}$.

Question 20

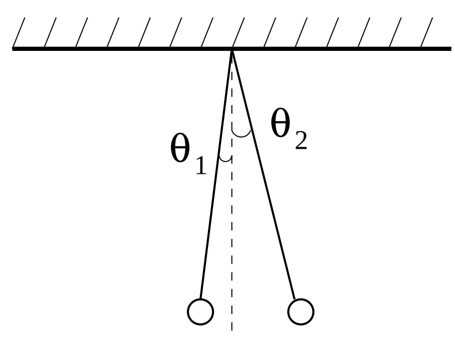
Four very long straight wires carry equal electric currents in the $+z$ direction. They intersect the x–y plane at $(x, y) = (-a, 0)$, $(0, a)$, $(a, 0)$, and $(0, -a)$. The magnetic force exerted on the wire at position $(-a, 0)$ is along

(a) $+y$. (b) $-y$.

(c) $+x$. (d) $-x$.

Question 21

Two masses, both equal to 100 g, are suspended at the ends of identical light strings of length $\ell = 1.0$ m, attached to the same point on the ceiling (see figure). At time $t = 0$, they are simultaneously released from rest, one at angle $\theta_1 = 10°$, the other at angle $\theta_2 = 20°$ from the vertical. The masses will collide



(a) at $\theta = 0.0°$, 0.50 s later.

(b) at $\theta = 5.0°$ to the right of the vertical, 0.16 s later.

(c) at $\theta = 0.0°$, 0.13 s later.

(d) at $\theta = 5.0°$ to the right of the vertical, 0.10 s later.

Question 22

Two trains are moving toward each other on two parallel tracks at the same speed with respect to the ground. When the whistle of one train blows, the frequency of the sound heard by a passenger on the other train will be

(a) greatest if the air is still.

(b) greatest if a wind blows in the same direction and at the same speed as the other train.

(c) greatest if a wind blows in the opposite direction and at the same speed as the other train.

(d) will not depend on whether there is a wind blowing.

Question 23

In an African wildlife reserve your jeep is travelling at a *constant* speed of 105 km/hr *alongside* a cheetah running at full speed. The animal, frightened by the jeep's noise, keeps at a constant distance of 60 m from the vehicle. Not having paid much attention to the surrounding landscape, you suddenly realize that your jeep is passing the very same spot where it was 60 seconds before. The speed of the cheetah is

(a) 105 km/hr. (b) 94 km/hr.

(c) 99 km/hr. (d) 82 km/hr.

Question 24

A jet-fighter pilot, wishing to experience weightlessness conditions, enters a parabolic trajectory closely approximating that of a projectile in free-fall. His aircraft must fly at a minimum speed of 400 km/hr to keep control, and he should not exceed 600 km/hr so as not to suffer a black-out when he pulls out of the trajectory at the end. There is no wind and air resistance can be compensated by the engine. In order to be weightless as long as possible, he should enter the parabola at an upward angle (with respect to the horizontal ground) of

(a) $33.7°$. (b) $41.8°$.

(c) $56.3°$. (d) $48.2°$.

Question 25

Which one of the following statements is **not** true? The phase difference between two light waves can change

(a) by reflection.

(b) by the waves travelling along paths of different lengths.

(c) by the waves travelling through media that have a different refraction index.

(d) by being transmitted through a boundary between two media with different refraction indices.

* * *

Part B

Problem 1

Passing a car on a two-lane road requires more caution as speeds increase. Apart from maintaining a safe distance between vehicles in the same lane, there should also be enough space between oncoming vehicles while the passing manœuvre takes place.

Take all cars initially, travelling at the same constant speed, to have a length 4.2 m. Suppose further that a passing manœuvre starts with the passing car at least 3.0 car lengths behind the back of the slower car, and that it ends when the driver has pulled in with the back of her car at least 3.0 car lengths ahead of the slower car (no fish-tailing!). The passing car takes 4.0 s at constant acceleration to reach its final passing speed, which it keeps thereafter. This final speed is 11 km/hr greater than the steady speed of the slower car.

(a) Calculate the total time it takes for such a passing manœuvre.

(b) Now suppose that at the instant the passing driver has reached her overtaking speed, she notices 500 m ahead (from the front of her car) a very large truck coming toward her. The speed limit for trucks on that road, which we shall assume the truck driver respects, is 90 km/hr. The slower car is driving at 100 km/hr. Otherwise, the same conditions as in part (a) apply. We assume that for minimal safety she should have completed her manœuvre at least 2.0 s before the truck and her pass each other, But she cannot abort the manoeuvre without braking unless the front of her car is at least 2.0 m behind the back of the slow car, and she cannot brake safely because another car right behind her is also trying to pass the slow car. Given that it takes her 1.6 s to react after she first sees the truck, discuss whether it is safe for her to continue, or whether she should pull back behind the slower car (if she can!). If it turns out that she is in trouble, could she continue accelerating at the same rate up to a higher speed not exceeding 120 km/hr and pass safely while still pulling in three car lengths in front of the slow car?

Problem 2

In Poe's famous horror tale, *The Pit and the Pendulum*, the hero, who finds himself strapped flat to a horizontal surface, notices an almost motionless pendulum hanging from the ceiling 12 m above him. After some time, he also realizes that the pendulum is descending very slowly toward him with, according to the story, the amplitude of the swings and the speed at the lowest point in each swing *both increasing inexorably*. What is swinging is in fact a small razor-edged crescent of steel which will eventually sweep across the man's heart.

Let's analyze this terrifying situation more closely to see if Poe's spell-binding description actually makes physical sense. To do this, we must make some assumptions which will not affect our final conclusion. The blade of mass m is suspended on the end of a massless cable of length l and is executing simple harmonic motion. Assume that the blade is also slowly lowered in very small increments of l which occur only when it goes through its lowest position in each swing, at $\theta = 0^\circ$, where θ is the angular position with respect to the vertical.

(a) Explain in one short sentence why the angular momentum of the razor at $\theta = 0^\circ$ remains constant all through the descent.

(b) Let the initial length of the cord be l_0 and the initial maximum speed of the razor be v_0. Obtain an algebraic expression for v as a function of l.

(c) Derive an algebraic expression for the kinetic energy of the pendulum at $\theta = 0^\circ$ as a function of l.

(d) Make a reasonable simplyfing assumption about the total mechanical energy. Find (always as a function of l) the maximum gravitational potential energy acquired by the pendulum in a given swing.

(e) Find the maximum angular displacement of the razor as a function of l.

(f) Discuss the accuracy of Poe's description of the descent of the pendulum.

Problem 3

One fine afternoon, as his family watches from some distance, a man walking barefoot on moist ground near a tower supporting electric transmission lines suddenly stops and collapses. His relatives call in emergency services, but these arrive too late to prevent the man's death.

The autopsy reveals that the man's heart went into ventricular fibrillation, a fatal condition if not stopped very quickly. His relatives believe that he was electrocuted by some stray electric current from the tower and sue the power company. The court orders a forensic investigation to determine if the company is responsible for the victim's death.

The investigation finds that there was indeed an electrical fault at the time the incident occurred: for about 1.0 s, an electric current of 100 A leaked into the ground from a vertical conducting rod whose rounded tip was just below ground, located 10 m away from where the man was standing. The resistivity of the moist ground was about 100 $\Omega \cdot \mathrm{m}$.

(a) Assuming that the current spread uniformly into the ground from the rounded tip, find the magnitudes of the current density, J (in A/m^2), and of the electric field, E, as a function of the distance r from the tip of the rod. If you don't know the relationship between J and E, you can derive it from Ohm's law and a bit of dimensional analysis.

(b) Calculate E and J at the victim's location.

(c) Under any suitable approximation, obtain a reasonable estimate of the potential difference between the man's feet.

(d) It is reasonable to suppose that the current went up one leg of the man, across the torso (and the heart), and down through the other leg. The commonly accepted values for the resistance of a leg is 300 Ω and that of a torso 1000 Ω in these conditions. From these data, estimate the current across the victim's torso.

(e) Using your expression for $E(r)$, obtain the potential difference, ΔV, between the tip of the rod and a point situated at distance r. Take the tip of the rod to have radius a. If the integral is a problem, you can still get the answer by noticing the similarity between $E(r)$ and the field of a well-known charge configuration.

(f) Currents ranging from 0.10 A to 1.0 A across a torso can trigger fibrillation in the heart. Could the accident have been caused by the current that leaked from the rod?

* * * *

Solutions to CAP Exam 2002

Part A

1 (c)	2 (b)	3 (c)	4 (d)	5 (a)
6 (a)	7 (c)	8 (d)	9 (d)	10 (b)
11 (a)	12 (d)	13 (d)	14 (b)	15 (d)
16 (c)	17 (d)	18 (a)	19 (b)	20 (c)
21 (a)	22 (d)	23 (d)	24 (d)	25 (d)

Part B

Problem 1

(a) It is convenient—but not essential to answer this—to work in the rest-frame of the slow car. Then we only have to deal with the relative final velocity v of the cars, and their relative acceleration a, the latter being also the acceleration of the passing car with respect to the ground. We are interested in the position of the front of the passing car with respect to the front of the slow car. Let $t_{\rm a}$ be the time it takes to reach speed v at acceleration $a = v/t_{\rm a}$. Furthermore, let $t_{\rm b}$ be the time it takes the front of the passing car to reach its final position, four car lengths ahead of the slow car, at speed v. Calling $T = t_{\rm a} + t_{\rm b}$ the total time it takes to pass the slow car, we can write for the position x of the front of the passing car:

$$x(T) = -4.0L + \frac{1}{2}a\,t_{\rm a}^2 + v\,t_{\rm b} = 4.0L$$

where $L = 4.2$ m is the length of a car. Eliminating $a = v/t_{\rm a}$ and solving for $t_{\rm b}$ yields

$$t_{\rm b} = \frac{8.0L}{v} - \frac{1}{2}\,t_{\rm a}$$

so that

$$T = \frac{8.0L}{v} + \frac{1}{2}\,t_{\rm a}$$

With $v = 11$ km/hr (3.06 m/s) and $t_{\rm a} = 4.0$ s, we obtain $T = 13$ s.

(b) The distance d that includes the distance the passing car covers *after* its driver notices the truck, plus the safety margin at the end, is

$$d = (v_{\rm car} + v_{\rm truck})(t_{\rm b} + 2.0\ {\rm s}) = (v_{\rm car} + v_{\rm truck})\left(\frac{8.0L}{v} - \frac{1}{2}\,t_{\rm a} + 2.0\ {\rm s}\right)$$

where the 2.0 s term accounts for the safety margin at the end of the manœuvre. With $v_{\rm car} = 111$ km/hr and (hopefully!) $v_{\rm truck} = 90$ km/hr (25 m/s) at the most, we obtain $d = 614$ m, which is greater than the initial distance of the truck. A collision seems inevitable if no action is taken.

Does the driver have time to pull in behind the slow car, given that braking would risk a collision with the car right behind her and that she must keep at a distance of at least 2.0 m behind the slow car if she pulls in? Well, when she sees the truck (at time $t_{\rm a} = 4.0$ s, she is at a distance $3.0L - vt_{\rm a}/2 = 6.5$ m behind the slow car. But during the 1.6 s it takes her to react, she has covered a further 4.9 m relative to the front car, so she is only about 1.6 m behind and would have to brake to keep at a safe distance from the car in front.

Her other alternative is to keep accelerating at the same rate. Now $t_{\rm a} = 4.0v'/v$, where v' is her new final relative speed which could be as large as 20 km/hr , giving $t_{\rm a} = 4.0 \times 20/11 = 7.3$ s. Taking into account her reaction time during which she travels at 111 km/hr, the distance she covers over the whole passing manœuvre, plus the safety margin at the end, is now

$$d = (v'_{\rm c} + v_i + v_{\rm t})\left(\frac{8.0L}{v'} - \frac{1}{2}t_{\rm a} + 2.0\ {\rm s}\right) + (v_{\rm c} + v_{\rm t}) \times 1.6\ {\rm s}$$

where v_i is the speed of the slow car. Inserting numbers gives $d = 351$ m, comfortably below the 500 m initial distance between her and the truck.

Setting d to be 500 m, one can actually solve the above equation (but the students are not expected to do this!) for the minimum v' that allows the driver to pass safely. It is a cubic whose solution is about 16 km/hr, so that the driver could accelerate to 116 km/hr only and still make it.

Problem 2

(a) At $\theta = 0°$, the force of gravity applies no torque on the razor, and neither does any other force, so that angular momentum there must be conserved throughout the descent.

(b) Since angular momentum is conserved, $mv_0 l_0 = mvl$, where v is the maximum speed of the razor when the cable has length l. Then

$$v = \frac{l_0}{l}\,v_0$$

(c) The maximum kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mv_0^2\left(\frac{l_0}{l}\right)^2$$

(d) Total mechanical energy being conserved, $K = U$, so that

$$U = \frac{1}{2}mv^2 = \frac{1}{2}mv_0^2\left(\frac{l_0}{l}\right)^2$$

(e) Let h be the maximum height reached by the razor measured from the lowest point of a given swing. Then $h = l(1 - \cos\theta_{\max})$, where $\theta_{\max}$ is the maximum angular displacement of the razor from the vertical. At that angle, the razor has its maximum potential energy, $U = mgl(1 - \cos\theta_{\max})$, relative to its potential energy at the lowest point of a swing before any change in l. Equating this to the result of part (d) and solving for $\theta_{\max}$ yields

$$\theta_{\max} = \cos^{-1}\left(1 - \frac{(v_0 l_0)^2}{2gl^3}\right)$$

(f) The results of parts (b) and (e) make it clear that Poe's description is inaccurate: far from increasing, as he claims, the maximum speed and maximum displacement (and therefore the amplitude) of the razor decrease with l.

Problem 3

(a) The current spreads out uniformly from the tip over a hemisphere of area $2\pi r^2$. With current $I = 100$ A, this gives a current density

$$J = \frac{I}{2\pi r^2} = \frac{15.9}{r^2}\ \text{A/m}^2$$

The current density is related to the electric field magnitude, E, by $E = \rho J$, where ρ is the resistivity of the soil. This expression is easily obtained from Ohm's law: $V = RI$, where V is the potential difference across a fairly small distance l in the ground, and R is the resistance of the ground over this distance. Then Ohm's law can be rewritten $El = (\rho l/A)(JA)$, with A the area across which the current flows. Dimensional analysis would yield the same answer.

With the numbers given,

$$E = \frac{\rho I}{2\pi r^2} = \frac{15.9}{r^2}$$

(b) With $r = 10$ m, $J = 0.16$ A/m^2, and $E = 16$ V/m (or N/C).

(c) Assume that one foot was at most $d = 0.5$ m further away from the rod than the other foot. Taking E to be roughly constant over this distance, we obtain a fairly good estimate for the potential difference: $V \approx Ed \approx 8$ V.

(d) The equivalent resistance of the body would be that of the two legs and the torso in a series combination, therefore 1600 Ω. From Ohm's law, the current flowing through the torso would be

$$i \approx 8/1600 = 0.005\ \text{A}$$

or 5 mA.

(e) We have for the potential difference between r and the tip:

$$\Delta V = V(r) - V(a) = -\int_a^r \left(\frac{\rho I}{2\pi r^2}\right) dr = \frac{\rho I}{2\pi}\left(\frac{1}{r} - \frac{1}{a}\right)$$

We could also have noted that E has the same $1/r^2$ distance fall-off as the well-known Coulomb field, so that the potential should have $1/r$ fall-off, just like the Coulomb potential. The term $1/a$ simply ensures that $\Delta V = 0$ when $r = a$.

[Incidentally, if we use this expression to check our answer to part (c), with one foot at $r = 10.25$ m, and the other at $r = 9.75$ m, we get the same potential difference to the accuracy used.]

(f) The estimated current through the torso is twenty times smaller than the threshold current for fibrillation. Even with our assumptions, we can safely discard any possibility that it caused the man's death.

CAP High School Prize Exam

11 April 2003
9:00 – 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name: ______________________ Given Name: ______________

Home Address: ____

____

______________________ Postal Code: ______________

Telephone: (   ) ________________ E-mail: ______________________

School: ________________________________ Grade: ______________

Physics Teacher: ______________________________

Date of Birth: ______________________________ Sex: ______________

Citizenship: __________________________

For how many years have you studied in a Canadian school? ____

Would you prefer further correspondence in French or English? ____

Sponsored by:

Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists
2003 Prize Exam

This is a three hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. Performance on the multiple-choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. The questions in part B have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.

Non-programmable calculators may be used. Please be very careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Radius of Earth	$R_E = 6.38 \times 10^3$ km
Mass of Earth	$M_E = 6.0 \times 10^{24}$ kg
Mass of Sun	$M_S = 2.0 \times 10^{30}$ kg
Radius of Earth's orbit	$R_{ES} = 1.50 \times 10^8$ km
Acceleration due to gravity	$g = 9.80$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ J·m/C^2
Speed of sound in air	$v_s = 343$ m/s
Energy conversion	1 eV = 1.6×10^{-19} J

Part A: Multiple Choice

Question 1

A parallel-plate capacitor holds charge q and is not connected to anything. The distance between the plates is now increased. The electrical energy stored on the capacitor

(a) decreases;
(b) remains the same;
(c) increases;
(d) can do any of the above, depending on how the capacitance changes.

Question 2

When a mechanical or electromagnetic wave goes from one medium to another, it undergoes a change in

(a) amplitude only; (b) both speed and wavelength;
(c) speed only; (d) wavelength only.

Question 3

Two identical rooms in a perfectly insulated house are connected by an open doorway. The temperature in the two rooms are maintained at different values. The room which contains more air molecules is

(a) the one with the higher temperature;
(b) the one with the lower temperature;
(c) the one with the higher pressure;
(d) neither, since both have the same volume.

Question 4

Three airplanes, A, B and C, each release an object from the same altitude and with the same initial speed v_0 with respect to the ground.. At the moment their object is released, A is flying horizontally, B is flying upward at an angle θ with respect to the horizontal, and C is flying at the same angle θ as B but downward with respect to the horizontal. Assuming the ground to be horizontal and neglecting any aerodynamical effect, the speeds v at which the three objects will hit the ground satisfy

(a) $v_A = v_B < v_C$; (b) $v_A > v_B = v_C$;
(c) $v_A < v_B < v_C$; (d) $v_A = v_B = v_C$.

Question 5

Two identical conducting spheres, A and B, carry equal electric charge. They are separated by a distance much larger than their diameter and exert an electrostatic force F on each other. A third identical conducting sphere C is initially uncharged and far away from A and B. Sphere C is then brought briefly into contact with sphere A, then with sphere B, and finally removed far away. The electrostatic force between A and B is now

(a) $3F/8$; (b) $F/2$;
(c) $F/4$; (d) $F/16$.

Question 6

On the ground, the Earth exerts a force F_0 on an astronaut. The force that the Earth exerts on this astronaut inside the Space Shuttle in low Earth orbit, 300 km above the ground, is

(a) a little less than F_0;
(b) a little more than F_0;
(c) exactly F_0;
(d) zero, since the astronaut is weightless when in orbit.

Question 7

A person is swinging a ball at the end of a string of length ℓ with constant speed v. The work done by the tension T in the string over one revolution is

(a) 0; (b) $mv^2/2$;
(c) $2\pi\ell T$; (d) undetermined by the information given.

Question 8

The pressure exerted by a gas on the walls of the vessel that contains it is due to the

(a) change in kinetic energy of the gas molecules as they strike the walls;

(b) collisions between the gas molecules;

(c) repulsive force between the gas molecules;

(d) change in momentum of the gas molecules as they strike the walls.

Question 9

A Martian creature similar to an Earth frog jumps with an initial speed v_0 and attains the range R over horizontal ground. The maximum possible height reached by the creature, neglecting friction in the tenuous Martian air, is (θ being the launch angle):

(a) $\dfrac{R}{4}\ \tan\theta$;

(b) $\dfrac{R}{4}\ \sin\theta$;

(c) $\dfrac{R}{2}\ \tan\theta$;

(d) undetermined because of missing data.

Question 10

The work done to accelerate a truck on a horizontal road from rest to speed v

(a) is less than that required to accelerate it from v to $2v$;

(b) is equal than that required to accelerate it from v to $2v$;

(c) is more than that required to accelerate it from v to $2v$;

(d) may be any one of the above since it depends on the force acting on the truck and the distance over which it acts.

Question 11

If the Earth did not rotate on its axis, the magnitude of the gravitational acceleration at the Equator would be about

(a) 0.003% larger; (b) 0.3% larger;

(c) 0.3% smaller; (d) 0.003% smaller.

Question 12

You want to apply a force on a box so that it moves with constant speed across a horizontal floor. The coefficient of kinetic friction between the box and the floor is $\mu_{\rm k}$. Of the four following cases, the force you apply on the box will be smallest when you

(a) push on it with a force applied at an angle $0 < \theta < 90°$ downward from the horizontal;

(b) pull on it with a force applied at the same angle θ as in (a), upward from the horizontal;

(c) do either (a) or (b) since the applied force is the same;

(d) push or pull with a force applied horizontally.

Question 13

An electric current runs counterclockwise in a rectangular loop around the outside edge of this page, which lies flat on your table. A uniform magnetic field is then turned on, directed parallel to the page from top to bottom. The magnetic force on the page will cause

(a) the left edge to lift up; (b) the right edge to lift up;

(c) the top edge to lift up; (d) the bottom edge to lift up.

Question 14

A car has a maximum acceleration of $3.0\,$m/s^2. Its maximum acceleration while towing another car twice its mass, assuming no skidding, would be

(a) $3.0\,$m/s^2 ; (b) $1.5\,$m/s^2 ;

(c) $1.0\,$m/s^2 ; (d) $0.5\,$m/s^2 .

Question 15

Two satellites of equal mass, A and B, are in concentric orbits around the Earth. The distance of B from Earth's centre is twice that of A. The ratio of the centripetal force acting on B to that acting on A is

(a) 1 ; (b) $\sqrt{1/2}$;

(c) 1/2 ; (d) 1/4 .

Question 16

A proton sits at coordinates $(x, y) = (0, 0)$, and an electron at (d, h), where $d \gg h$. At time $t = 0$, a uniform electric field $\boldsymbol{E}$ of unknown magnitude but pointing in the positive y direction is turned on. Assuming that d is large enough that the proton-electron interaction is negligible, the y coordinates of the two particles will be equal (at equal time)

(a) at about $y = d/2000$;

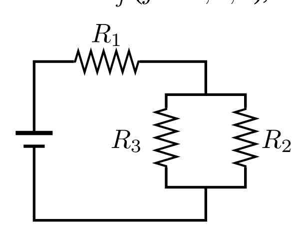
(b) at an undetermined value since E is unknown;

(c) at about $y = d/43$;

(d) nowhere: they move in opposite directions.

Question 17

In the circuit below we increase the resistance R_2. If I_j is the current through resistor R$_j$ ($j = 1, 2, 3$), then



(a) I_1 and I_2 both increase;

(b) I_1 decreases and I_2 increases;

(c) I_1 and I_2 both decrease;

(d) I_1 increases and I_2 decreases.

Question 18

Two carts, A and B, are placed on an air track. They are made of the same material and *look* identical. B is given a constant speed and collides elastically with A at rest. After the collision, both carts move in the same direction. One concludes that

(a) A is hollow;

(b) B is hollow;

(c) A and B are identical;

(d) any of the first three answers is possible.

Question 19

The smallest length scale known in physics is the Planck length. It is an important ingredient in some currrent cosmological theories. Which of the following expressions could represent this Planck length? (see Data table.)

(a) $\sqrt{e^2/hc}$; (b) $\sqrt{hc/G}$;
(c) $\sqrt{Ghc}$; (d) $\sqrt{hG/c^3}$.

Question 20

The Webb space telescope, scheduled to be launched in 2010, will have a mirror $6\,$m in diameter. Compared with the Hubble space telescope, whose mirror has a $2.4\,$m diameter, it will be able to resolve objects whose angular separation is about

(a) 2.5 times smaller;

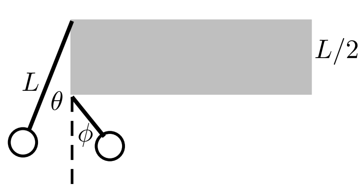
(b) 5 times smaller;

(c) an order of magnitude smaller;

(d) the same: the larger mirror only increases the amount of light gathered.

Question 21

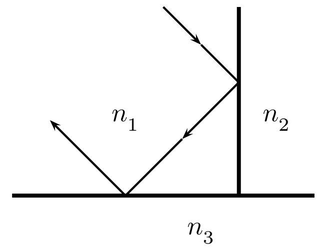
A simple pendulum of length L is suspended from the top of a flat beam of thickness $L/2$. The bob is pulled away from the beam so that it makes an angle $\theta < 30°$ with the vertical, as shown in the figure. It is then released from rest. If ϕ is the maximum angular deflection to the right, then



(a) $\phi = \theta$; (b) $\phi < \theta$;
(c) $\theta < \phi < 2\theta$; (d) $\phi \geq 2\theta$.

Question 22

In the diagram below, light is incident on the interface between media 1 and 2 at exactly the critical angle, and is totally reflected. The light is then also totally reflected at the interface between media 1 and 3, after which it travels in a direction opposite to its initial direction. The two interfaces are perpendicular. The media must have a refraction index n such that



(a) $n_1 < n_2 < n_3$; (b) $n_1^2 - n_3^2 \geq n_2^2$;
(c) $n_1^2 - n_2^2 \geq n_3^2$; (d) $n_1^2 + n_2^2 \geq n_3^2$.

Question 23

For the sake of science a physicist jumps attached to the end of a bungee cord, carrying sound measuring equipment. As he swings up and down vertically with a period of $6.0\,$s, he monitors the frequency of a sound source on the ground directly below him, and observes a difference of $84\,$Hz between the maximum and minimum frequency of the source. If the source emits at a constant $1370\,$Hz, and assuming no significant attenuation of his oscillations over the duration of the measurements, the amplitude of his oscillations is closest to

(a) $10\,$m; (b) $20\,$m;
(c) $32\,$m; (d) $15\,$m.

Question 24

A person pulls a box along the ground at constant speed. Considering the Earth and the box together as a system, which of the following is true about the net force F exerted by the person on this system and the work W she does on it?

(a) $F = 0$ and $W = 0$; (b) $F \neq 0$ and $W = 0$;
(c) $F = 0$ and $W \neq 0$; (d) $F \neq 0$ and $W \neq 0$.

Question 25

A magnet moves inside a coil. Which of the following factors can affect the emf induced in the coil?

I. the speed at which the magnet moves

II. the strength of the magnet

III. the number of turns in the coil

(a) I only; (b) I and II only;
(c) II and III only; (d) I, II and III.

* * *

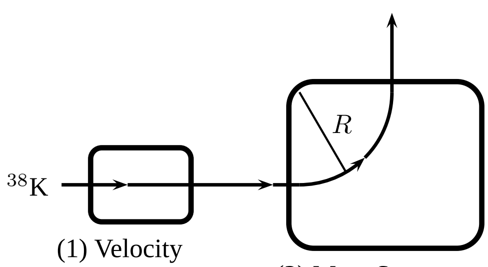
Part B

Problem 1

At TRIUMF, a large experimental particle and nuclear physics research facility on the campus of the University of British Columbia, one major programme involves the production of intense beams of unstable isotopes of alkali atoms (potassium K, rubidium Rb, francium Fr). These have the advantage that since their valence shell contains only one electron, their closed shell structure when they are ionised simplifies calculations.

Many isotopes are produced when bombarding a calcium oxide target with 0.5 GeV protons from the TRIUMF accelerator. Until recently, the desired isotope was selected by means of the TRIUMF Isotope Separator On-Line (TISOL)—now decommissioned and replaced by a combined separator/accelerator called ISAC—and sent as a low-speed beam to experimental areas.

You are asked to design a (much) simplified version of TISOL. More specifically, you want to select ^{38}K ions whose energy is 20 keV. ^{38}K has a mass of 6.3×10^{-26} kg. Separation should proceed in two steps, as illustrated below.



The figure shows the desired path of a 20 keV ^{38}K ion through the system. This path is to be achieved by means of suitable *uniform* time-independent electromagnetic fields. Interactions between ions can be neglected here.

(a) In the first step, out of all ions (^{38}K or not) entering the velocity selector from the left, only those that have a speed corresponding to a 20 keV ^{38}K ion should be undeflected. Suggest a field configuration that can do this, draw a sketch showing the direction of the field(s), and derive as much information as you can about the magnitude of the field(s).

(b) In the second step, only ^{38}K ions should be deflected so that the radius R of their trajectory is 2.1 m. Again, suggest a suitable field configuration for this, draw a sketch showing the direction of the field(s), and derive as much information as you can about the magnitude of the field(s).

Problem 2

Tides are mainly caused by the gradient (or variation) of the gravitational force of the Moon and of the Sun across the Earth's diameter. Large water masses, such as Earth's oceans, bulge along the direction of the gradient and are pinched in the perpendicular direction. As the Earth rotates, the bulges (high tide regions) move across the surface of the Earth.

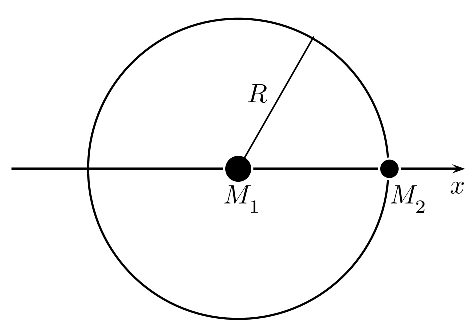
Now the Bay of Fundy, between New Brunswick and Nova Scotia, is reputed to have the highest tides in the world. Their amplitude is only about a metre at the mouth, or entrance, of the bay, whereas at the other end, 260 km away, the amplitude reaches up to 16 m. You may assume that the relevant speed of water waves in the bay (for *very long* wavelengths and shallow enough depth) is about 25 m/s. The bay is narrow compared to its length.

Using the above data, investigate whether the unusually high tides could be the result of a resonance excited by the Moon in the bay. Assume that the depth of the bay is uniform, and neglect the influence of the Sun. Hint: calculate the period of the water oscillations in the bay.

Problem 3

The next-generation large space telescope is scheduled for launch in 2010. It will be put in orbit around the Sun, in a special zone where its distance relative to the Earth and to the Sun can remain constant. The location of such zones was first calculated in the XVIII$^{\mathrm{th}}$ century by the French-Italian mathematician Joseph-Louis Lagrange.

Even though it relies on approximations, Lagrange's full solution is fairly involved, but you should still be able to make a good qualitative guess at a partial solution. Consider two point masses, M_1 and M_2, referring respectively to the Sun and the Earth. Both orbit around their common centre of mass at angular velocity ω and with a period of one year. These orbits are *circular* to a good approximation, and the distance R between M_1 and M_2 is constant. Since $M_1 \gg M_2$, the motion of M_1 is not detectable at the scale of the figure below and can be neglected.



We wish to find where, on the line that joins M_1 and M_2, an object of mass m can sit so that it also orbits the centre of mass (which we can take to be at M_1's position) with the *same*, constant, angular velocity ω. We can also safely assume that m is so small that it does not influence the motion of M_1 and M_2.

(a) Write down the equation that must be satisfied by the forces acting on the orbiting m in terms of ω, M_1, M_2, R and x, where x is the distance between m and M_1.

(b) Show that in the limit $M_2 \ll M_1$, this condition can be written

$$u^3 - 1 \; = \; \pm \frac{\alpha\, u^2}{(1 \pm u)^2}$$

where $u \equiv x/R$, and $\alpha \equiv M_2/M_1$.

(c) Do not attempt to solve this algebraic equation. Instead, use physical arguments to find how many solutions there are for x and where roughly the Lagrange zones are positioned on the x axis. Provide a qualitative sketch based on the above figure, and explain your reasoning.

(d) The Webb space telescope (as it is called) will operate at a temperature of about 35 K and, therefore, it should be shielded from heat sources at all times, while having as unobstructed a view of the sky as possible. Discuss which (if any) of your solutions is most suitable for the Webb telescope.

(e) If there are other Lagrange zones off the x axis, they cannot be found from the equation in (b), but what would be their minimum number? Justify your answer.

Hint: To a good approximation Kepler's Third Law for the system is $\omega^2 R^3 \; = \; GM_1$.

* * * *

Solutions to 2003 CAP High School Exam

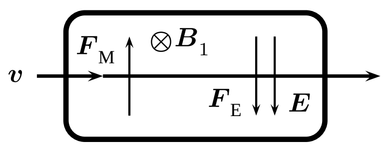
Part A

1 (c)	2 (b)	3 (b)	4 (d)	5 (a)
6 (a)	7 (a)	8 (d)	9 (a)	10 (a)
11 (b)	12 (b)	13 (c)	14 (c)	15 (d)
16 (a)	17 (c)	18 (a)	19 (d)	20 (a)
21 (c)	22 (c)	23 (a)	24 (c)	25 (d)

Part B

Problem 1

(a) As shown in the figure below, we can use a uniform electric field, $\boldsymbol{E}$, **directed downward in the plane of the page**, and a uniform magnetic field, $\boldsymbol{B}_1$, **directed into the page**. ($\boldsymbol{E}$ directed upward and $\boldsymbol{B}_1$ out of the page will also work.)



(1) Velocity Selector

Since the alkali ions all have positive charge e, they experience

- an electric force $\boldsymbol{F}_E = e\boldsymbol{E}$, directed downward in the plane of the page;
- a magnetic force $\boldsymbol{F}_M = e\boldsymbol{v}\times\boldsymbol{B}_1$, where $\boldsymbol{v}$ is the velocity of an ion. From the right-hand rule for vector products, it is directed upward in the plane of the page; for $\boldsymbol{v}$ directed to the right, the magnitude of $\boldsymbol{F}_M$ is simply evB.

For a given velocity $\boldsymbol{v}_0$, there exist magnitudes of $\boldsymbol{E}$ and $\boldsymbol{B}_1$ such that the magnetic and electric forces balance each other. This condition is achieved when $\boldsymbol{F}_E = -\boldsymbol{F}_M$, or $eE = ev_0B_1$. Then all ions which have speed

$$v_0 = \frac{E}{B}$$

when they enter the velocity selector will not be deflected. Ions with other speeds will be deflected and can be extracted from the initial beam. Only the ratio E/B is determined by the condition:

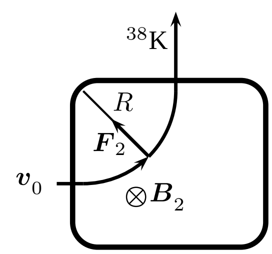
$$\frac{E}{B} = \sqrt{\frac{2K_0}{m_0}}$$

where m_0 is the mass of a ^{38}K ion and $K = m_0v_0^2/2$. Then

$$\frac{E}{B} = \sqrt{\frac{2(2.0\times10^4\,\text{eV})(1.6\times10^{-19}\,\text{C})}{6.3\times10^{-26}\,\text{kg}}} = \underline{3.2\times10^5\,\text{m/s}}$$

Note that this value is much smaller than the speed of light, consistent with our ignoring relativistic effects.

(b) In this case, the uniform magnetic field $\boldsymbol{B}_2$ **directed into the page**, as shown in the figure below, exerts a centripetal force $\boldsymbol{F}_2$ on incoming ions, giving them a circular trajectory of radius R. It does not change their energy (and speed) since magnetic fields do no work. The direction of deflection of the ions is determined from the right-hand rule applied to $\boldsymbol{F}_2 = e\boldsymbol{v}_0\times\boldsymbol{B}_2$.



(2) Mass Separator

Since $\boldsymbol{v}_0$ and $\boldsymbol{B}_2$ are perpendicular, we have $F_2 = ev_0B_2$. Using the general expression for a centripetal force, $F_2 = mv_0^2/R$, where m is the mass of an ion, we obtain

$$R = \frac{mv_0}{eB_2}$$

All incoming ions having the same speed, to each ionic mass in the beam will correspond one value of R in a given magnetic field. If only ^{38}K ions are to have a trajectory with radius of curvature R_0, we should use a field

$$B_2 = \frac{mv_0}{eR_0} = \frac{1}{R_0}\sqrt{\frac{2m_0K_0}{e^2}}$$

With the data supplied, we have

$$B_2 = \frac{1}{R_0}\sqrt{\frac{2K_0(\text{eV})}{e/m_0}} \qquad K_0 \text{ in eV!}$$

$$= \frac{1}{2.1\,\text{m}}\sqrt{\frac{2(2.0\times10^4\,\text{eV})}{(1.6\times10^{-19}\,\text{C})/(6.3\times10^{-26}\,\text{kg})}}$$

$$= \underline{0.060\,\text{T}}$$

Problem 2

If the high tides in the Bay of Fundy are caused by a resonance mechanism, it must be because the tidal forces from the Moon driving seawater in and out of the bay are exciting one of its standing-wave modes. The rise and fall of the tide can be modelled by a wave with extremely long wavelength propagating on the water. The boundary conditions at the mouth and the far end of the bay mimic those of a standing wave on a string with on end fixed (mouth), where the vertical displacement is minimum, and the other free (far end), where displacement is maximum. This mode would have a wavelength $\lambda = 4L$, where L is the length of the string or, here, the length of the bay: $L = 260\,\text{km}$.

With $v = 25\,\text{m/s}$ the speed of the waves, their period is

$$\begin{aligned} T &= \frac{\lambda}{v} = \frac{4L}{v} \\ &= \frac{4(2.60 \times 10^5\,\text{m})}{25\,\text{m/s}} = 4.16 \times 10^4\,\text{s} \\ &= \underline{11.6\text{ hours}} \end{aligned}$$

Now we expect the period of the tides to be about 12.4 hours. Indeed, if the Moon had a fixed position with respect to the Earth, the two tidal bulges (high tides) on Earth would move in the direction opposite the Earth's rotation with one passing through a given position every 12 hours. But the Moon moves in its own orbit *with* the Earth's rotation with an average period of about 29 days, ie. 6.2° in the sky every 12 hours or 720 minutes. Therefore, it takes 25 minutes for the Moon to cover that angular distance, and this increases the tidal period to 12.4 hours.

We have found that the period with which the water sloshes back and forth in the bay due to the Moon's tidal force is comparable to the period of the fundamental resonance mode for our admittedly crude model of the Bay of Fundy. Given the approximations involved, this is quite a good match, and it is likely that this resonance mechanism can explain why the tides are amplified.

Problem 3

(a) We are looking for a configuration where the telescope, the Earth and the Sun maintain the same *relative* positions in a coordinate system rotating at constant angular velocity ω. In other words, in that rotating system, the telescope has the following net acceleration:

$$a = \omega^2 x - \left[\frac{GM_1}{x^2} \pm \frac{GM_2}{(R \pm x)^2}\right]$$

where the + sign is to be used when $x > R$ and the $-$ sign when $x < R$. (Technical point: since we are working in the *non-inertial* frame rotating with angular velocity ω, an extra pseudo-acceleration (Coriolis) term dependent on $\mathrm{d}x/\mathrm{d}t$ should also be present. With our assumption of a circular orbit, however, $\mathrm{d}x/\mathrm{d}t = 0$, and the Coriolis term does not contribute.)

The condition that a vanishes then reads

$$\underline{\omega^2 x = \frac{GM_1}{x^2} \pm \frac{GM_2}{(R \pm x)^2}}$$

(b) Using Kepler's Third Law, $GM_1 = \omega^2 R^3$, and introducing the parameter $\alpha \equiv M_2/M_1$, we can eliminate ω^2 and M_1:

$$x = \frac{R^3}{x^2} \pm \frac{\alpha\, R^3}{(R \pm x)^2}$$

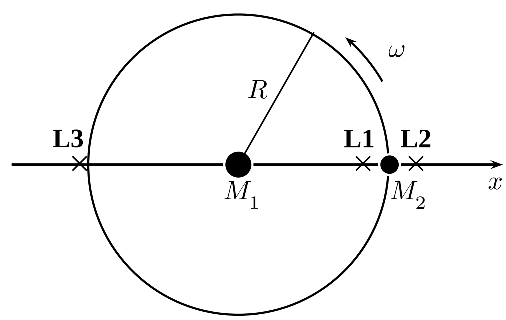
Multiplying by x^2/R^3 and rearranging yields, with $u \equiv x/R$:

$$\underline{u^3 - 1 = \pm \frac{\alpha\, u^2}{(1 \pm u)^2}}$$

(c) From the result in (a), there should be a point L1 on the axis between the Earth and the Sun where the condition that the centrifugal acceleration $\omega^2 x_1$ equals the net gravitational acceleration (which is smaller than that due to the Sun) can be satisfied for a value of ω equal to the Earth's, so that $x_1 < R$. Since the mass of the Earth is over 300 000 times smaller than the Sun's, L1 should not be very far from Earth. It is actually 1.5 million km away, one hundred times closer than the Sun.

Conversely, the same condition will be satisfied at a point L2 a bit further than the Earth's orbit, where this time the gravitational attractions of Sun and Earth add, so that $x_2 > R$. L2 should be (and is) at about the same distance from the Earth's orbit as L1.

Finally, at a point L3 on the other side of Sun, the condition is also satisfied where the gravitational acceleration due to the Sun's and the Earth's gravity reinforcing each other equals $\omega^2 x_3$. Since Earth is much further from L3 than from L2, L3 should be much closer to R than L2. It is a good place to hide from Earth.



(d) The point labelled L2 in the figure will be suitable for the telescope. A heat screen attached to the telescope will shield it from heat radiated by both Earth and Sun since they are always aligned as viewed from L2, leaving the rest of the sky unobstructed for observation.

(e) Since the system shown in the figure is symmetric about the x axis, Lagrange points off the axis must come in pairs. So the minimum number is two. In fact, Lagrange found one pair of solutions off the axis, for a total of five.

CAP High School Prize Exam

8 April 2004
9:00 – 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name: ______________________ Given Name: ______________

Home Address: ____

____

______________________ Postal Code: ______________

Telephone: (   ) ______________ E-mail: ______________________

School: ______________________________ Grade: ______________

Physics Teacher: ______________________________

Date of Birth: ______________________________ Sex: ______________

Citizenship: ______________________

For how many years have you studied in a Canadian school? ____

Would you prefer further correspondence in French or English? ____

Sponsored by:

Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists
2004 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. Performance on the multiple-choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. The questions in part B have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.

Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Acceleration due to gravity	$g = 9.80$ m/s^2
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ J·m/C^2
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K

Part A: Multiple Choice

Question 1

You are standing upright in a room in front of a vertical mirror. In this mirror, you can see from your position only the upper two-thirds of your body. You wish to see the entire length of your body reflected in the mirror. Which combination of the following three courses of action will achieve this?

(I) Move away from the mirror;

(II) move toward the the mirror;

(III) use a mirror whose height will allow you to see your whole image when you are at your initial position.

(a) (I) only; (b) (II) only;
(c) (III) only; (d) either (I) or (III).

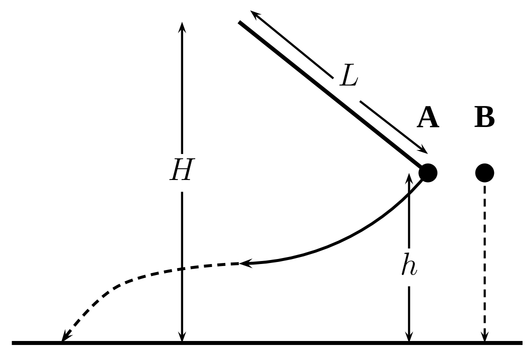
Question 2

A 5 kg cart collides on a horizontal surface with a 20 kg cart. Which cart experiences the smaller force because of the collision?

(a) The 5 kg cart; (b) the forces are equal;
(c) the 20 kg cart; (d) it depends if the collision is elastic.

Question 3

Two monkeys with the same mass stand on a branch at height h above the horizontal jungle floor. Monkey A steps off the branch holding the end of an inextensible rope of length L whose other end is tied to another branch at height H, lets go at the bottom of the swing, and falls freely to the floor, as shown below. Monkey B steps off and falls straight downward. Then, neglecting air resistance but not the tension in the rope, the total work W done on each monkey and the speed v with which each hits the floor are as follows:



(a) $W_A < W_B$, $v_A < v_B$; (b) $W_A = W_B$, $v_A < v_B$;
(c) $W_A = W_B$, $v_A = v_B$; (d) $W_A < W_B$, $v_A = v_B$.

Question 4

A person, standing on a train that is accelerating forward at 3.3 m/s^2, throws a ball vertically upward. Neglecting air resistance, the magnitude of the ball's acceleration relative to the train is

(a) 9.8 m/s^2; (b) 10.3 m/s^2;
(c) 7.0 m/s^2; (d) 13.1 m/s^2.

Question 5

I must cross a river in the shortest possible time. Water flows downstream at a constant 5.0 m/s between the two parallel shores. Taking the direction of the flow as reference, if my boat has a maximum speed of 10 m/s, it should head at

(a) 90°; (b) 120°;
(c) 150°; (d) 27°.

Question 6

A hoop and a solid cylinder have the same mass and radius. They both roll, without slipping, on a horizontal surface. If their kinetic energies are equal,

(a) the hoop has a greater translational speed than the cylinder;

(b) the cylinder has a greater translational speed than the hoop;

(c) the hoop and the cylinder have the same translational speed;

(d) the hoop has a greater rotational speed than the cylinder.

Question 7

A string clamped at both ends is vibrating. At the moment the string looks flat, the instantaneous transverse velocity of points along the string, excluding its end-points, must be

(a) zero everywhere.

(b) dependent on the location along the string.

(c) not zero anywhere.

(d) non-zero and in the same direction everywhere.

Question 8

In vacuum, a potential difference V is maintained between points a distance d apart. The corresponding electric field $\boldsymbol{E}$ accelerates an electron from rest to a speed v over that distance. Which one of the following statements is true?

(a) E does not depend on d; (b) E depends on V, not on d;

(c) E depends only on d: (d) v depends on V, not on d.

Question 9

Light waves propagate along the length of a pipe containing a perfect vacuum and whose end caps are perfect reflectors. A standing-wave pattern is formed with frequency f and wavelength λ. When a gas, with index of refraction n, is introduced into the pipe, which of the following changes occurs?

(a) λ increases; (b) f increases;

(c) λ decreases; (d) f decreases.

Question 10

Students claim that in a lab experiment they witnessed a head-on *elastic* collision between two balls on a horizontal surface which resulted in the balls being both at rest. No external horizontal force was acting at any time on the masses. Which of these comments is the most appropriate about this process?

(a) Initial speeds and masses must have been different;

(b) initial speeds and masses must have been identical;

(c) initial speeds, but not necessarily masses, were identical;

(d) the process cannot have occurred as claimed.

Question 11

The potential across a $3\,\mu$F capacitor is $12\,$V when it is not connected to anything. It is then connected in parallel with an uncharged $6\,\mu$F capacitor. At equilibrium, the charge q on the $3\,\mu$F capacitor and the potential difference V across it are

(a) $q = 12\,\mu$C, $V = 4\,$V; (b) $q = 24\,\mu$C, $V = 8\,$V;

(c) $q = 36\,\mu$C, $V = 12\,$V; (d) $q = 12\,\mu$C, $V = 6\,$V;

Question 12

In subatomic physics, one often associates a characteristic wavelength λ with a particle of mass m. If $\hbar = h/2\pi$ (h being Planck's constant) and c the speed of light, which of the following expressions is most likely to be the correct one?

(a) $\lambda = \hbar c/m$; (b) $\lambda = \hbar/mc^2$;

(c) $\lambda = \hbar m/c$; (d) $\lambda = \hbar/mc$;

Question 13

At equilibrium, the electric field at a point on a closed conducting surface, whether the surface is charged or neutral, can never be

(a) tangent to the surface;

(b) perpendicular to the surface;

(c) zero;

(d) directed inward since it must vanish inside.

Question 14

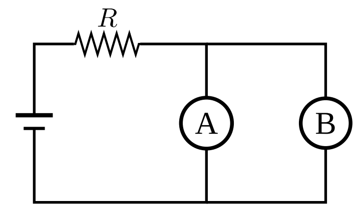
A uniform cubical box of mass M rests on a horizontal floor with one edge against a small obstruction fixed to the floor. Can a horizontal force of magnitude F applied on the box at the centre of the side opposite the obstruction tip the box?

(a) No, never; (b) yes, only if $F > mg$;

(c) yes, $F > mg/2$ is sufficient; (d) yes, only if $F > 2mg$.

Question 15

A circuit consists of a battery, a resistor R , and two light bulbs A and B as shown below:



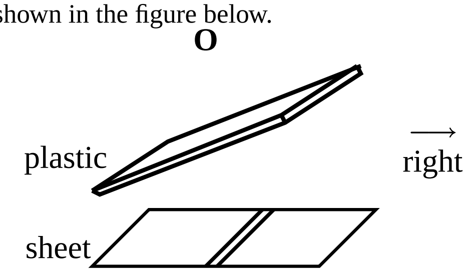
If the filament in lightbulb A burns out, then the following is true for light bulb B:

(a) It is turned off; (b) its brightness does not change;

(c) it gets dimmer; (d) it gets brighter.

Question 16

An observer at O views two closely spaced lines on the bottom sheet through an angled slab of plastic with parallel faces, exactly as shown in the figure below.



Compared to when there is no plastic, the lines appear to the observer

(a) the same but shifted to the right;

(b) shifted to the left and spaced further apart;

(c) shifted to the right and spaced closer together;

(d) exactly as they do without the plastic slab.

Question 17

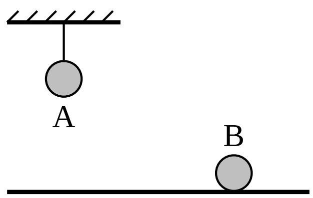
If all of Earth's polar ice were suddenly to melt into the oceans, in the short term the length of the day would

(a) increase; (b) remain the same;

(c) decrease; (d) first decrease, then increase.

Question 18

Two identical balls, A and B, of uniform composition and initially at the same temperature, each absorb exactly the same amount of heat. A is hanging down from the ceiling while B rests on the horizontal floor in the same room. Assuming no subsequent heat loss by the balls, which of the following statements is correct about their final temperatures, T_A and T_B, once the balls have reached their final dimension?



(a) $T_A < T_B$; (b) $T_A > T_B$;

(c) $T_A = T_B$; (d) $T_A \leq T_B$.

Question 19

Two uncharged balls A and B, each very light and coated with a conducting material, hang vertically side by side just touching each other. A positively-charged glass rod is brought near ball A without touching it. Now A and B are separated and then the glass rod is removed. If Q_A and Q_B represent the electric charges on A and B, respectively, you conclude that

(a) $Q_A < 0$ and $Q_B < 0$; (b) $Q_A < 0$ and $Q_B > 0$;

(c) $Q_A > 0$ and $Q_B < 0$; (d) $Q_A > 0$ and $Q_B > 0$;

Question 20

You are travelling at constant speed in an airtight car with a balloon floating motionless next to you. Suddenly, you slam on the brakes so as to stop the car quickly. During decceleration, with respect to the car the balloon

(a) moves forward; (b) remains motionless;

(c) moves backward; (d) can move forward or backward.

Question 21

Due to tidal friction on Earth, the radius R of the Moon's orbit is increasing at the rate of a few centimetres per year. During this process, the Moon's angular momentum

(a) remains constant since its speed decreases;

(b) remains constant but its total energy increases;

(c) increases as $\sqrt{R}$ while its total energy increases;

(d) decreases as $\sqrt{R}$ while its kinetic energy decreases.

Question 22

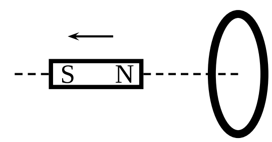
In order to measure the speed v of blood flowing through an artery, a uniform magnetic field $\boldsymbol{B}$ is applied in a direction perpendicular to the flow and a voltmeter measures the voltage across the diameter D of the artery, at right angles to $\boldsymbol{B}$. If positive and negative ions in the blood are longitudinally at rest with respect to the flow, the speed of the flow is closest to

(a) $v = V/BD$; (b) $v = BD/V$;

(c) $v = VD/B$; (d) $v = B/VD$.

Question 23

A bar magnet with its north (N) and south (S) poles as shown below is initially moving to the left, along the axis of, and away from a circular conducting loop. A current I is induced in the loop, with $\boldsymbol{a}$ the acceleration of the magnet due to this current. As seen from the magnet looking in the direction of the loop,



(a) I runs clockwise and $\boldsymbol{a}$ points to the left;

(b) I runs counterclockwise and $\boldsymbol{a}$ points to the right;

(c) I runs clockwise and $\boldsymbol{a}$ points to the right;

(d) I runs counterclockwise and $\boldsymbol{a}$ points to the left.

Question 24

You are moving a negative charge $q < 0$ at a small constant speed away from a conducting spherical shell on which resides a negative charge $Q < 0$. The electrostatic field of Q is $\boldsymbol{E}$. Let U be the total energy of q, W_a the work done by the force $\boldsymbol{F}_a$ you exert on q, and W_E the work done by the electrostatic force $\boldsymbol{F}_E$ on q. Then, as q is being moved,

(a) $W_a = -W_E$, and therefore U remains constant;

(b) $\boldsymbol{F}_a = -\boldsymbol{F}_E$, and therefore U remains constant;

(c) U increases;

(d) U decreases;

Question 25

An ice cube of pure fresh water floats on pure fresh water in a glass. A huge ice shelf, also of pure fresh water, floats on the ocean away from Antarctica. Neglecting the contribution due to the density of air, as the ice cube and the iceberg melt,

(a) the water level rises both in the glass and in the ocean;

(b) the water level does not change in either case;

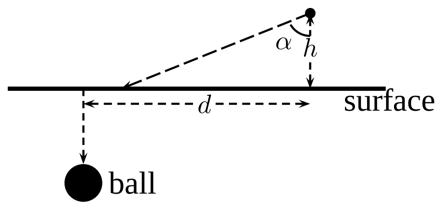
(c) the water level stays the same in the glass but rises in the ocean;

(d) the water level decreases in the glass but stays the same in the ocean.

Part B

Problem 1

A uniform ball of mass $M = 1.00\,\text{kg}$ is released from rest at the surface of a pond and sinks vertically into the water. An observer, wishing to determine the density of this ball, measures the time-dependence of the angle α between the vertical at his position, a distance h above the surface, and the direction of the ball's apparent instantaneous position (see figure). The horizontal distance between the observer and the initial position of the ball is d.



The drag force acting on the ball as it sinks is well approximated at low speed v by $F_{\text{drag}} = -bv$, where b is a constant. We also take the density of water to be $\rho_{\text{w}} = 1000\,\text{kg/m}^3$ and its refraction index as $n_{\text{w}} = 1.33$.

(a) Derive an expression for the instantaneous depth y of the ball as a function of α, h, and d.

(b) Having calculated the depth of the ball as a function of time, the observer can then construct the following table for the speed of the ball as a function of time:

Time [s]	Speed [m/s]	Time [s]	Speed [m/s]
0.000	0.000	0.225	0.8425
0.010	0.0910	0.275	0.8767
0.025	0.2086	0.325	0.8966
0.075	0.5081	0.375	0.9082
0.125	0.6823	0.425	0.9145
0.175	0.7836	0.475	0.9155

From these data, find the initial acceleration of the ball and estimate at which value its speed will stabilise. Then calculate the density of the ball and obtain a value for the drag coefficient b.

Problem 2

Bicycle headlights are often powered by a generator, with the rim of the generator's shaft rolling against the tire's rim due to the rotation of the wheel. The shaft is rigidly connected to a coil within the generator which rotates in a magnetic field $\boldsymbol{B}$.

In one such generator, the coil has 125 turns of wire and a cross-sectional area of $0.0010\,\text{m}^2$, immersed in a field of magnitude 0.080 T. At the area of contact with the tire, the rim of the generator's shaft has a radius of 1.25 cm. The tire's diameter is 66 cm.

(a) If the lightbulb in the headlight needs 5.0 W of average electrical power, corresponding to a voltage amplitude of 4.0 V, to produce a decent amount of light, calculate the linear speed of the bicycle needed, assuming no slipping anywhere in the problem.

(b) Calculate the torque that must be supplied by the bicycle wheel to produce the required average electrical power for the bulb. State briefly two other simplifying assumptions that must be made.

(c) What is the amplitude of the current induced in the coil under these conditions?

(d) To the extent that the lightbulb obeys Ohm's Law, and that its resistance is constant, by what factor would the power delivered to the bulb increase if the cyclist tripled the bicycle's speed (presumably downhill or at the Tour de France)?

Problem 3

The long jump as an Olympic sport dates back at least as far as 700 BC. Unlike in the modern Olympics, it was practised not only from a running start, but also from a standing start. The length of a jump was measured (as it still is) from the point where the back of the feet touched the ground on landing. Paintings on ancient Greek vases depict athletes jumping from a standing start while holding compact weights in both hands. Examples of these have been dug up by archaeologists, and typical ones, made of stone or lead, each have a mass of around 3 kg.

(a) Write a short paragraph explaining qualitatively how athletes could boost their performance, ie. jump further from a standing start, by carrying such weights. More precisely, describe what they would have done with the weights during the jump.You can accept that, as biomechanics research has shown, a loaded body could take off at the same speed and angle as an unloaded one.

(b) Estimate by how much the length of a 3 m jump could be increased for a 65 kg athlete jumping at an angle of 50°. Remember that this takes place from a standing start, not a running start like today's long jump. You can take the centre of mass of the body to be at a height of 1 m above ground and the length of the arms to be 65 cm. Also, at the moment of take-off, the athlete is leaning forward so that his shoulders are about 15 cm in front of the centre of mass of his body.

* * * *

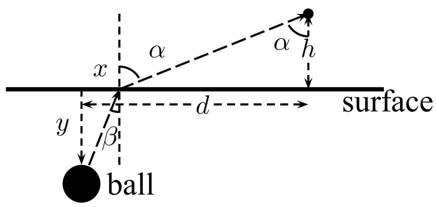
Solutions to 2004 CAP High School Exam

Part A

1 (c)	2 (b)	3 (c)	4 (b)	5 (a)
6 (b)	7 (b)	8 (d)	9 (d)	10 (d)
11 (a)	12 (d)	13 (a)	14 (b)	15 (d)
16 (b)	17 (a)	18 (b)	19 (b)	20 (c)
21 (c)	22 (a)	23 (c)	24 (d)	25 (c)

Part B

Problem 1



(a)

Let x be the horizontal distance from the point where the ball was released, to the point where a ray originating from the instantaneous position of the ball at depth y crosses the surface and reaches the observer.

From Snell's law we have $\sin\alpha = n_w \sin\beta$, where $n_w = 1.33$ is the index of refraction of water.

From geometry, we can also write:

$$x = d - h\tan\alpha$$
$$\tan\beta = x/y$$

Then

$$y = \frac{x}{\tan\beta} = x\frac{\cos\beta}{\sin\beta} = x\sqrt{\frac{1-\sin^2\beta}{\sin^2\beta}}$$
$$= (d - h\tan\alpha)\sqrt{\left(\frac{n_w}{\sin\alpha}\right)^2 - 1}$$

(a) The initial acceleration a_0 can be calculated from the firs velocity point in the data table. Velocity changed from 0 to 0.091 m/s in 0.01 s, which gives $a_0 = 9.10\,\text{m/s}^2$.

The terminal speed can be read off the table as approximately $v_f = 0.916\,\text{m/s}^2$.

The total instantaneous downward force F acting on the ball when it has speed v is

$$F = Mg - \rho_w V g - bv \qquad (1)$$

where Mg is the gravitational force, $\rho_w V g$ is the upward buoyancy force if the ball has volume V, and bv is the drag force. By Newton's second law, $F = Ma$, where a is the acceleration of the ball..

Initially, v is small enough that the drag force can be ignored, and we can write

$$F = \rho_{\text{ball}} V g - \rho_w V g = \rho_{\text{ball}} V a_0$$

with ρ_{ball} the density of the ball. Thus,

$$\rho_{\text{ball}} = \frac{\rho_w}{1 - a_0/g}$$
$$= \frac{1000\,\text{kg/m}^3}{1 - 9.10/9.80}$$
$$= 1.4\times 10^4\,\text{kg/m}^3$$

On the other hand, when the speed v has reached its fina value v_f, the acceleration vanishes and eq. (1) becomes

$$0 = Mg - \rho_w V g - bv_f = (1 - \frac{\rho_w}{\rho_{\text{ball}}})Mg - bv_f$$

Solving for b yields

$$b = \frac{Mg}{v_f}\left(1 - \frac{\rho_w}{\rho_{\text{ball}}}\right)$$
$$= \frac{(1.00\,\text{kg})(9.80\,\text{m/s}^2)}{0.916\,\text{m/s}}\left(1 - \frac{1.00\times 10^3\,\text{kg/m}^3}{1.4\times 10^4\,\text{kg/m}^3}\right)$$
$$= 9.9\,\text{kg/s}$$

Problem 3

(a) The centre of mass of the body of the athlete loaded with the weights follows a parabolic trajectory. Then, if the jumper thrusts the weights forward at arm's length as he jumps, the centre of mass lies a bit forward and higher from its position if there were no weights. In mid-flight the athlete then swings the weights down and backward before landing, which puts the point of contact of his feet at a position slightly further ahead with respect to the centre of mass then if no weights were carried. There is also an extra enhancement coming from the upward vertical motion of the body with respect to the centre of mass as the weights are swung down before landing, in effect extending the parabolic trajectory a bit further downward, and therefore further forward.

(b) Let M be the mass of the body and m the total mass of the weights. The position of the centre of mass is given by $\boldsymbol{R} = (X, Y)$:

$$\boldsymbol{R} = \frac{M\boldsymbol{r}_1 + m\boldsymbol{r}_2}{M+m} = \frac{\boldsymbol{r}_1 + \frac{m}{M}\boldsymbol{r}_2}{1 + m/M}$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the positions of the centre of mass of the body and the centre of mass of the weights, respectively. The shift in the position of the centre of mass of the body due to the weights is then

$$\begin{aligned}\Delta\boldsymbol{R} &= \frac{M\boldsymbol{r}_1 + m\boldsymbol{r}_2}{M+m} = \frac{\boldsymbol{r}_1 + \frac{m}{M}\boldsymbol{r}_2}{1+m/M} - \boldsymbol{r}_1 \\ &= \frac{\Delta\boldsymbol{r}}{1+M/m}\end{aligned}$$

where $\Delta\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$. With $M = 65\,\text{kg}$ and $m = 6\,\text{kg}$, this gives $\Delta\boldsymbol{R} = 0.085\,\Delta\boldsymbol{r}$.

Let us break down the effect into three parts, two horizontal ones and one vertical.

- At take-off, the weights are held at arm's length (arms horizontal). Assuming 65 cm as the length of the arms, plus another 15 cm for the horizontal position of the shoulders relative to the centre of mass of the body, we have $\Delta x = x_2 - x_1 = 80\,\text{cm}$, which gives $\Delta X = 6.8\,\text{cm}$.
- At landing, the arms are swung backwards. We assume that at landing the arms make an angle of 40° with the vertical. Then the centre of mass of the body is $0.0845\,\sin 40°(60\,\text{cm}) = 3.3\,\text{cm}$ ahead of the centre of mass of the system.
- Vertically, if the athlete lands in a crouching position, it is reasonable to assume that the vertical position of the weights has shifted by up to one metre between take-off and landing. Therefore, when the trajectory of the system's centre of mass is back to its starting height above ground, the centre of mass of the body is higher by about $\Delta Y = 0.085 \times 1\,\text{m} = 8.5\,\text{cm}$. The angle of the trajectory being about $50°$, the corresponding horizontal extension of the length of the jump is $\Delta X = \Delta Y/\tan 50° = 7.1\,\text{cm}$.

Totalling these three contributions gives an extension of the jump's length of about 17 cm. This is of course quite dependent on the assumptions we made, but gives a reasonable estimate.

Problem 2

(a) Since the generator shaft contacts the tire very close to its edge, we can take its tangential speed v to be that of the bicycle wheel, and therefore the linear speed of the bicycle itself. This is related to its rotational speed ω by $v = \omega\, r$, where r is the radius of the shaft.

As is readily derived from Faraday's law applied to the rotating coil with N turns and area A, immersed in a uniform magnetic fiel with magnitude B, the voltage output of the generator as a function of time is:

$$\mathcal{E} = \omega NBA\,\sin\omega t$$

and its amplitude is therefore $\mathcal{E}_{\text{max}} = \omega NBA$. Combining these expressions leads to

$$\begin{aligned}v &= \frac{r\,\mathcal{E}}{NBA} \\ &= \frac{(1.25\times 10^{-2}\,\text{m})(4.0\,\text{V})}{(125)(0.080\,\text{T})(1.0\times 10^{-3}\,\text{m}^2)} \\ &= 5.0\,\text{m/s}\end{aligned}$$

(b) The average torque supplied by the tire's rotation is given by $\tau = P/\omega$, with P the average mechanical power. If we ignore any loss due to (1) friction and (2) the resistance of the coil, P is also the average electrical power produced by the generator. Then

$$\begin{aligned}\tau &= rP/v \\ &= \frac{(1.25\times 10^{-2}\,\text{m})(5.0\,\text{W})}{5.0\,\text{m/s}} \\ &= 1.25\times 10^{-2}\,\text{N}\cdot\text{m}\end{aligned}$$

(c) The power amplitude P_{max} is twice the average power, and the maximum current is therefore:

$$I_{\text{max}} = P_{\text{max}}/\mathcal{E}_{\text{max}} = (10\,\text{W})/(4.0\,\text{V}) = 2.5\,\text{A}$$

(d) Assuming that the lightbulb obeys Ohm's Law, the power it consumes goes like the square of the voltage supplied at constant resistance. Since that voltage goes like the linear speed of the bicycle, power is proportional to v^2. If that speed triples (to 15 m/s, or 54 km/hr), the power supplied is multiplied by 9. The bulb may well burn out!

CAP High School Prize Exam

April 8 2005

9:00 – 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name: ________________________ Given Name: ________________

Home Address: ____

____

________________________ Postal Code: ________________

Telephone: ( ) ____________________ E-mail: ________________________

School: ________________________________ Grade: ________________

Physics Teacher: ________________________________

Date of Birth: ________________________________ Sex: ________________

Citizenship: ________________________

For how many years have you studied in a Canadian school? ____

Would you prefer further correspondence in French or English? ____

Sponsored by:

Canadian Association of Physicists

Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists
2005 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. Performance on the multiple-choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. The questions in part B have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.
Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ N·m²/kg²
Radius of Earth	$R_\text{E} = 6.37 \times 10^3$ km
Mass of Earth	$M_\text{E} = 6.0 \times 10^{24}$ kg
Radius of Earth's orbit	$R_\text{ES} = 1.50 \times 10^8$ km
Acceleration due to gravity	$g = 9.80\,\text{m/s}^2$
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_\text{e} = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_\text{p} = 1.673 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ J·m/C²

Part A: Multiple Choice

Question 1

A child throws a ball toward the front end of an approaching train. The collision between the ball and the train is perfectly elastic. Let v be the speed of the ball with respect to the train and V its speed with respect to the ground. If the labels"i" and "f" refer to those speeds just before and just after, respectively, the ball hits the train, then

(a) $v_\text{i} = v_\text{f}$ and $V_\text{i} < V_\text{f}$; (b) $v_\text{i} < v_\text{f}$ and $V_\text{i} < V_\text{f}$;

(c) $v_\text{i} > v_\text{f}$ and $V_\text{i} < V_\text{f}$; (d) $v_\text{i} = v_\text{f}$ and $V_\text{i} > V_\text{f}$.

Question 2

A physics textbook of mass m rests flat on a horizontal table of mass M placed on the ground. Let $\boldsymbol{N}_{\text{a}\to\text{b}}$ be the contact force exerted *by* body "a" *on* body "b". According to Newton's 3rd Law, which of the following is an action-reaction pair of forces?

(a) $m\boldsymbol{g}$ and $\boldsymbol{N}_{\text{table}\to\text{book}}$;

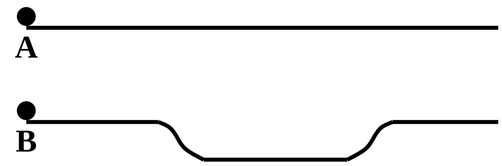
(b) $(m + M)\boldsymbol{g}$ and $\boldsymbol{N}_{\text{table}\to\text{book}}$;

(c) $\boldsymbol{N}_{\text{ground}\to\text{table}}$ and $M\boldsymbol{g} + \boldsymbol{N}_{\text{book}\to\text{table}}$;

(d) $\boldsymbol{N}_{\text{ground}\to\text{table}}$ and $\boldsymbol{N}_{\text{table}\to\text{ground}}$.

Question 3

At some time t, two identical balls, A and B, are set rolling without slipping at the same speed from one end of two tracks which are identical with the same horizontal length, except that track B has a dip in the path of ball B, as shown in the figure. The straight portions of the tracks are horizontal. Gravity is uniform throughout. Which ball reaches the other end first?



(a) Ball A;

(b) neither, since both arrive at the same time;

(c) ball B;

(d) ball A, but only if the dip is deep enough.

Question 4

To a good approximation, Earth and Jupiter move around the Sun in circular orbits of 1.49×10^8 km and 7.79×10^9 km radius, respectively. What is the maximum error that can arise in the prediction of solar eclipse times (as observed from Earth) on Jupiter caused by one of its moons if one fails to take into account the variation of the relative position of the two planets?

(a) 2.6×10^3 s; (b) 3.1×10^3 s;

(c) 5.0×10^2 ; (d) 9.9×10^2 s.

Question 5

Experiment shows that two perfectly neutral parallel metal plates separated by a small distance d, attract each other via a very weak force, known as the Casimir force. The force per unit area of the plates, $\mathcal{F}$, depends only on the Planck constant h, on the speed of light c, and on d. Which of the following has the best chance of being correct for $\mathcal{F}$?

(a) $\mathcal{F} = hc/d^2$; (b) $\mathcal{F} = hc/d^4$;

(c) $\mathcal{F} = hd^2/c$; (d) $\mathcal{F} = d^4/hc$.

Question 6

A simple pendulum of length L with a bob of mass m is taken into Earth orbit on the International Space Station. Its frequency of oscillation with respect to that on the ground is

(a) greater; (b) smaller but non-zero;

(c) the same; (d) zero.

Question 7

When you turn on a battery-operated portable music-playing device, how long must you leave it on for electrons leaving the negative terminal of the battery to reach the positive terminal if their path lies within good conductors?

(a) A few milliseconds; (b) a few tenths of a second;

(c) a few microseconds; (d) a few minutes.

Question 8

Circuit A is made of resistors connected in series to a battery; circuit B is made of resistors connected in parallel to a battery. Let P be the power drawn from the batteries. As the number of resistors in each circuit is increased,

(a) P_{A} increases and P_{B} decreases;

(b) both P_{A} and P_{B} increase;

(c) P_{A} decreases and P_{B} increases;

(d) P_{A} and P_{B} remain the same.

Question 9

According to a simplified but still useful model, the drag force due to air resistance on a moving car goes like the square of the car's speed v. Suppose that the maximum speed of a car is limited only by this drag force. If the power of the car's engine were increased by 50%, the top speed of the car would increase by about

(a) 50%; (b) 15%;

(c) 22%; (d) 30%.

Question 10

A projectile is launched with an initial velocity $\boldsymbol{v}_{\text{i}}$, with $v_{\text{i}x}$ and $v_{\text{i}y}$ the horizontal and vertical velocity components, respectively. When is there a point on its trajectory after launch where its velocity is perpendicular to its acceleration?

(a) Always; (b) only if $v_{\text{i}x} \neq 0$ and $v_{\text{i}y}$ points upward;

(c) only if $v_{\text{i}x} \neq 0$; (d) always, except if $v_{\text{i}x} = 0$.

Question 11

Three charged conducting metal spheres, of radius R_1, R_2, and R_3, are connected together by wires. Let $R_1 < R_2 < R_3$. At equilibrium, which of the following sets of relations involving the electric field strength E generated by a sphere, its potential V, and its charge Q, must hold between the spheres?

(a) $V_1 = V_2 = V_3,\ E_1 < E_2 < E_3,\ Q_1 < Q_2 < Q_3$;

(b) $V_1 = V_2 = V_3,\ E_1 > E_2 > E_3,\ Q_1 < Q_2 < Q_3$;

(c) $V_1 < V_2 < V_3,\ E_1 < E_2 < E_3,\ Q_1 = Q_2 = Q_3$;

(d) $V_1 > V_2 > V_3,\ E_1 < E_2 < E_3,\ Q_1 > Q_2 > Q_3$;

Question 12

A ball of mass m attached to an inextensible string of length R in swung around a vertical circle just fast enough so that the string is always fully stretched. Let ΔT denote the difference between the tension in the string at the bottom and at the top of the circle, v_b and v_t the speed of the ball at the bottom and at the top, respectively. Then, taking "dependence" to be with respect to a set of independent variables,

(a) ΔT is independent of R , v_b and v_t;

(b) ΔT is independent of R , but depends on $v_b^2 - v_t^2$;

(c) ΔT depends on R , but on neither v_b nor v_t;

(d) ΔT depends on R and $v_b^2 - v_t^2$.

Question 13

An object of mass m hangs motionless from a vertical spring. When the object is pulled down to a new rest position, the total mechanical energy of the system

(a) increases; (b) remains the same;

(c) decreases; (d) may increase or decrease depending on the new position.

Question 14

As more and more negative electric charge is being brought to a conducting sphere, inside the sphere

(a) the electric field and potential increase;

(b) the electric field stays constant and the potential increases;

(c) the electric field stays constant and the potential decreases;

(d) the electric field increases and the potential decreases.

Question 15

A static magnetic field of about 0.01 T in strength can erase data on the magnetic strip of a credit card. What would be roughly the minimum diameter of a long straight wire carrying a 100 A current for which your card would be safe no matter how close you take it to the wire?

(a) 0.2 mm; (b) 1 mm;

(c) 2 mm; (d) 4 mm.

Question 16

A string of length L is composed of two segments of equal length. One segment has linear mass density μ_1 and the other other $\mu_2 \neq \mu_1$. One segment is tied to a wall, and the string is stretched by a force, applied to the other segment, which is much greater than the total weight of the string. If T_i is the tension in the i^th segment, and v_i the speed of a transverse wave propagating along that segment,

(a) $v_1 = v_2$ and $T_1 = T_2$; (b) $v_1 \neq v_2$ and $T_1 = T_2$;

(c) $v_1 = v_2$ and $T_1 \neq T_2$; (d) $v_1 \neq v_2$ and $T_1 \neq T_2$.

Question 17

A perfectly straight portion of a uniform rope has mass M and length L. At end A of the segment, the tension in the rope is T_{A}; at end B it is $T_{\text{B}} > T_{\text{A}}$. The tension in the rope at a distance $L/5$ from end A is

(a) $T_{\text{B}} - T_{\text{A}}$; (b) $(T_{\text{A}} + T_{\text{B}})/5$;

(c) $(4T_{\text{A}} + T_{\text{B}})/5$; (d) $(T_{\text{A}} - T_{\text{B}})/5$.

Question 18

Two spheres are identical except that sphere A is white whereas sphere B is black. After they have been in thermal contact long enough with each other and their surroundings, in the visible range,

(a) A radiates less than B;

(b) both emit the same amount of radiation;

(c) A radiates more than B;

(d) A radiates more than B only if its temperature is high enough.

Question 19

An aircraft bound for Vancouver and coming from Montréal is flying due west. Its body and wings are covered in aluminium. At some point on its flight path, the Earth's magnetic field points north and downward. The point on the plane's exterior which is then at the highest potential is

(a) the nose (front); (b) the tail (back);

(c) the tip of the right wing; (d) the tip of the left wing.

Question 20

You are on the shore of a canal of uniform width d and want to reach a point a distance $L > d$ away *along* the other shore as quickly as possible. To achieve this, you first run along the shore at constant speed v_1, then jump in the canal and swim directly toward your target at constant speed $v_2 < v_1$, both being your maximum speeds. The water in the canal is motionless. The angle of your trajectory in the water with respect to the shore must obey:

(a) $\cos\theta = v_1/v_2$; (b) $\cos\theta = \sqrt{v_1^2/v_2^2 - 1}$;

(c) $\cos\theta = 1 - d/L$; (d) $\cos\theta = v_2/v_1$.

Question 21

Exactly half of a rectangular conducting loop lies in a uniform magnetic field perpendicular to the plane of the loop. At some point in time, the magnitude of the magnetic field starts rapidly decreasing. While this is happening, which of the following statements most accurately describes the effect on the loop?

(a) The loop is pulled into the magnetic field;

(b) the loop is pushed out of the magnetic field;

(c) the loop starts rotating;

(d) the behaviour of the loop cannot be determined unless the direction of the magnetic field is completely specified.

Question 22

Two otherwise identical spaceships have different solar sails: sail A is a perfect reflector, sail B is a perfect absorber. Each starts at the same distance from the Sun and travels radially outward. Let $\Delta p_{\rm A}$ and $\Delta p_{\rm B}$ be the momentum gained by the ships after travelling equal distances. Then

(a) $\Delta p_{\rm A} = \Delta p_{\rm B}$; (b) $\Delta p_{\rm A} > \Delta p_{\rm B}$;

(c) $\Delta p_{\rm A} < \Delta p_{\rm B}$; (d) $\Delta p_{\rm A} = \Delta p_{\rm B} = 0$.

Question 23

If there were only one transmitter, and you were separated from this transmitter by the many tall buildings to be found in downtown Toronto, spaced about 30 m apart on average, with which of the following would you be most likely to experience dead spots (places with very poor or no reception)?

(a) AM radio stations (frequency 1 MHz);

(b) FM radio stations (frequency 100 MHz);

(c) cell phones (frequency 1000 MHz);

(d) all of the previous equally.

Question 24

A horizontal cathode ray tube (CRT) is set so that its electron beam produces a spot of light at the centre of the screen when no external electromagnetic field is present. When you look straight at the screen, however, you discover that the spot, instead of being at the centre as it should, is shifted a bit to the right. Suspecting what the cause of this deflection may be, you rotate the CRT by 180° around its vertical axis. Facing the screen, you find that the spot of light is still shifted to right of centre by the same distance as before. You conclude that the CRT is immersed in

(a) an electric field directed horizontally to the left with respect to the screen's initial position;

(b) an electric field directed horizontally to the right with respect to the screen's initial position;

(c) a magnetic field directed vertically upward with respect to the screen's initial position;

(d) a magnetic field directed vertically downward with respect to the screen's initial position;

Question 25

Light rays from a very distant source travel along the $+x$ direction. Two identical thin lenses with focal length $f > 0$ and their optical axis along x, sit, one at $x = 0$, and the other at $x = d < f$. Where do the rays focus?

(a) $d + \dfrac{f(f-d)}{2f-d}$; (b) $d - \dfrac{f(f-d)}{d}$;

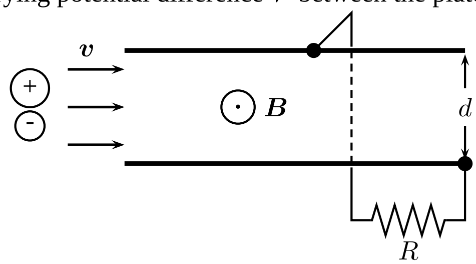
(c) $d + \dfrac{f(f-d)}{2(f+d)}$; (d) $d + \dfrac{f^2}{2f-d}$.

* * *

Part B

Problem 1

A magnetohydrodynamic (MHD) generator is a device that converts part of the kinetic energy of a streaming hot gas into electrical energy. At its operating temperature of between 2000 and 3000 K, the gas is readily ionised. As schematically shown in the figure, the ions and electrons enter a region between two electrodes (here, parallel conducting plates of area A separated by a gap d) in which a uniform and constant magnetic field $\boldsymbol{B}$ has been set up, pointing straight out of the page. Their initial velocity $\boldsymbol{v}$ is parallel to the plates. A resistor R is connected to the plates. The magnetic field is strong enough that many charged particles will hit the electrodes and charge them before they can exit. This creates a time-varying potential difference V between the plates.



(a) On a diagram, display all the forces acting on a positive ion at some generic position between the plates. Use your diagram to show that the ion experiences a braking force a soon as its velocity has a component transverse to the plates.

(b) Then, as a crude approximation, assume that any electric field $\boldsymbol{E}$ arising from the process is uniform everywhere between the plates. Assume also that the transverse component of the velocity of the charges remains everywhere much smaller than its longitudinal component, and that the latter therefore is pretty uniform. The gas obeys a generalised Ohm's law, according to which $\boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$, where $\boldsymbol{J}$ is the transverse current density between the plates and σ is the conductivity of the gas.

Obtain an expression for the current I flowing through the resistor in terms of v, B, and R.

(c) Derive an expression for the output voltage V that does not explicitly contain R.

Problem 2

A 1000 kg satellite is orbiting the Earth at an altitude of 400 km. It receives electrical power from a solar panel of area $A = 10\,\text{m}^2$. At this altitude, Earth's atmosphere is very tenuous, with a density $\rho \approx 10^{-11}$ kg/m^3. Nevertheless, over time, the friction force generated by collisions of the molecules and the panel might cause the satellite to lose altitude.

(a) Assume that in such collisions, the molecules become embedded in the solar panel. If the satellite is moving at speed v, find an expression for the maximum retarding force on the solar panel in terms of ρ and of the radius of the satellite's orbit. Make any other reasonable assumption.

(b) Estimate how much altitude the satellite might lose over one week because of this friction. If you make assumptions, do not forget to justify them briefly.

(c) Somebody claims that as the satellite loses altitude it also loses speed because of the friction. Comment briefly.

Problem 3

In a mood for some physics as you look at a sailboat on a lake, you wonder about its stability when a strong wind blows from the side. The sailboat leans over in the wind and the question is whether its keel can prevent it from being blown over completely.

Analyze the stability of the sailboat using the following over-simplified model.

Consider the hull of the sailboat as an airtight hollow cylinder. On top of this cylinder and perpendicular to it sits a mast carrying a square sail, assumed to be always parallel to the length of the hull. Attached to the bottom of the hull is a keel in the shape of a square always parallel to the sail. A heavy lead weight forms the bottom of the keel under water. Other than the lead weight, consider mast, sail, hull, and keel to be weightless. Somewhat unrealistically (it would make tacking difficult), you can also assume that the sail extends all the way from the bottom to the top of the mast.

(a) Establish a relationship between wind speed and the angle θ the mast will tilt away from its initial upright position when a wind with speed v blows perpendicular (initially) to the sail. Other assumptions may be made so long as they are explicitly stated and, as much as possible, justified.

In this simplified model, is this boat stable in all wind speeds? Since you may not be able to find a general solution for the angle θ as a function of v, you are welcome to obtain solutions valid only for small θ or for large θ. The expansion $(1 + x)^n \approx 1 + nx + \ldots$ for $x \ll 1$ may be useful.

(b) The following data, loosely based on the *Catalina Capri* – 16, are given for a small sailboat. Area A of sail is 12 m^2, mass M of lead weight 190 kg, depth d of keel below water level, technically known as draft, 0.75 m, height h of mast 6.6 m, specific gravity of lead, supposed to be the part at the bottom of the keel that acts as a stabiliser, 11.3. Also, air density is 1.2 kg/m^3.

With these data, at what wind speed will θ be 30°? 60°?

Solutions to 2005 CAP High School Exam

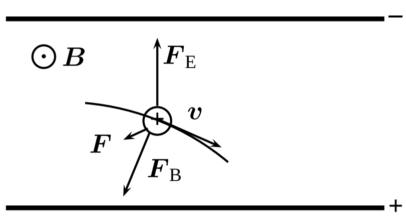
Part A

1 (a)	2 (d)	3 (c)	4 (a)	5 (b)
6 (d)	7 (d)	8 (c)	9 (b)	10 (b)
11 (b)	12 (a)	13 (a)	14 (c)	15 (d)
16 (b)	17 (c)	18 (a)	19 (d)	20 (d)
21 (a)	22 (b)	23 (c)	24 (c)	25 (a)

Part B

Problem 1

(a) In the figure below, the electric force $\boldsymbol{F}_{\mathrm{E}}$ and the magnetic force $\boldsymbol{F}_{\mathrm{B}}$ on the ion are displayed. The resultant net force $\boldsymbol{F}$ has a centripetal component (perpendicular to the trajetory), which does not change v, and a tangential component opposite $\boldsymbol{v}$ which slows down the ion.



(b) Assuming that $\boldsymbol{v}$ remains approximately longitudinal, in a first approximation the transverse current density is

$$J \;=\; \sigma\,(vB \,-\, E)$$

Since the potential difference V between the plates is equal to Ed as well as to RI, we can write the current I as

$$I \;=\; J\,A \;=\; A\sigma\,(vB - IR/d)$$

Solving for I gives:

$$\boxed{I \;=\; \frac{vBd}{d/\sigma A \,+\, R}}$$

(c) Since $V = RI$, we can solve for RI in the next-to-last equation and obtain

$$\boxed{V \;=\; vBd \,-\, Id/\sigma A}$$

Problem 2

(a) The force on the panel, or the rate of momentum change, is equal to the product of the number of molecules that strike the panel per unit of time with the change of momentum of individual particles when they collide with the panel. Since the molecules are absorbed, they come to rest and the change is equal to the incident momentum itself.

If n is the number of molecules per unit volume, the number that strike the panel per unit of time, or flux, is equal to the product of n with the surface A of the panel and the component of the molecules' velocity perpendicular to the panel. The momentum of an individual incident molecule perpendicular to the panel is its mass m times the component of of its velocity perpendicular to the panel. Since $nm = \rho$, the mass density of the molecules, this gives $F = \rho A v^2$, with v the average speed of the molecules with respect to the panel, when the flux is perpendicular to it.

We assume that this average speed is that of the satellite in its orbit, ie. we neglect the intrinsic motion of the molecules, whose average speed with respect to the Earth is an order of magnitude lower than the satellite's speed ($\approx 7\,\mathrm{km/s}$). Then, with r the orbital radius of the satellite,

$$\begin{aligned}
F &= \rho A\omega^2 r \\
&= \rho A\big(4\pi^2/T^2\big)\,r \qquad (T \text{ is the period of the orbit}) \\
&= \rho A\big(GM/r^2\big)\,r \qquad (\text{Kepler's third law}) \\
&= \boxed{\rho AGM/r}
\end{aligned}$$

(b) The rate what which energy is lost because of this retarding force is $P = \boldsymbol{F}\cdot\boldsymbol{v} = -F\omega r$. Again, from Kepler's third law, $\omega = \sqrt{GM/r^3}$, and we obtain

$$\boxed{P \;=-\;\rho A(GM/r^3)^{3/2}}$$

To find the resulting rate of change in the radius, we observe that the total energy of the satellite is $E = -GMm/2r$. If we calculate $\Delta E = E_2 - E_1$ corresponding to $\Delta r = r_2 - r_1$, we see that if we can assume that $\Delta r \ll r_1,\, r_2$, then we can approximate $r_2 \approx r_1 = r$, so that $\Delta E = (GMm/2r^2)\Delta r$, from which immediately follows

$$\Delta r \;=\; (2r^2/GMm)P\Delta t$$

so long as we can assume that the rate of energy loss is small and constant on the time scale of interest.

Combining with the second boxed equation above yields:

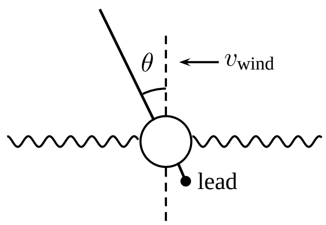
$$\boxed{\Delta r \;=-\frac{2\rho A}{m}\,\sqrt{GMr}\,\Delta t}$$

Inserting numbers and $\Delta t \approx 6 \times 10^5$ s (a week), gives $\boxed{\Delta r \approx 6\,\text{km}}$.

The answer is consistent with our assumption of reasonably slow orbit decay, but it also shows that the satellite would not stay very long in orbit unless it gives itself a small compensating boost quite often.

Problem 3

(a) Similar to problem 2, the initial force exerted by the wind on the sail is $F = \rho A v^2$, where ρ is air density, A the area of the sail, and v the wind speed.



When the mast is tilted at angle θ with respect to the vertical, the component of the force perpendicular to the sail is $\rho A v^2 \cos^2\theta$. This force exerts a torque about the hull, with moment arm $\frac{1}{2} h \cos\theta$. The total torque of the wind force is then

$$\tau_{\text{w}} = \frac{1}{2}\rho A h v^2 \cos^3\theta$$

A counter-torque is provided by the lead mass at the bottom of the keel. We should take into account the buoyancy of the lead. If M is the mass of the lead, then its net weight is $(1 - \rho_{\text{water}}/\rho_{\text{lead}})Mg$. The moment arm is $d\sin\theta$, so that the opposing torque from the keel is

$$\tau_{\text{k}} = \left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{lead}}}\right) Mgd\sin\theta$$

Imposing the condition for equilibrium, $\tau_{\text{w}} = \tau_{\text{k}}$, and solving for v, we obtain

$$\boxed{v = \left[\frac{2\left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{lead}}}\right) Mgd\sin\theta}{\rho A h \cos^3\theta}\right]^{1/2}}$$

where θ is now the equilibrium angle.

Writing the answer in this form makes it easy to check that dimensions are correct, since the denominator in the bracket is obviously a mass, and it is well known that v has the same dimensions as $\sqrt{gd}$.

Let us rewrite the previous expression after inserting $\cos^2\theta = 1 - \sin^2\theta$:

$$(1 - \sin^2\theta)^3 = -\frac{B^2}{v^4}\sin^2\theta$$

where

$$B = \frac{2\left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{lead}}}\right) Mgd}{\rho A h}$$

This is a cubic equation which is not that easily solved if you don't have a good calculator or a computer at hand. Nevertheless, one can look at the small-angle solution using the binomial expansion: $(1 - \sin^2\theta)^3 \approx 1 - 3\sin^2\theta$. Then, in the limit of small θ angles,

$$\sin\theta \approx \frac{1}{\sqrt{(3 + B^2/v^4)}}$$

One sees that θ increases smoothly with v, as expected.

In the large-angle approximation, for θ approaching 90°, it is best to recast our result for v in terms of $\cos\theta$:

$$\frac{v^4}{B^2} = \frac{1 - \cos^2\theta}{(\cos^2\theta)^3} \approx \frac{1}{(\cos^2\theta)^3} \quad \text{in the limit of large } \theta$$

Then we get immediately:

$$\sin\theta \approx \sqrt{1 - (B^2/v4)^{1/3}}$$

This is seen to approach 1 smoothly; more important, there exists a solution. Thus, the range $0 \le v < \infty$ maps smoothly to the range $0 \le \theta \le 90°$, so that equilibrium can be maintained at any finite speed.

(b) Inserting the data yields $\boxed{v = 4.5\,\text{m/s}}$ for $\theta = 30°$ and $\boxed{v = 13.6\,\text{m/s}}$ for $\theta = 60°$

CAP High School Prize Exam

7 April 2006

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name:________________________________Given Name:_______________________

Home Address:____

_________________________________ Postal Code:_______________________

Telephone: (   )____________________ E-mail: __________________________________

School: ____ Grade:_____

Physics Teacher: ____

Date of Birth: ___________________________ Sex: Male ☐ Female ☐

Citizenship: ____

For how many years have you studied in a Canadian school? ____________________________

Would you prefer the further correspondence in French or English? _______________________

Sponsored by:

Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists
2006 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on a student's performance on sections A, B, and C of the exam. Part A consists of twenty multiple-choice questions; part B consists of five questions that require graphic solutions. The problems in part C can also require graphing.

Performance on questions in parts A and B will be used to determine whose written work in part C will be marked for prize consideration by the CAP Exam National Committee. The questions in part C have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.

Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck!

Data

Speed of light $c = 3.00\text{x}10^{8}$ m/s
Gravitational constant $G = 6.67\text{x}10^{11}$ $\text{N·m}^2/\text{kg}^2$
Acceleration due to gravity $g = 9.80$ m/s^2
Fundamental charge $e = 1.60\text{x}10^{-19}$ C
Mass of electron $m_e = 9.11\text{x}10^{-31}$ kg
Mass of proton $m_p = 1.67\text{x}\ 10^{-27}$ kg
Planck's constant $h = 6.63\text{x}10^{-34}$ J·s
Coulomb's constant $1/(4\pi\varepsilon_0) = 8.99\text{x}10^{9}$ $\text{N·m}^2/\text{C}^2$
Boltzmann constant $k = 1.38\text{x}10^{-23}$ J/K

Part A: Multiple Choice

Question 1

A uniform meter stick is supported on a fulcrum at the 25-cm mark. A 0.50-kg object is hung from the 0-cm end of the meter stick, and the stick is balanced horizontally. The mass of the meter stick is:

(a) 0.25 kg; (b) 0.50 kg;
(c) 0.75 kg; (d) 0.75 kg;

Question 2

The radius of the Earth is $6.37\cdot10^{6}$m. Approximately how fast is a person on the Equator moving due to the Earth rotation?

(a) 5 m/s
(b) 50 m/s
(c) 500 m/s
(d) 5000 m/s

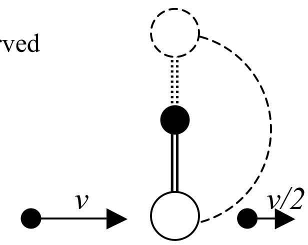
Question 3

In a certain region of space, the electric field is zero. From this we can conclude that the electric potential in this region is:

(a) constant; (b) zero;
(c) positive; (d) negative;

Question 4

A bullet of mass m and speed v hits a pendulum bob of mass M at time t_1, and passes completely through the bob. The bullet emerges at time t_2 with a speed of $v/2$. The pendulum bob is suspended by a stiff rod of length l and negligible mass. After the collision, the bob can barely swing through a complete vertical circle. At time t_3, the bob reaches the highest position.

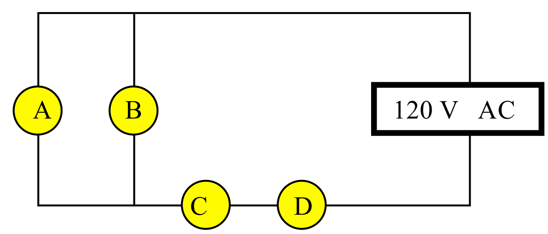


What quantities are conserved in this process?

(a) Total kinetic energy of the bob and the bullet during the time interval $\Delta t = t_2 - t_1$.
(b) Total momentum of the bob and the bullet during the time interval $\Delta t = t_2 - t_1$.
(c) Total mechanical energy of the bob and the bullet during the time interval $t_3 - t_1$.
(d) Momentum of the bob after t_2.

Question 5

The diagram shows a circuit with four identical light bulbs.



When we remove bulb A from the circuit, the light intensity of bulb C:

(a) remains the same;
(b) increases;
(c) decreases;
(d) becomes zero.

Question 6

The amplitude of an electromagnetic wave in vacuum is doubled with no other changes made to the wave. As a result of this doubling of the amplitude, which of the following statements is correct?

(a) The speed of propagation of the wave changes.
(b) The frequency of the wave changes.
(c) All of the above are true.
(d) None of the above is true.

Question 7

Two objects are initially at rest on a frictionless surface. Object 1 has a greater mass than object 2. The same constant force starts to act on each object. The force is removed from each object after it accelerates over a distance d. After the force is removed from both objects, which statement is correct (p: momentum; K: kinetic energy)?

(a) $p_1 < p_2$; (b) $p_1 > p_2$; (c) $K_1 > K_2$; (d) $K_1 < K_2$.

Question 8

A dart is loaded into a spring-loaded toy dart gun by pushing the spring in by a distance *d*. For the next loading, the spring is compressed a distance 2*d*. How much faster does the second dart leave the gun compared to the first?

(a) four times as fast;
(b) two times as fast;
(c) the same;
(d) half as fast;
(e) one-fourth as fast.

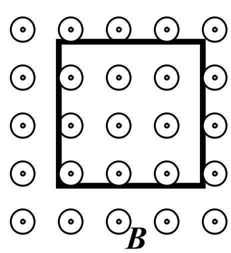
Question 9

One of two parallel metallic plates is uniformly charged with charge + *q*, and the other one is charged with charge -*q*. In this case, the electric field between them is *E*. When the negatively charged plate is discharged then recharged with a positive charge 4*q*, the electric field between the plates becomes:

(a) 0;
(b) 1.5 *E*;
(c) 2.5 *E*;
(d) 3*E*;
(e) 5*E*.

Question 10

A uniform magnetic field ***B*** is directed out of the page. A metallic wire has the shape of a square frame and is placed in the field as shown. While the shape of the wire is steadily transformed into a circle in the same plane, the current in the frame:



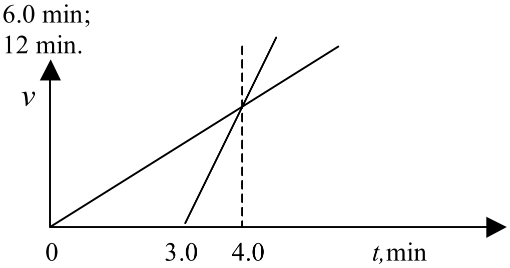
(a) is directed clockwise;
(b) does not appear;
(c) is directed counterclockwise.

Question 11

The drawing shows velocity (*v*) versus time (*t*) graphs for two cyclists moving along the same straight segment of a highway from the same point. The second cyclist starts moving at $t = 3.0$ min.

At what time do the two cyclists meet?

(a) 4.0 min;
(b) 6.0 min;
(c) 12 min.



Question 12

Two solid spheres manufactured of the same material freely fall down in the air. One sphere has a diameter twice as large as the other. The force due to air resistance is proportional to the cross-sectional area of a moving object and is a quadratic function of the speed of an object. Some time after the beginning of motion in the presence of air resistance, the velocity of each sphere becomes constant. It is called the terminal velocity. The ratio of terminal velocities of the spheres v_{big}/v_{small} is:

(a) 2; (b) $\sqrt{2}$; (c) $\frac{1}{2}$; (d) $\frac{1}{\sqrt{2}}$

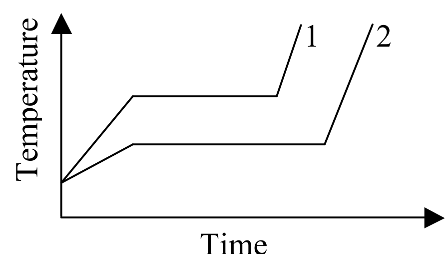
Question 13

Astronauts are weightless in orbit because:

(a) they are beyond the pull of gravity;
(b) they travel in the accelerated frame of reference where fictitious centrifugal force is equal to centripetal force, and the net force on an astronaut is zero;
(c) they are in free fall together with their spacecraft;
(d) of another cause.

Question 14

Two solid objects of the same mass are supplied with heat at the same rate ΔQ / Δt. The temperature of the first object with latent heat L_1 and specific heat capacity c_1 changes according to graph 1 on the diagram. The temperature of the second object with latent heat L_2 and specific heat capacity c_2 changes according to graph 2 on the diagram.

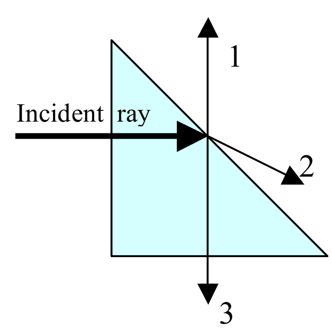


Based on what is shown on the graph, the latent heats L_1 and L_2, and the specific heat capacities c_1 and c_2 in solid state obey which of the following relationships:

(a) $L_1 > L_2$; $c_1 < c_2$;
(b) $L_1 < L_2$; $c_1 < c_2$;
(c) There is not enough information to determine these relations;
(d) $L_1 > L_2$; $c_1 > c_2$;
(e) $L_1 < L_2$; $c_1 > c_2$.

Question 15

A light ray strikes a prism as shown in the drawing. The angles of the prism are 90°, 45°, and 45°. The critical angle of the prism material is 49°. What rays are the possible continuations of the incident ray?



(a) The ray 1 only.
(b) The ray 2 only.
(c) The ray 3 only.
(d) Both rays 1 and 3.
(e) Both rays 2 and 3.

Question 16
You stand on a platform at a train station and listen to a train horn as the train approaches the station at a constant velocity. While the train approaches, but before it arrives, you hear:
(a) the intensity and the frequency of the sound both increasing;
(b) the intensity and the frequency of the sound both decreasing;
(c) the intensity increasing and the frequency decreasing;
(d) the intensity decreasing and the frequency increasing;
(e) the intensity increasing and the frequency remaining the same;

Question 17
A linear conductor with current I_1 is placed along the axis of a circular conductor, which carries current I_2.
The magnetic force acting on each of the conductors is:
(a) zero;
(b) directly proportional to the product of currents I_1 and I_2, and inversely proportional to the radius of the circular conductor;
(c) directly proportional to the product of currents I_1 and I_2, inversely proportional to the square of the radius of the circular conductor;
(d) directly proportional to the product of current I_1, current I_2, and the area of a circular conductor;

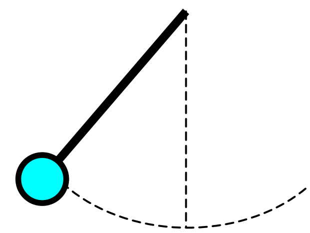
Question 18
An aquarium is filled with water. The lateral wall of the aquarium is 40 cm long and 30 cm high. Using 10 m/s^2 for the acceleration due to gravity, and 1 g/cm^3 for the density of water, the force on the lateral wall of the aquarium is:
(a) 36 N;
(b) 90 N;
(c) 180 N;
(d) 1500 N.

Question 19
A satellite is moved from one circular orbit around the earth, to another of lesser radius. Which of the following is true?
(a) The kinetic energy increases and the potential energy increases;
(b) The kinetic energy increases, and the potential energy decreases;
(c) The kinetic energy decreases and the potential energy decreases;
(d) The kinetic energy decreases and the potential energy increases.

Question 20
A point particle carries a positive charge $+q$ and is maintained at an a distance h above a large conductive uncharged plate. The electric force of interaction between the charge and the plate is **best** described as:
(a) zero force;
(b) an attractive force proportional to q/h^2;
(c) an attractive force proportional to q^2/h^2;
(d) an attractive force proportional to $q^2/4h^2$;
(e) a repulsive force proportional to q^2/h^2.

Part B: Problems that require graphic solutions

Question 21
A heavy pendulum bob is swinging back and forth when the string, supporting it, suddenly breaks. Ignoring the mass of the string and air resistance, draw the path of the subsequent motion of the bob if the string breaks when the bob is at its highest point.



Question 22
A long vertical tube with a weightless movable piston inside has its lower end under water. The tube is motionless throughout the experiment. The cross-sectional area of the piston is A. The initial position of the piston is just over the surface of the water. Then it is slowly moved upwards. Sketch a graph of the force necessary to lift the piston as a function of the height of the piston.

Question 23

On the same graph, draw the following two functions: 1) kinetic energy versus speed of a particle according to classical mechanics and 2) the same function for Einstein's relativistic theory.

Question 24

An electron is moving at a constant velocity in a region of space with a uniform electric field of $1.0 \cdot 10^3$ N/C and a uniform magnetic field of $1.0 \cdot 10^{-3}$T. The electric field has only an x-component while the magnetic field has only a y-component.

Sketch a vector diagram for the velocity of the electron, the magnetic field, and the electric field in the Cartesian system of coordinates.

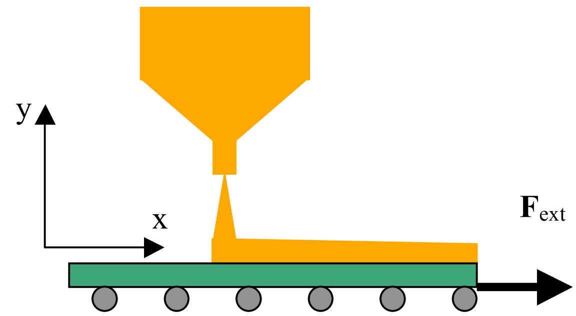
Question 25

A brick is on an incline whose angle of inclination can be changed from 0 to 90 degrees.

Sketch a graph of the force of friction acting on the brick versus the angle of inclination in the given range.

Part C: Problems

Problem 1



Sand from a stationary hopper falls onto a moving conveyor belt at a rate of 5.00 kg/s as shown above. The conveyor belt is supported by frictionless rollers and moves at a constant speed of 0.75 m/s under the action of a constant horizontal external force $\mathbf{F}_{ext}$ supplied by the motor that drives the belt. Find:

1) the force of friction exerted by the belt on the sand;
2) the external force $\mathbf{F}_{ext}$;
3) the work done by $\mathbf{F}_{ext}$ in 1 s;
4) the kinetic energy acquired by the falling sand each second due to the change in its horizontal motion;
5) compare and analyse the answers to 3) and 4).

Problem 2

A source of electric power with electromotive force E and internal resistance r, is connected to a resistor.

1) Find the power delivered to the resistor as a function of the resistance of the resistor.
2) Find the power produced by the source as a function of the resistance of the resistor.
3) Sketch graphs of these two functions.
4) Treating the efficiency of the source as the ratio of power consumed to power produced by the source, find the efficiency of the source of electric power as a function of the resistance of the resistor and sketch the graph of this function.
5) Find the resistance of the resistor that corresponds to the maximum power delivered to the resistor, and calculate the efficiency for this resistance.

Problem 3

A point source of light is moving uniformly along a straight line, which intersects the optical axis of a converging lens under a small angle α at a distance $2F$ from the lens with a focal distance F. The speed of the source of light is v_0.

i. Draw the lens, the trajectory of the source of light, and the trajectory of its image produced by the lens.
ii. Draw the vector diagram for the velocity of the source of light, the velocity of the image, and the relative velocity of the image with respect to the source.
iii. Determine the minimum relative speed of the image of the source of light with respect to the source.

CAP High School Prize Exam

7 April 2006

Marker's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

Sponsored by:

Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists 2006 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on a student's performance on sections A, B, and C of the exam. Performance on the questions in parts A and B will be used to determine whose written work in part C will be marked for prize consideration by the CAP Exam National Committee. Part A consists of twenty multiple-choice questions; part B consists of five questions that require graphic solution. The problems in part C can also require graphing.
Non-programmable calculators may be used. The multiple-choice questions must be **on the answer card/sheet** provided. Solutions to the three long problems must be written on **three separate** sheets, as they will be marked by people in different parts of Canada.

Data

Speed of light c = 3.00×10^{8} m/s
Gravitational constant G = 6.67×10^{11} $N \cdot m^2/kg^2$
Acceleration due to gravity g = 9.80 m/s^2
Fundamental charge e = 1.60×10^{-19} C
Mass of electron m_e = 9.11×10^{-31} kg
Mass of proton m_p = 1.67×10^{-27} kg
Planck's constant h = 6.63×10^{-34} J·s
Coulomb's constant $1/(4\pi\varepsilon_0)$ = 8.99×10^{9} $N \cdot m^2/C^2$
Boltzmann constant k = 1.38×10^{-23} J/K

Part A: Multiple Choice

Question 1

A meter stick is supported on a fulcrum at the 25-cm mark. A 0.50-kg object is hung from the 0-cm end of the meter stick, and the stick is balanced horizontally. The mass of the meter stick is:

(a) 0.25 kg;
(b) 0.50 kg; *
(c) 0.75 kg;
(d) 1.0 kg

Solution

Condition of equilibrium for torques with respect to the fulcrum is: 0.50x0.25 =Mx0.25→M=0.50 kg

---

Question 2

The radius of the Earth is $6.37 \cdot 10^{6}$m. Approximately, how fast is a person on the Equator moving due to the Earth rotation?

1. 5m/s
2. 50m/s
3. 500m/s *
4. 5000m/s

Solution

The relationship between the angular speed (ω) and the linear speed (v) gives: $v = \omega R$, where R is the radius of the Earth. If T is a period of rotation, $\omega = 2\pi / T$, and T = 24 x 3600 s.
$v = 2\pi R / T = 4.63 \times 10^{2} \approx 500$ m/s

Question 3

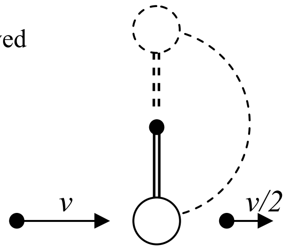
In a certain region of space, the electric field is zero. From this we can conclude that the electric potential in this region is:

(a) constant; *
(b) zero;
(c) positive;
(d) negative.

Solution

In general, only (a) is the correct answer, because in the absence of field its work on a charge is zero; work is charge multiplied by the potential difference. Therefore, the potential difference must be zero, and the potential is everywhere the same.

Question 4
A bullet of mass m and speed v hits a pendulum bob of mass M at time t_1, and passes completely through the bob. The bullet emerges at time t_2 with a speed of $v/2$. The pendulum bob is suspended by a stiff rod of length l and negligible mass. After the collision, the bob can barely swing through a complete vertical circle. At time t_3, the bob reaches the highest altitude.

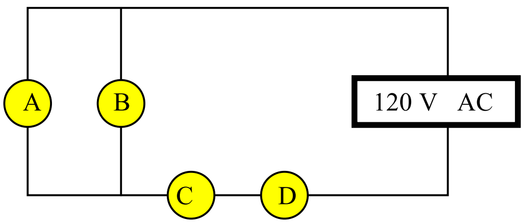


What quantities are conserved in this process?

(a) Total kinetic energy of the bob and the bullet during the time interval $\Delta t = t_2 - t_1$.
(b) Total momentum of the bob and the bullet during the time interval $\Delta t = t_2 - t_1$. *
(c) Total mechanical energy of the bob and the bullet during the time interval $t_3 - t_1$.
(d) Momentum of the bob after t_2.

Solution
(a) and (c) are wrong, because of the energy loss during the motion of the bullet inside the bob;
(d) is wrong, because an object experiences the external force of gravity that changes momentum.
(b) is the correct answer.

Question 5
The diagram shows a circuit with four identical light bulbs.



When we remove bulb A from the circuit, the light intensity of bulb C:
(a) remains the same;
(b) increases;
(c) decreases;*
(d) becomes zero.

Solution
The equivalent resistance of the part of circuit with the bulb B increases after the removal of the bulb A. This entails the reduction in current through the bulb C.

Question 6
The amplitude of an electromagnetic wave in vacuum is doubled with no other changes made to the wave. As a result of this doubling of the amplitude, which of the following statement is correct?
(a) The speed of propagation of the wave changes.
(b) The frequency of the wave changes.
(c) All of the above are true.
(d) None of the above is true.

(d) is correct because an amplitude, a speed of propagation, and a frequency of the wave are independent variables.

Question 7
Two objects are initially at rest on a frictionless surface. Object 1 has a greater mass than object 2. The same constant force starts to act on each object. The force is removed from each object after it accelerates over the distance d. After the force is removed from both objects, what statement is correct (p is a momentum; K is a kinetic energy)?
(a) $p_1 < p_2$; (b) $p_1 > p_2$; * (c) $K_1 > K_2$; (d) $K_1 < K_2$.

Solution
(b) Object 2 has a greater acceleration because of its smaller mass. Therefore, it takes less time to travel the distance d. Even though the force applied to objects 1 and 2 is the same, the change in momentum is less for object 2 because Δt is smaller.

Question 8
A dart is loaded into a spring-loaded toy dart gun by pushing the spring in by a distance d. For the next loading, the spring is compressed a distance $2d$. How much faster does the second dart leave the gun compared to the first?
(a) four times as fast;
(b) two times as fast;*
(c) the same;
(d) half as fast;
(e) one-fourth as fast.

Solution
According to the law of conservation of energy, the potential energy of a spring transforms into the kinetic energy of a dart. The potential energy is proportional to the square of the compression, and kinetic energy is proportional to the square of the initial speed of the dart. Therefore, the speed increases the same number of times as the compression does.

Question 9
One of two parallel metallic plates is uniformly charged with the charge $+q$, and the other one is charged with the charge $-q$. In this case, the electric field between them is E. When the negatively charged plate is recharged with a positive charge $4q$, the electric field between the plates becomes:
(a) 0;
(b) 1.5 E;*
(c) 2.5 E;
(d) 3E;
(e) 5E.

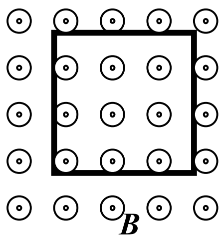
Solution
Initially, the field between two plates was $E_i = q / (A\varepsilon_0)$, where A is the area of the plate, and ε_0 is the permittivity of free space. This was a result of addition of the fields of two plates. After the replacement of the negatively charged plate, the field can be calculated as:

$$E_f = E_1 + E_2 = \frac{q}{2A\varepsilon_0} - \frac{4q}{2A\varepsilon_0} = -\frac{3q}{2A\varepsilon_0} = -1.5E_i$$

The magnitude of the field increases by the factor 1.5.

Question 10

A uniform magnetic field ***B*** is directed out of the page. A metallic wire has the shape of a square frame and is placed in the field as shown. The shape of the wire is steadily transformed into a circle in the same plane.



The current in the frame:

(a) is directed clockwise;*
(b) does not appear;
(c) is directed counterclockwise.

Solution

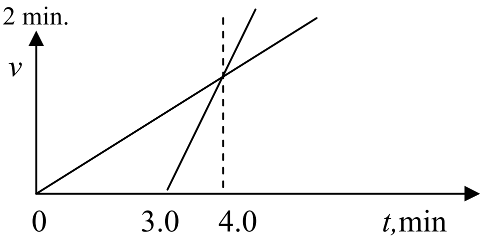
The area of a circle is greater than the area of a square with the same perimeter. During the transformation of the shape of the frame, the magnetic flux through the frame is increasing. It results in the induction of electric current in the loop. According to the Lenz's law, the current produces the magnetic field of the opposite direction to the external field whose flux is increasing. Therefore, the direction of the induced current is clockwise.

Question 11

The drawing shows velocity (v) versus time (t) graphs for two cyclists moving along the same straight segment of a highway from the same point. The second cyclist starts moving at $t = 3.0$ min.

At what time do the two cyclists meet?

(a) 4.0 min;
(b) 6.0 min;*
(c) 12 min.



Solution

From the slopes on the graph, we can obtain the ratio of accelerations of the two cyclists:

$a_2 / a_1 = 4$. The distances traveled by the two cyclists must be equal:

$$\frac{a_1 t^2}{2} = \frac{a_2 (t-3)^2}{2}; \frac{a_2}{a_1} = \left(\frac{t}{t-3}\right)^2;$$

$$\frac{t}{t-3} = 2; \Rightarrow t = 6\,\text{min}$$

Question 12

Two solid spheres manufactured of the same material freely fall down in the air. One sphere has a diameter twice as large as the other does. The force due to air resistance is proportional to the cross-sectional area of a moving object and is a quadratic function of the speed of an object. In some time after the beginning of motion in the presence of air resistance, the velocity of each sphere becomes constant. It is called the terminal velocity. The ratio of terminal velocities of the spheres v_{big}/v_{small} is:

(a) 2; (b) $\sqrt{2}$;* (c) $\frac{1}{2}$; (d) $\frac{1}{\sqrt{2}}$

Solution

The uniform motion with the terminal velocity is possible when the force of gravity of the object is equal to the force of the air resistance:

$$Mg = kv^2 \frac{\pi d^2}{4}; \qquad D\frac{\pi d^3}{6} g = kv^2 \frac{\pi d^2}{4} \quad (1)$$, where

M is the mass of a sphere; d is its diameter; D is the density of the material of the sphere; v is the velocity; and k is the constant coefficient for both spheres. From the equation (1), v is proportional to $\sqrt{d}$.

Question 13

Astronauts are weightless in orbit because:

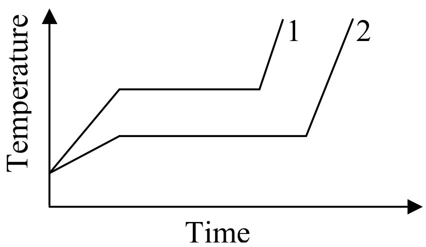
(a) they are beyond the pull of gravity;
(b) they travel in the accelerated frame of reference where fictitious centrifugal force is equal to centripetal force, and the net force on an astronaut is zero;
(c) they are in free fall together with their spacecraft;*
(d) of another cause (specify).

Solution

The single force acting on the spacecraft and the astronauts inside is the force of gravity. Therefore, astronauts are in the free fall.

Question 14

Two solid objects of the same mass are supplied with heat at the same rate $\Delta Q / \Delta t$. The temperature of the first object with latent heat L_1 and specific heat capacity c_1 changes according to the graph 1 on the diagram. The temperature of the second object with latent heat L_2 and specific heat capacity c_2 changes according to the graph 2 on the diagram.



Based on what is shown on the graph, the latent heats L_1 and L_2, and the specific heat capacities c_1 and c_2 in solid state obey the following relationships:

(a) $L_1 > L_2$; $c_1 < c_2$;
(b) $L_1 < L_2$; $c_1 < c_2$;*

(c) There is no information to obtain the relationship between latent heats;
(d) $L_1 > L_2$; $c_1 > c_2$;
(e) $L_1 < L_2$; $c_1 > c_2$.

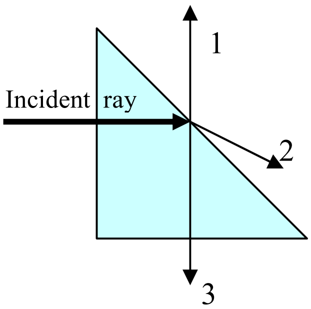
Solution

(b) The horizontal segment of the graph corresponds to the melting of the solids. Under the same condition, the two identical masses have different time of phase transition. The time of the second object is greater, therefore, its latent heat is also greater.

(e) Comparing the slopes of the diagrams for the solid state of the two objects, we see that the same heat causes less change of temperature for the second object. Hence, the second object has the greater specific heat capacitance.

Question 15

A light ray strikes a prism as shown in the drawing. The angles of the prism are 90°, 45°, and 45°. The critical angle of the prism material is 49°. What rays are the possible continuations of the incident ray?



1. The ray 1 only.
2. The ray 2 only.
3. The ray 3 only.
4. Both rays 1 and 3.
5. Both rays 2 and 3.*

Solution

The angle of incidence of the initial ray on the inclined side of the prism is less than the critical angle, therefore, the Snell's law gives the possibility for the ray to be refracted in the direction of the ray 2. In addition, there is also a possibility of reflection along ray 3.

Question 16

You stand on a platform at a train station and listen to a train horn as the train approaches the station at a constant velocity. While the train approaches, but before it arrives, you hear:

(a) the intensity and the frequency of the sound both increasing;
(b) the intensity and the frequency of the sound both decreasing;
(c) the intensity increasing and the frequency decreasing;
(d) the intensity decreasing and the frequency increasing;
(e) the intensity increasing and the frequency remaining the same;*
(f) the intensity and the frequency remaining the same.

Solution

The intensity of sound is increasing because the train approaches. However, the frequency does not change if the speed of the train is the same. Due to the Doppler's effect this frequency is shifted.

Question 17

A linear conductor with current I_1 is placed along the axis of a circular conductor, which carries current I_2.

The magnetic force acting on each of the conductors is:

(a) zero;*
(b) directly proportional to the product of currents I_1 and I_2, and inversely proportional to the radius of the circular conductor;
(c) directly proportional to the product of currents I_1 and I_2, inversely proportional to the square of the radius of the circular conductor;
(d) directly proportional to the product of current I_1, current I_2, and the area of a circular conductor;

Solution

Each conductor is parallel to the direction of magnetic field lines of another conductor. If the wire with current is parallel to the magnetic field line, the sine of the angle between this two directions is zero, and the magnetic force on the wire is zero.

Question 18

An aquarium is filled with water. The lateral wall of the aquarium is 40 cm long and 30 cm high. Using 10 m/s^2 for the acceleration due to gravity, and 1 g/cm^3 for the density of water, the force on the lateral wall of the aquarium is:

(a) 36 N;
(b) 90 N;
(c) 180 N;*
(d) 1500 N.

Solution

The pressure P of water on the base of the aquarium is given by: $P = DgH$, where D is the density of water; H is the height of the wall. Pressure is a linear function of the height. Therefore, the average pressure is half of the maximum one. The force on the lateral wall can be found from: $F = P_{av}HL$, where L is the length of the wall.

$$F = [(1000x10x0.3) / 2] \text{ x } 0.3 \text{ x } 0.4 = 180 \text{ N.}$$

Question 19

A satellite is moved from one circular orbit around the earth, to another of lesser radius. Which of the following is true?

(a) The kinetic energy increases and the potential energy increases;
(b) The kinetic energy increases, and the potential energy decreases;*
(c) The kinetic energy decreases and the potential energy decreases;
(d) The kinetic energy decreases and the potential energy increases.

Solution

In the circular motion around the Earth, the centripetal force on the satellite is a gravitational force. Therefore, $v^2 = GM/R$, where M is the mass of the Earth, R is the radius of the Earth G is the universal gravitational constant. Therefore, the kinetic energy increases with the decrease in the radius of the orbit. The gravitational potential energy is negative and decreases with the decrease in radius.

Question 20

A little particle carries a positive charge $+q$ and is maintained at an altitude h over a large conductive uncharged plate. The electric force of interaction between the charge and the plate is **better** described as:

(a) zero force;
(b) an attractive force proportional to q/h^2;
(c) an attractive force proportional to q^2/h^2;
(d) an attractive force proportional to $q^2/4h^2$;*
(e) a repulsive force proportional to q^2/h^2.

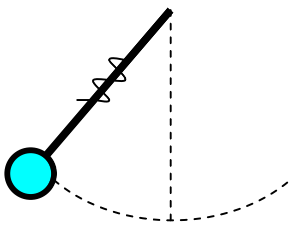
Solution

The lines of field of the point charge are everywhere perpendicular to the surface of the conductive plane. The pattern of the electric field lines over the plane is absolutely the same as for the system of two identical and oppositely charged particles separated by the distance *2h*. Therefore, we can apply the Coulomb's law to calculate the force.

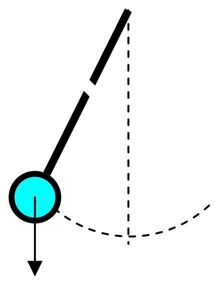
Part B: Questions that require graphical solutions

Question 21

A heavy pendulum bob is swinging back and forth when the string, supporting it, suddenly breaks. Ignoring the mass of the string and air resistance, draw the path of the subsequent motion of the bob if the string breaks when the bob is at its highest point.



Solution



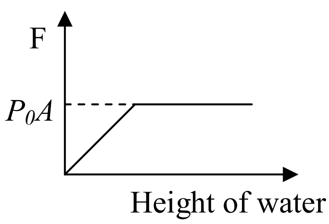
The motion will be vertically down under the force of gravity. No other forces are applied to the bob.

Question 22

A long vertical tube with a weightless movable piston inside has its lower end under the water. The tube is motionless throughout the experiment. The cross-sectional area of the piston is *A*. The initial position of the piston is just over the surface of the water. Then it is slowly moved upwards. Sketch a graph of the force applied to lift the piston during its motion as a function of the height of water under the piston.

Solution

The force will increase linearly until the pressure of water under the piston becomes equal to the atmospheric pressure P_0. After this moment, the force becomes constant.

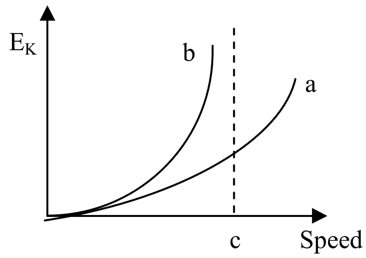


Question 23

On the same graph, draw the following two functions: 1) kinetic energy versus speed of a particle according to classical mechanics and 2) the same function in relativistic theory.

Solution

c = 3x10^{8}m/s



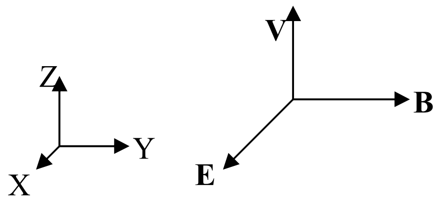
(a) Classical Mechanics (b) Relativistic Mechanics

Question 24

An electron is moving at a constant velocity in a region of space with a uniform electric field of $1.0{\cdot}10^3$ N/C and a uniform magnetic field of $1.0{\cdot}10^{-3}$T. The electric field has only an x-component while the magnetic field has only a y-component.

Sketch a vector diagram for the velocity of the electron, magnetic field, and electric field in the Cartesian system of coordinates.

Solution

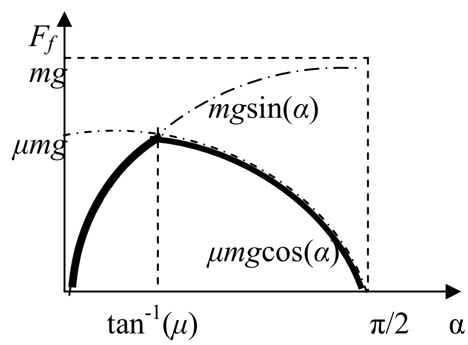


Question 25

A brick is on an incline whose angle of inclination can be changed from 0 to 90 degrees.

Sketch a graph of the force of friction acting on the brick versus the angle of inclination in the given range.

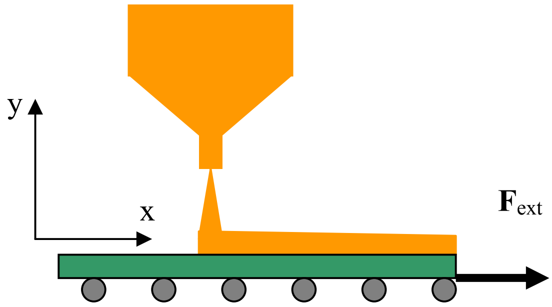
Solution



There are two segments of the graph: 1) when the brick is motionless, and static friction exists; and
2) when the brick moves and the force of kinetic friction acts on it. The motion starts when $\alpha=\tan^{-1}(\mu)$

1) For the motionless brick: $F_f = mg\sin(\alpha)$
2) For the moving brick: $F_{fk} = \mu mg\cos(\alpha)$

Part C: Problems
Problem 1



Sand from a stationary hopper falls onto a moving conveyor belt at a rate of 5.00 kg/s as in Fig.1. The conveyor belt is supported by frictionless rollers and moves at a constant speed of 0.750 m/s under the action of a constant horizontal external force $\mathbf{F}_{ext}$ supplied by the motor that drives the belt. Find:

1) the force of friction exerted by the belt on the sand;
2) the external force $\mathbf{F}_{ext}$;
3) the work done by $\mathbf{F}_{ext}$ in 1 s;
4) the kinetic energy acquired by the falling sand each second due to the change in its horizontal motion;
5) compare and analyse the answers to 3) and 4).

Solution

Introducing the vertical and horizontal axes of coordinates, we can find the rate of change of the x-component of momentum for the sand:

(1) $\dfrac{\Delta p_x}{\Delta t} = \dfrac{(5.00\ \text{kg})(0.750\ \text{m/s})}{1.00\ \text{s}} = \boxed{3.75\ \text{N}}$

This change occurs due to the only horizontal force on the sand – the force of friction: $f = \dfrac{\Delta p_x}{\Delta t} = \boxed{3.75\ \text{N}}$

(2) The belt is in equilibrium. It experiences the external force and the force of friction from the sand:

$\sum F_x = m a_x:\quad +F_{ext} - f = 0$ and $F_{ext} = \boxed{3.75\ \text{N}}$

(3) $W = F\Delta r\cos\theta = 3.75\ \text{N}\,(0.750\ \text{m})\cos 0° = \boxed{2.81\ \text{J}}$

(4) $\dfrac{1}{2}(\Delta m)\,v^2 = \dfrac{1}{2}5.00\ \text{kg}(0.750\ \text{m/s})^2 = \boxed{1.41\ \text{J}}$

(5) Friction between the sand and the belt converts half of the input work into internal energy of both objects.

Problem 2

A source of electric power with electromotive force E and internal resistance r, is connected to a resistor.

1) Find the power delivered to the resistor as a function of the resistance of the resistor.
2) Find the power produced by the source as a function of the resistance of the resistor.
3) Sketch graphs of these functions.
4) Treating the efficiency of the source as a ratio of the consumed power to the power produced by the source, find the efficiency of the source of electric power as a function of the resistance of the resistor and sketch the graph of this function.
5) Find the resistance of the resistor that corresponds to the maximum power delivered to the resistor, and calculate the efficiency for this resistance.

Solution

1)The power, delivered to a resistor, is given by:

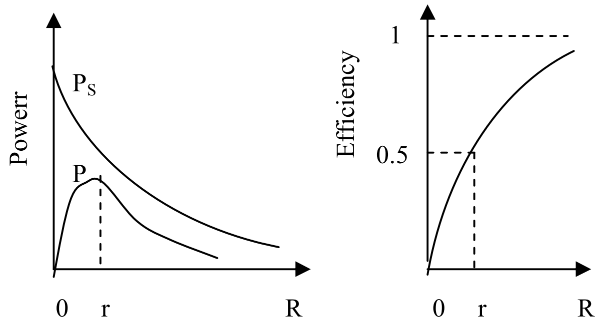
$P = \dfrac{E^2}{(R+r)^2}R$. This function has a maximum at $R = r$.

$$P_{max} = E^2/(4r).$$

2) The power, produced by the source is :

$P_S = \dfrac{E^2}{(R+r)^2}(R+r) = \dfrac{E^2}{(R+r)}$. This function is steadily decreasing with increasing of the resistance R (when $R = 0$, $P_S = E^2/r$).

4) The efficiency is given by: $e = R/(R+r)$



Resistance of an external resistor

Problem 3

A point source of light is moving uniformly along a straight line, which intersects the optical axis of a converging lens under a small angle α at a distance 2F from the lens with a focal distance F. The speed of the source of light is v_0.

i. Draw the lens, the trajectory of the source of light, and the trajectory of its image produced by the lens.
ii. Draw the vector diagram for the velocity of the source of light, the velocity of the image, and the relative velocity of the image with respect to the source.
iii. Determine the minimum relative speed of the image of the source of light with respect to the source.

Solution

First, we must investigate the trajectory of the image of the source of light. The path AB of the source during some time interval is shown on the diagram (fig 3.1) as an object. Its image A'B' is a trajectory of the image of the point source of light during the same time interval.

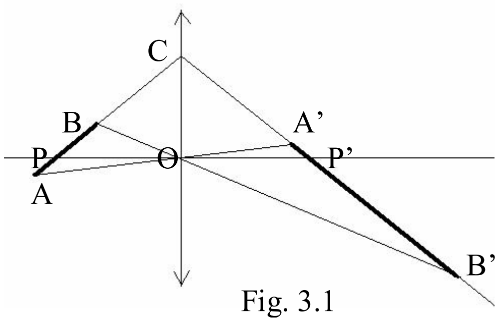


Fig. 3.1

Due to the properties of the point at the double-focal distance from the lens, the image of the object is also at the double-focal distance from the lens. Therefore, the trajectory of the image of the source of light A'B' crosses the optical axis at the point P' at the distance 2F from the lens. The triangles POC and P'OC are equivalent right angle triangles, and thus, the trajectory of the source and the trajectory of its image cross the optical axis of the lens under the same angle α. The diagram shows the different distances AB and A'B', covered by the source and its image during the same time interval, which permits to state that the speeds of the source and the image are different.
While the object moves at the velocity $\boldsymbol{v_0}$, its image moves at the velocity $\boldsymbol{u}$ along the line that crosses the optical axis at distance 2F from the vertex of lens and has the same magnitude of inclination α to the axis (fig.3.2).

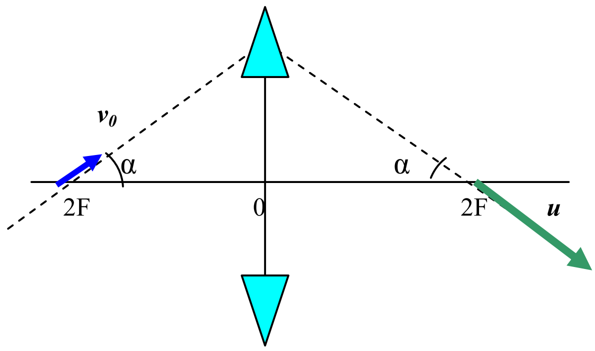


Fig.3.2

The vector diagram of the velocities (fig.3.3) shows that the relative velocity $\boldsymbol{V}$ has its minimum magnitude when it is perpendicular to the vector $\boldsymbol{u}$. From this condition, we can obtain the result: $|\boldsymbol{V}|_{\min} = v_0 \sin 2\alpha$.

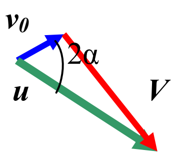


Fig.3.3

The obtained condition, however, does not permit us to conclude about the instantaneous position of the source of light at the moment that matches the result for the relative velocities, as it is clear from fig. 3.1. The only general conclusion is that the position of the source must guaranty the following relationship between the instantaneous speed of the source and the instantaneous speed of its image:

$$v_0 < u$$

CAP High School Prize Exam

April 5th 2007
9:00 – 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name:________________________________ Given Name:_______________________

Home Address:____

____ Postal Code:______________

Telephone: (   )____________________ E-mail: __________________________________

School: ____ Grade:_____

Physics Teacher: ____

Date of Birth: ___________________________ Sex: Male ☐ Female ☐

Citizenship: ____ or

Immigration Status:____

For how many years have you studied in a Canadian school? ___________________________

Would you prefer the further correspondence in French or English? _______________________

Sponsored by:

Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists
2007 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on a student's performance on sections A, B, and C of the exam. Performance on the questions in parts A and B will be used to determine whose written work in part C will be marked for prize consideration by the CAP Exam National Committee. Part A consists of twenty multiple-choice questions; part B consists of five questions that require graphic solution. The problems in part C can also require graphing. The questions in part C have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.
Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

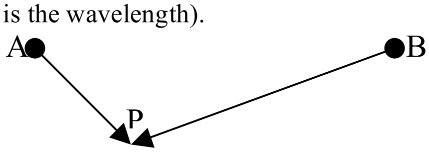
Speed of light $c = 3.00\text{x}10^8$ m/s
Gravitational constant $G = 6.67\text{x}10^{-11}$ N·m^2/kg^2
Acceleration due to gravity $g = 9.80$ m/s^2
Density of fresh water $\rho = 1.00\text{x}10^3$ kg/m^3
The normal atmospheric pressure $P_0 = 1.01\text{x}10^5$Pa
Fundamental charge $e = 1.60\text{x}10^{-19}$ C
Mass of electron $m_e = 9.11\text{x}10^{-31}$ kg
Mass of proton $m_p = 1.67\text{x}\ 10^{-27}$ kg
Planck's constant $h = 6.63\text{x}10^{-34}$ J·s
Coulomb's constant $1/(4\pi\varepsilon_0) = 8.99\text{x}10^9$ N·m^2/C^2
Boltzmann constant $k = 1.38\text{x}10^{-23}$ J/K

Part A: Multiple Choice

Question 1
A book is placed on a chair. Then a videocassette is placed on the book. The floor exerts a normal force:

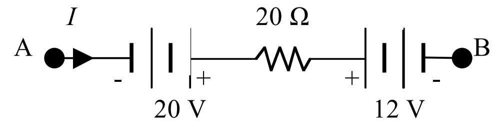
(a) on all three;
(b) only on the book;
(c) only on the chair;
(d) upwards on the chair and downwards on the book.

Question 2
The figure shows two point sources (A and B) of coherent mechanical waves of the same wavelength. Source B emits waves that are $+\pi$ radians out of phase with the waves from source A. Source A is 3λ distant from P and source B is 5λ distant from P (λ is the wavelength).



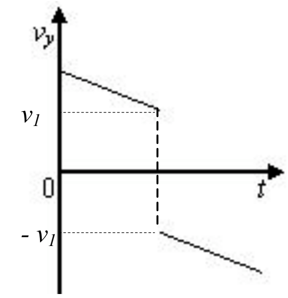
The phase difference between the waves arriving at P from A and B is
(a) 0 rad (b) $\pi/2$ rad (c) 2π rad (d) 3π rad (e) 4π rad

Question 3
What is the potential difference $V_B - V_A$ in the circuit segment below if the current $I = 1.5$ A?



(a) +22 V (b) –22 V (c) –38 V (d) +38 V (e) +2.0 V

Question 4
The graph on the right shows a velocity versus time graph for a ball.
Which explanation best fits the motion of the ball as shown by the graph?



(a) The ball falls, is caught, and is thrown down with a greater velocity.
(b) The ball rises, hits the ceiling, and falls down.
(c) The ball falls, hits the floor, and bounces up.
(d) The ball rises, is caught, and then is thrown down.

Question 5
A light bulb A is rated at 60 W and a light bulb B is rated at 100 W. Both are designed to operate at 110 V. Which statement is correct?

(a) The 60 W bulb has a greater resistance and greater current than the 100 W bulb.
(b) The 60 W bulb has a greater resistance and smaller current than the 100 W bulb.
(c) The 60 W bulb has a smaller resistance and smaller current than the 100 W bulb.
(d) The 60 W bulb has a smaller resistance and greater current than the 100 W bulb.
(e) We need to know the resistivities of the filaments to answer this question.

Question 6
When a light ray travels between any two points, the path it takes is the one that

(a) covers the greatest distance;
(b) avoids travel in more than one medium;
(c) covers the least distance;
(d) takes the least time;
(e) is the mean between the longest and the shortest paths.

Question 7
A fountain sends water to a height of 100 metres. What must be the pressurization (above atmospheric pressure) of the water system driving the fountain? 1 ATM = 10^5 N/m^2.

(a) 10.8 ATM; (b) 9.80 ATM;
(c) 8.80 ATM; (d) 1.00 ATM.

Question 8
A bead of mass m, attached to a string, is pushed and starts to rotate in the vertical plane. Only the gravitaional force influences the rotation of the bead. The difference between the tension of the string when the bead is at the lowest point (T_L) and when the bead is at the upper point (T_U) is:

(a) $T_U - T_L = 2mg$;
(b) $T_U - T_L = 6mg$;
(c) $T_L - T_U = 2mg$;
(d) $T_L - T_U = 5mg$;
(e) $T_L - T_U = 6mg$.

Question 9
Two identical particles, each with a mass of 4.5 mg and charge of 30 nC, are moving directly toward each other with equal speeds of 4.0 m/s at the instant when the distance separating the two particles is equal to 25 cm. How far apart will they be from each other at the point of closest approach?

(a) 7.8 cm; (b) 9.8 cm; (c) 12 cm;
(d) 15 cm; (e) 20 cm.

Question 10
A straight wire of length L carries a current I in the positive z direction in a region where the magnetic field is uniform and specified by $B_x = 3B$, $B_y = -2B$, and $B_z = B$, where B is a constant. What is the magnitude of the magnetic force on the wire?
(a) 1.0 ILB; (b) 3.2 ILB; (c) 3.6 ILB;
(d) 4.2 ILB; (e) 5.0 ILB

Question 11
A spaceship of mass m circles a planet of mass M in a circular orbit of radius R. How much energy is required to transfer the spaceship to a circular orbit of radius $3R$?

(a) $GmM/(2R)$; (b) $GmM/(3R)$; (c) $GmM/(4R)$;
(d) $GmM/(6R)$; (e) $3GmM/(4R)$

Question 12
Three simple pendulums with strings of different lengths and bobs of different masses are pulled out from their position of equilibrium to angles of θ_1, $\theta_2 = 2\theta_1$, and $\theta_3 = 3\theta_1$, respectively. All angles θ_1, θ_2 and θ_3 are very small. The bobs are then released and start oscillating freely. Which answer better matches the results of measurements of the frequencies of the three pendulums?

(a) $f_1 = 2f_2$ and $f_1 = 3f_3$;
(b) $f_3 = 3f_1$ and $f_2 = 2f_1$;
(c) $f_1 = f_2 = f_3$;
(d) We need to know the mass of each bob to find the relationship between the frequencies;
(e) We need to know the length of each pendulum to find the relationship between the frequencies.

Question 13
The electric potential inside a charged solid spherical conductor in equilibrium:

(a) is always zero;
(b) decreases from its value at the surface to a value of zero at the centre;
(c) is constant and is equal to its value at the surface;
(d) increases from its value at the surface to a higher value at the centre.

Question 14
For which process below will the internal energy of a system NOT change?

(a) An adiabatic expansion or compression of an ideal gas;
(b) An isothermal expansion or compression of an ideal gas;
(c) An isobaric expansion or compression of an ideal gas;
(d) The freezing of a quantity of a liquid at its melting point;
(e) The evaporation of a quantity of a liquid at its boiling point.

Question 15
A diver shines an underwater searchlight at the surface of a pond whose water has an index of refraction $n = 1.33$. At what angle of incidence relative to the surface will the light be totally reflected?

(a) 41^o (b) 47^o (c) 49^o
(d) 51^o (e) 58^o

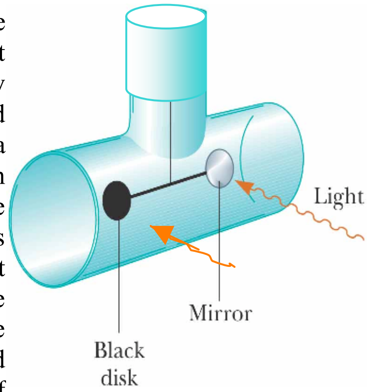
Question 16
According to the Bohr model of the atom, an electron can undergo a transition from one orbit that is closer to the nucleus to another which is farther from the nucleus, by absorbing a photon whose energy E depends on its frequency f as $E = hf$, where h is Planck's constant. An energy of 13.6 eV is needed to ionize a hydrogen atom by ejccting an electron from the lowest energy level. What is the longest wavelength of a photon that can eject the electron from the lowest energy level of the atom?

(a) 40 nm;
(b) 60 nm;
(c) 70 nm;
(d) 80 nm;
(e) 90 nm.

Question 17
One reason why we know that magnetic fields are not the same as electric fields is because the force they exert on a charge $+q$, is:
(a) in opposite directions in electric and magnetic fields if the charge is moving;
(b) parallel to the magnetic field and perpendicular to the electric field if the charge is moving;
(c) parallel to the electric field and perpendicular to the magnetic field if the charge is moving;
(d) zero in the electric field and nonzero in the magnetic field if the charge is not moving;

Question 18

In an experiment to prove the existence of the light pressure, two vertically oriented disks are attached as shown to the ends of a horizontal beam in an evacuated tube. The horizontal beam is suspended at its central point by a vertical wire. The surfaces of the disks are simultaneously illuminated by a parallel beam of light of high intensity.

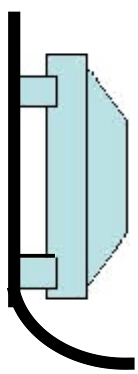


Which explanation best fits the behaviour of the two-disk system?

(a) The horizontal beam is displaced from its equilibrium position due to the equal light pressure on both discs.
(b) The horizontal beam rotates around the vertical wire with the mirror moving in the direction of light propagation and the black disk moving in the opposite direction.
(c) The horizontal beam rotates around the vertical wire with the black disk moving in the direction of light propagation and the mirror moving in the opposite direction.
(d) The experiment cannot show the existence of light pressure because light has no mass.

Question 19

You are shown a photo of a car driven on a vertical inside wall of a huge cylinder with a radius of 50 m. The coefficient of static friction between the car tires and the cylinder is $\mu_s = 0.8$. The minimum speed, at which the car can be driven like that, is:



(a) 20 km/h; (b) 70 km/h; (c) 90 km/h; (d) 120 km/h.

Question 20

An emf may be induced in

(a) a piece of linear wire when it moves in a uniform static magnetic field;
(b) a closed loop of wire moving with a fixed orientation at a constant velocity in a non-uniform static magnetic field;
(c) a closed loop of wire moving with a fixed orientation and accelerating in a uniform static magnetic field;
(d) the cases described in (a) and (b) only;
(e) the cases described in (b) and (c) only.

Part B: Questions that require graphical solutions

Question 21

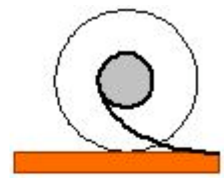
Two identical dielectric balls supported by insulating threads hang side by side, touching each other. The two balls are initially electrically neutral.
Sketch the position of the balls on their threads after one of the balls is positively charged and the other stays neutral.
Sketch the electric field lines near the balls.

Question 22

A 71-kg man stands on a spring scale in an elevator. Starting from rest, the elevator ascends, attaining its maximum speed of 1.2 m/s in 0.80 s. It travels with this constant speed for the next 2.0 seconds. The elevator then undergoes a uniform deceleration downwards (in the negative y direction) for 1.9 s and comes to rest.
Draw a diagram for the reading of the scale versus time during the motion of the elevator.

Question 23

A spool of thread on a horizontal surface may roll without slipping if someone pulls it by the free end of the thread. The spool may roll toward or away from the person who pulls the free end of the thread depending on the direction of the applied force.



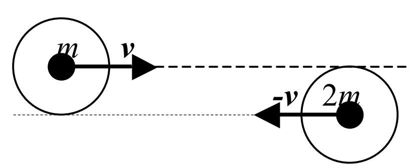
Sketch a side-view diagram of the spool of thread on the horizontal surface which shows the direction of the force applied to the free end of the thread in such a way that this force does not cause the rolling of the spool in either direction. The force belongs to the plane of the drawing.

Question 24

Consider the motion of a mass attached to a spring.
Sketch the following two graphs one below the other using the same scale for the time axis (in units of period):
1) Displacement of the mass versus time;
2) Kinetic energy of the mass versus time.

Question 25

A system consists of two ideal spheres with equal diameters that move toward each other as shown.



The spheres undergo a glancing collision.
Sketch a vector diagram for the linear momentum of each of the spheres and for the centre of mass of the system before and after the perfectly elastic collision of the spheres.

Part C: Problems

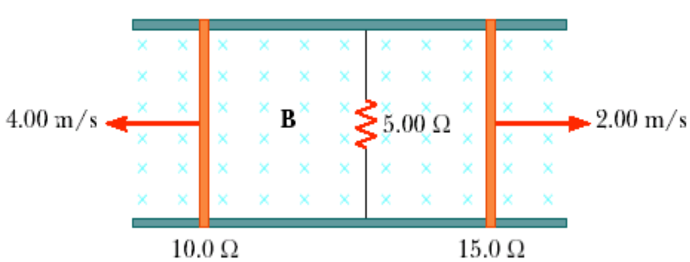
Problem 1

A student working in a laboratory has to remove a heavy box with new equipment from its initial position on the floor to the opposite wall of the room, which is d metres away. She may choose: either to pick up the box, to carry it in her hands across the room, and then to drop it on the floor in the new place; or to attach a rope to one of the corners of the box and pull the box along the floor to the new place. The mass M of the box and its contents is printed on the package. Acceleration due to gravity g, the coefficients of static and kinetic friction μ_s and μ_k for the box material and the floor, and other constants are available to the student in a handbook. After performing some calculations, the student notes that $\mu_k < \mu_s < 1$ and chooses the second method.

1) What physical quantities should the student compare to choose the proper method of moving the heavy box?
2) What parameter related to a quantity above shows an advantage for one of the two methods? Estimate this parameter for the two methods.
3) Draw a free-body diagram (i.e. a vector diagram) for the chosen method of pulling the box with the attached rope along the horizontal floor.
4) Dragging the box may be easier or harder. Express the condition that makes dragging easiest in terms of physics. Identify the parameter responsible for this condition using your free-body diagram.
5) Using the parameter found, give a numerical condition for the easiest way of moving the box by the student.

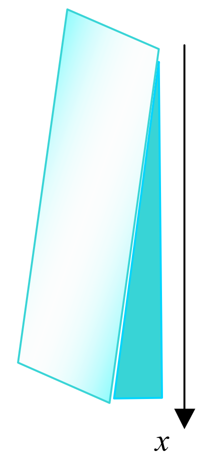
Problem 2

Two parallel rails with negligible resistance are 10.0 cm apart and are connected by a 5.00-Ω resistor. The circuit also contains two metal rods having resistances of 10.0 Ω and 15.0 Ω sliding along the rails as shown in the figure. The rods are pulled away from the resistor at constant speeds of 4.00 m/s and 2.00 m/s, respectively. A uniform magnetic field of magnitude 0.0100 T is applied perpendicular to the plane of the rails.



1) Determine the direction and the value of the current in the 5.00 Ω resistor.
2) Find the forces applied to the 10.0 Ω and the 15.0 Ω rods.
3) What is the force applied to the segment of the circuit with the 5.00 Ω resistor?
4) If the segment with the 5.00 Ω rod could slide along the rails and was released at a given time, how would this segment move (qualitatively)?

Problem 3



A soap film with an index of refraction $n = 1.33$ is contained within a rectangular wire frame. The frame is held vertically so that the film drains downwards and approximates the shape of a wedge with flat faces near the top. The thickness of the film at the very top is essentially zero. The film is viewed in reflected white light with near-normal incidence, and the first violet (λ = 420 nm) interference band is observed 3.00 cm from the top edge of the film.

1) Locate the first red (λ = 680 nm) interference band.
2) Determine the film thickness at the positions of the first violet and the first red bands.
3) What is the wedge angle of the film?

CAP High School Prize Exam

5 April 2007

Marker's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

Sponsored by:

Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists 2007 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on a student's performance on sections A, B, and C of the exam. Performance on the questions in parts A and B will be used to determine whose written work in part C will be marked for prize consideration by the CAP Exam National Committee. Part A consists of twenty multiple-choice questions; part B consists of five questions that require a graphic solution. The problems in part C can also require graphing.
Non-programmable calculators may be used. The multiple-choice questions must be **on the answer card/sheet** provided. Solutions to the three long problems must be written on **three separate** sheets, as they will be marked by people in different parts of Canada.

Data

Speed of light $c = 3.00\text{x}10^8$ m/s
Gravitational constant $G = 6.67\text{x}10^{-11}$ N·m^2/kg^2
Acceleration due to gravity $g = 9.80$ m/s^2
Density of fresh water $\rho = 1.00\text{x}10^3$ kg/m^3
The normal atmospheric pressure $P_0 = 1.01\text{x}10^5$Pa
Fundamental charge $e = 1.60\text{x}10^{-19}$ C
Mass of electron $m_e = 9.11\text{x}10^{-31}$ kg
Mass of proton $m_p = 1.67\text{x}\ 10^{-27}$ kg
Planck's constant $h = 6.63\text{x}10^{-34}$ J·s
Coulomb's constant $1/(4\pi\varepsilon_0) = 8.99\text{x}10^9$ N·m^2/C^2
Boltzmann constant $k = 1.38\text{x}10^{-23}$ J/K

Part A: Multiple Choice

Question 1

A book is placed on a chair. Then a videocassette is placed on the book. The floor exerts a normal force

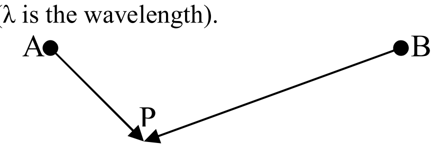
(a) on all three;
(b) only on the book;
(c) only on the chair;
(d) upwards on the chair and downwards on the book.

Solution

Answer **(c)** is correct because the floor is in contact only with the chair (the normal force is a force of reaction).

Question 2

The figure shows two point sources (A and B) of coherent mechanical waves of the same wavelength. Source B emits waves that are $+\pi$ radians out of phase with the waves from source A. Source A is 3λ distant from P and source B is 5λ distant from P (λ is the wavelength).



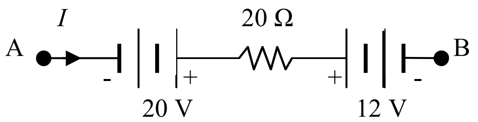
The phase difference between the waves arriving at P from A and B is

(a) 0 rad (b) $\pi/2$ rad (c) 2π rad (d) 3π rad (e) 4π rad

Solution

If two points are separated by the distance equal to the wavelength λ, the phase difference between oscillations of the wave in these points is equal to 2π rad. Without the initial phase difference between the waves from two sources, the paths difference in the point P would be 2λ, which is equivalent to the phase difference of 4π rad. As the wave, emitted from the point B, is π rad ahead relative to the phase of the wave from the point A, the resultant phase difference in the point P is equal to $(4\pi - \pi)$ rad $= 3\pi$ rad. Answer **(d).**

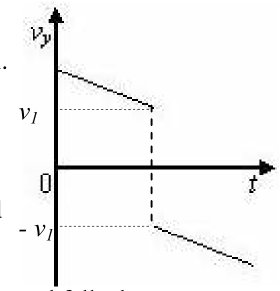
Question 3
What is the potential difference $V_B - V_A$ in the circuit segment below if the current I = 1.5 A?



(a) +22 V (b) –22 V (c) –38 V (d) +38 V (e) +2.0 V

Solution
$V_B - V_A$ = - 12 V + 20Ω x (- 1.5 A) + 20 V = - 22 V.
Answer **(b)** is correct.

Question 4
The graph on the right shows the velocity versus time graph for a ball. Which explanation best fits the motion of the ball as shown by the graph?



(a) The ball is falling, is caught, and is thrown down with greater velocity.
(b) The ball is rising, hits the ceiling, and falls down.
(c) The ball is falling, hits the floor, and bounces up.
(d) The ball is rising, is caught, and then is thrown down.

Solution
At the beginning, the motion must be decelerated. So, answers (a) and (c) are wrong. As in (d) it is said that the ball is caught, it means that the ball is stopped. Some time interval must be shown for the state of the ball in the hands of a person, who has caught it.
(b) is the correct answer.

Question 5
A light bulb A is rated at 60 W and a light bulb B is rated at 100 W. Both are designed to operate at 110 V. Which statement is correct?

(a) The 60 W bulb has a greater resistance and greater current than the 100 W bulb.
(b) The 60 W bulb has a greater resistance and smaller current than the 100 W bulb.
(c) The 60 W bulb has a smaller resistance and smaller current than the 100 W bulb.
(d) The 60 W bulb has a smaller resistance and greater current than the 100 W bulb.
(e) We need to know the resistivities of the filaments to answer this question.

Solution
All answers suppose the calculation of a ratio of resistances and currents for two bulbs. For resistances we use the formula for power P as: $P = V^2/R$; for currents the other formula is more convenient: P = IxV.
So, $R_{60} / R_{100} = 100 / 60 = 5/3$; and $I_{60} / I_{100} = 60 /100 = 3/5$.
Therefore, the bulb A has greater resistance and smaller current through it.
Answer **(b)** is correct.

Question 6
When a light ray travels between any two points, the path it takes is the one that

(a) covers the greatest distance;
(b) avoids travel in more than one medium;
(c) covers the least distance;
(d) takes the least time;
(e) is the mean between the longest and the shortest paths.

Answer **(d)** is correct according to the Fermat's principle.

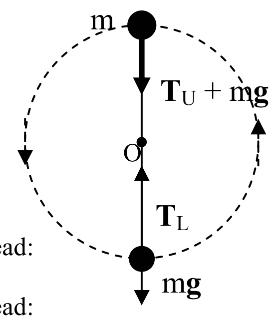
Question 7
A fountain sends water to a height of 10 meters. What must be the pressurization (above atmospheric pressure) of the water system driving the fountain? 1 ATM = 10^5 N/m^2.

(a) 10.8 ATM; (b) 9.80 ATM;
(c) 8.80 ATM; (d) 0.980 ATM.

Solution
$P_x + P_0 = \rho gh + P_0$, where P_0 is the atmospheric pressure; ρ is the density of water; g is acceleration to gravity; h is given to be 100 m. $P_x = \rho gh = 1.00 \times 10^3 \times 9.80 \times 10 / 10^5$ = 0.980 ATM.
(d) is the correct answer.

Question 8
A bead of mass m, attached to a string, is pushed and starts to rotate in the vertical plane. Only the gravitational force influences the rotation of the bead. The difference between the tension of the string when the bead is at the lowest point (T_L) and when the bead is at the upper point (T_U) is:

(a) $T_U - T_L = 2mg$;
(b) $T_U - T_L = 6mg$;
(c) $T_L - T_U = 2mg$;
(d) $T_L - T_U = 5mg$;
(e) $T_L - T_U = 6mg$.



Solution
For the lower position of the bead:
$T_L - mg = mv_L^2/R$ (1)
For the upper position of the bead:
$T_U + mg = mv_U^2/R$ (2)
The law of conservation of energy gives the following:
$mv_L^2/2 = mv_U^2/2 + 2mgR$ (3)
Subtracting equation (2) from (1) and taking into account (3), we can obtain that
$T_L - T_U = 2mg + 4mg = 6mg$
Answer **(e)**

Question 9

Two identical particles, each with a mass of 4.5 mg and charge of 30 nC, are moving directly toward each other with equal speeds of 4.0 m/s at the instant when the distance separating the two particles is equal to 25 cm. How far apart will they be from each other at the point of closest approach?

(a) 7.8 cm; (b) 9.8 cm; (c) 12 cm;
(d) 15 cm; (e) 20 cm.

Solution

According to the law of conservation of energy:

$$2\frac{mv^2}{2} + k\frac{q^2}{r_i} = k\frac{q^2}{x}$$, where $k = 9.0\text{x}10^9\text{Nm}^2/\text{C}$.

Solving the equation for x, we obtain answer **(a)**

Question 10

A straight wire of length L carries a current I in the positive x direction in a region where the magnetic field is uniform and specified by $B_x = 3B$, $B_y = -2B$, and $B_z = B$, where B is a constant. What is the magnitude of the magnetic force on the wire?

(a) 1.0 ILB; (b) 2.2 ILB; (c) 3.6 ILB;
(d) 4.2 ILB; (e) 5.0 ILB

Solution

x-component of the magnetic field B_x does not act on the wire where current is directed in x-direction. A magnitude of a vector sum of y- and z-components of the magnetic field is equal to

$$B_{\perp} = \sqrt{(B)^2 + (-2B)^2} = \sqrt{5}B = 2.2B$$

The magnitude of the magnetic force on the wire is given by

$F = B_{\perp} IL = 2.2\ ILB$

Answer **(b)**

Question 11

A spaceship of mass m circles a planet of mass M in an orbit of radius R. How much energy is required to transfer the spaceship to a circular orbit of radius $3R$?

(a) $GmM/(2R)$; (b) $GmM/(3R)$; (c) $GmM/(4R)$;
(d) $GmM/(6R)$; (e) $3GmM/(4R)$

Solution

According to the law of conservation of energy:

$$\frac{mv_i^2}{2} - G\frac{Mm}{R} + W = \frac{mv_f^2}{2} - G\frac{Mm}{3R}$$

The centripetal force is the gravitational force:

$$\frac{mv_i^2}{R} = G\frac{Mm}{R^2}$$, therefore the total energy on each orbit is:

$$E_i = -\frac{1}{2}G\frac{Mm}{R}; \qquad E_f = -\frac{1}{2}G\frac{Mm}{3R}$$

$$W = -G\frac{Mm}{6R} - \left(-G\frac{Mm}{2R}\right) = G\frac{Mm}{3R}$$

Answer **(b)**

Question 12

Three simple pendulums with strings of different lengths and bobs of different masses are pulled out from the position of equilibrium to angles of θ_1, $\theta_2 = 2\theta_1$, and $\theta_3 = 3\theta_1$, respectively. All angles θ_1, θ_2 and θ_3 are very small. The bobs are then released and start oscillating freely. Which answer better matches the results of measurements of the frequencies of the three pendulums?

(a) $f_1 = 2f_2$; and $f_1 = 3f_3$;
(b) $f_3 = 3f_1$ and $f_2 = 2f_1$;
(c) $f_1 = f_2 = f_3$;
(d) Need to know the mass of each bob to find the relationship among the frequencies;
(e) Need to know the length of each pendulum to find the relationship among the frequencies.

Solution

For a simple pendulum oscillating at small angles of deflection, the frequency depends only on the length of the pendulum and on the value of g.
Therefore, the correct answer is **(e).**

Question 13

The electric potential inside a charged solid spherical conductor in equilibrium:

(a) is always zero;
(b) decreases from its value at the surface to a value of zero at the centre;
(c) is constant and is equal to its value at the surface;
(d) increases from its value at the surface to a higher value at the centre.

Solution

The potential difference between the surface of a conductor and any point inside the conductor is calculated according to: $\Delta V = -\int E dr = 0$, because the electric field inside the sphere is 0. Therefore, the potential remains constant inside the charged conductive sphere.
Correct answer is **(c).**

Question 14

For which process below will the internal energy of a system NOT change?

(a) An adiabatic expansion or compression of an ideal gas;
(b) An isothermal expansion or compression of an ideal gas;
(c) An isobaric expansion or compression of an ideal gas;
(d) The freezing of a quantity of a liquid at its melting point;
(e) The evaporation of a quantity of a liquid at its boiling point.

Solution

For an ideal gas, the internal energy is proportional to the temperature and remains constant for the isothermal process only, because in this process the temperature is constant.
Answers (d) and (e) are also applicable to processes with the constant temperature. However, the change of state is

accompanied by the change in the internal energy. In example (d) the internal energy decreases; in the example (e) the internal energy increases.
The correct answer is **(b).**

Question 15
A diver shines an underwater searchlight at the surface of a pond whose water has an index of refraction n =1.33. At what angle of incidence relative to the surface will the light be totally reflected?

(a) 41^o (b) 47^o (c) 49^o (d) 51^o (e) 58^o

Solution
The angle of incidence of the initial ray on the surface is counted from the normal line to the surface. The critical angle of incidence α, is given by: $\sin \alpha = 1/n$; and $\alpha = 49^o$. The angle of the incident beam relative to the surface is given by $90^o - \alpha = 41^o$.
The correct answer is **(a).**

Question 16
According to the Bohr model of the atom, an electron can undergo a transition from one orbit that is closer to the nucleus to another which is farther from the nucleus, by absorbing a photon whose energy E depends on its frequency f as $E = hf$, where h is Planck's constant. Energy of 13.6 eV is needed to ionize a hydrogen atom by ejecting an electron from the lowest energy level. What is the longest wavelength of the photon that can eject the electron from the lowest energy level of the atom?

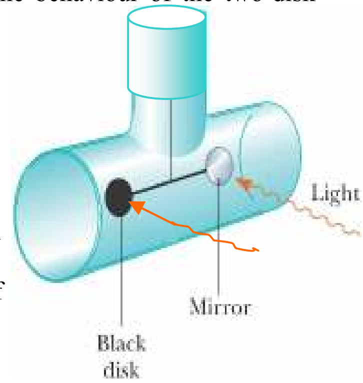
(a) 40 nm; (b) 60 nm; (c) 70 nm; (d) 80 nm; (e)90 nm.

Solution
To answer the problem, we need to convert electron volts in joules and use the relationship:
$E = hc/\lambda \rightarrow \lambda = hc/E = 90$ nm
Answer **(e).**

Question 17
One reason why we know that magnetic fields are not the same as electric fields is because the force exerted on a charge $+q$, is:
(a) in opposite directions in electric and magnetic fields if the charge is moving;
(b) parallel to the magnetic field and perpendicular to the electric field if the charge is moving;
(c) parallel to the electric field and perpendicular to the magnetic field if the charge is moving;
(d) zero in the electric field and nonzero in the magnetic field if the charge is not moving;

The correct answer is **(c).**

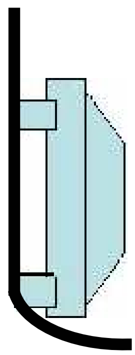
Question 18
In an experiment to prove the existence of the light pressure, two vertically oriented disks are attached as shown to the ends of a horizontal beam in an evacuated tube. The horizontal beam is suspended at its central point by a vertical wire. The surfaces of the disks are simultaneously illuminated by a parallel beam of light of high intensity.
Which explanation best fits the behaviour of the two-disk system?



(a) The horizontal beam is displaced from its equilibrium position due to the equal light pressure on both disks.
(b) The horizontal beam rotates around the vertical wire with the mirror moving in the direction of light propagation and the black disk moving in the opposite direction.
(c) The horizontal beam rotates around the vertical wire with the black disk moving in the direction of light propagation and the mirror moving in the opposite direction.
(d) The experiment cannot show the existence of light pressure because light has no mass.

Solution
The correct answer is **(b)** , because the mirror experiences a greater force from the light beam changing its momentum by $2p_0$ due to the reflection (the analogue of the perfectly elastic collision), while the black disk experiences only one half of this force absorbing the incident momentum totally (the analogue of the perfectly inelastic collision).

Question 19
You are shown a photo of a car driven on a vertical inside wall of a huge cylinder with a radius of 50 m. The coefficient of static friction between the car tires and the cylinder is $\mu_s = 0.8$. The minimum speed, at which the car can be driven like that, is:



(a) 20 km/h; (b) 70 km/h; (c) 90 km/h;
(d) 120 km/h.

Solution
Force of friction f_S is directed upwards and relates to the normal force N as: $f_S = N\mu_S$. Minimum velocity v corresponds to $f_S = mg$.
Normal force has to provide centripetal force:
$N = ma_r = mv^2/r$
$\mu_S mv^2/r = mg$
$v^2 = g\, r/\mu_S = 9.8 \cdot 50/0.8\ m^2/s^2$
$v = 25 m/s = 90 km/h$
The correct answer is **(c).**

Question 20
An emf may be induced in
(a) a piece of linear wire when it moves in a uniform static magnetic field;

(b) a closed loop of wire moving with a fixed orientation at a constant velocity in a non-uniform static magnetic field;
(c) a closed loop of wire moving with a fixed orientation and accelerating in a uniform static magnetic field;
(d) the cases described in (a) and (b) only;
(e) the cases described in (b) and (c) only.

The correct answer is **(d)**

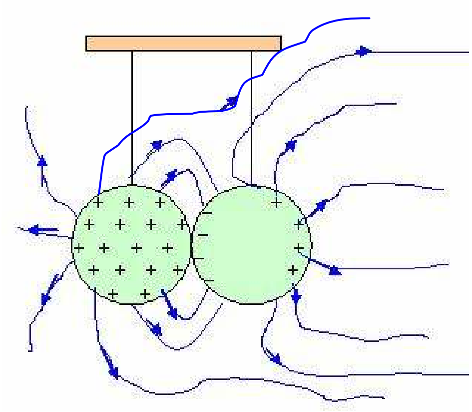
Part B: Questions that require graphical solutions

Question 21

Two identical dielectric balls supported by insulating threads hang side by side, touching each other. The two balls are initially electrically neutral.
Sketch the position of the balls on their threads after one of the balls is positively charged and the other stays neutral.
Sketch the electric field lines near the balls.

Solution

The material of both balls is an insulator. Therefore, charges cannot travel inside the balls. However, the electric field of the charged ball induces the polarization of molecules in the neutral ball and the neutral ball may be treated as a dipole. However, the electric field of the dipole is weaker than the field between the charged ball and the negative "end" of the neutral ball. Therefore, the field lines everywhere have the horizontal component from the charged ball to the neutral ball. The position of the balls and their threads does not change relatively to the initial one, as the balls attract each other.

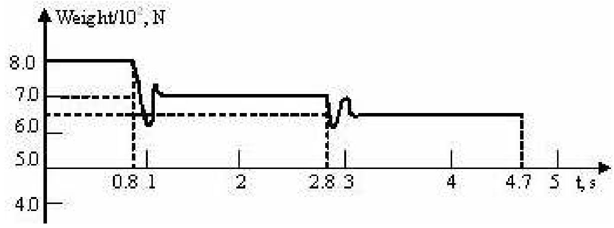


Question 22

A 71-kg man stands on a spring scale in an elevator. Starting from rest, the elevator ascends, attaining its maximum speed of 1.2 m/s in 0.80 s. It travels with this constant speed for the next 2.0 seconds. The elevator then undergoes a uniform acceleration downwards (in the negative y direction) for 1.9 s and comes to rest.
Draw a diagram for the reading of the scale versus time during the motion of the elevator.

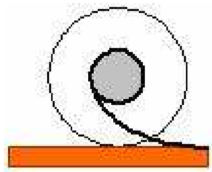
Solution

During the first 0.80 seconds, the elevator was moving upwards with constant acceleration; the scale showed the constant weight = 71 x (9.8 + 1.2/0.80) N = 8.0×10^2N.
During the next 2.0 seconds, the man traveled in an inertial frame of reference; and the scale showed the weight = 71 x 9.8 N = 7.0 x 10^2N.
During the last 1.9 seconds, the scale showed the weight = 71 x (9.8 – 1.2/1.9) N = 6.5 x 10^2 N.
At the moment the acceleration of the elevator changes, the spring of the scale starts to vibrate. We have no data about the elastic properties of the spring and should assume that vibrations damp very fast, so that a tiny time interval is needed to change the value of the spring's expansion or compression. Therefore, the real process cannot be shown as three perfect steps on the graph.



Question 23

A spool of thread on a horizontal surface may roll without slipping if someone pulls it by the free end of the thread. The spool may roll toward or away from the person who pulls the free end of the thread depending on the direction of the applied force.



Sketch a side-view diagram of the spool of thread on the horizontal surface which shows the direction of the force applied to the free end of the thread in such a way that this force does not cause the rolling of the spool in either direction. The force belongs to the plane of the drawing.

Solution

Rolling is instantaneous rotation about the contact point P. The gravitational and normal forces produce no torque about this point. In Figure 23.1, the force $\boldsymbol{F_0}$, applied at the critical angle θ_C,, exerts zero torque about the spool's contact point with the ground and so will not make the spool roll.

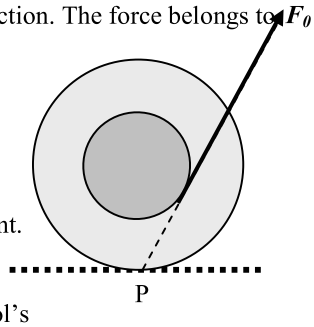


Fig. 23.1

Students are expected to draw only one diagram like in Fig. 23.1.

From the right-angle triangle, shown in the supplementary Figure 23.2, observe that

θ_C, = 90° – ϕ = 90° – (90°- γ) = γ

Thus, cos θ_C, = cos γ = r / R.

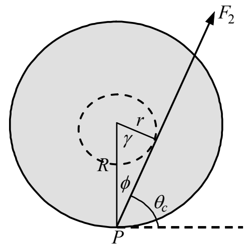
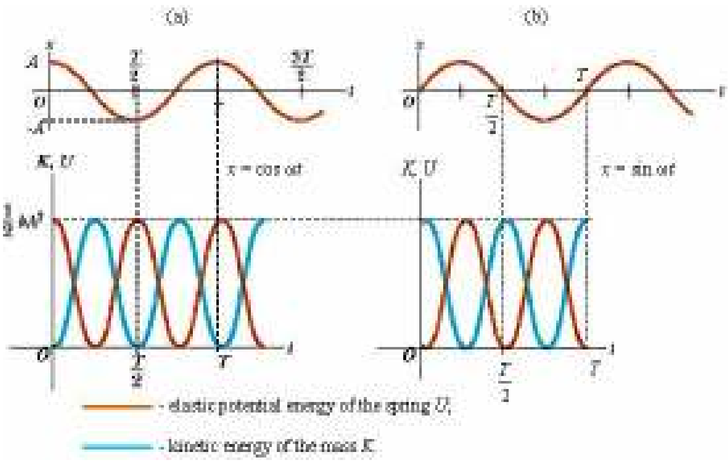


Fig. 23.2

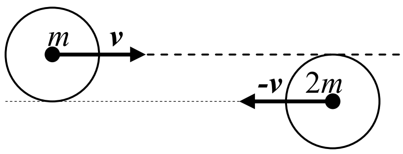
The direction of $\boldsymbol{F_0}$ depends on the radius of the layer of threads on the spool (r) and the external radius of the spool (R).

Question 24
Consider the motion of a mass attached to a spring.
Sketch the following two graphs one below the other using the same scale for the time axis (in units of period):
1) Displacement of the mass versus time;
2) Kinetic energy of the mass versus time.

Solution
Before sketching the graphs, we should write the formulae for (a) the displacement vs. time; (b) the elastic potential energy of the spring vs. time and the kinetic energy of the mass vs. time. Let us assume that the mass starts to move from the position with maximum displacement from the position of equilibrium (graph (a)).
(a) $x = A \cos \omega t$;
(b) $U = ½\ kx^2 = ½\ m\omega^2A^2\cos^2(\omega t)$;
$K = E - U = ½\ kA^2 - ½\ kx^2 = ½m\omega^2A^2\sin^2(\omega t)$.
Both diagrams (a) or (b) are correct. Students are expected to draw only the blue graphs for kinetic energy without graphs for potential energy.

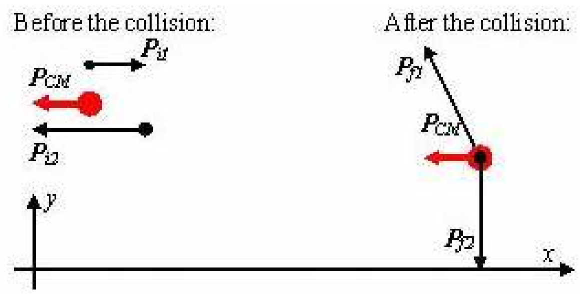


Question 25
A system consists of two ideal spheres with equal diameters that move toward each other as shown.



The spheres undergo a glancing collision.
Sketch a vector diagram for the linear momentum of each of the spheres and for the centre of mass of the system before and after the perfectly elastic collision of the spheres.

Solution
There are only internal forces between two spheres, and therefore the linear momentum of the system is conserved. This means that at any instant of time the linear momentum of the center of mass must be same. The collision is not head-on; therefore, the spheres scatter after the collision. The diagram for the momenta before the collision must show the relationship between two initial momenta ($p_2 = 2p_1$); the magnitude and the direction of the total momentum of the system. The system of coordinate is essential.
The diagram after the collision may vary if students have not enough time to solve the problem with two laws of conservation. However, it must show (1) that y-components of the two momenta are equal in magnitude and oppositely directed; (2) that the sum of x-components is equal to the initial total momentum of the system; and (3) that the less massive particle experiences the greater change in direction of motion and has the greater angle of scattering back.



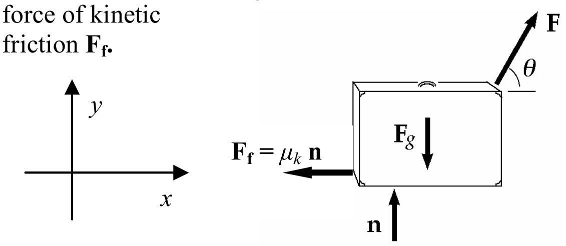
Part C: Problems
Problem 1
A student, working in a laboratory, has to remove a heavy box with new equipment from its initial position on the floor to the opposite wall of the room, which is *d* metres away. She may choose: either to pick up the box, to carry it in her hands across the room, and then to drop it on the floor in the new place; or to attach a rope to one of the corners of the box and pull the box along the floor to the new place. The mass *M* of the box and its contents is printed on the package. Acceleration due to gravity *g*, the coefficients of static and kinetic friction μ_s and μ_k for the box material and the floor, and other constants are available to the student in a handbook. After performing some calculations, the student notes that $\mu_k < \mu_s < 1$ and chooses the second method.

1) What physical quantities may the student compare to choose the proper method of moving the heavy box?
2) What parameter related to a quantity above shows an advantage for one of the two methods? Estimate and compare this parameter for the two methods.
3) Draw a free-body diagram (i.e. a vector diagram) for the chosen method of pulling the box with the attached rope along the horizontal floor.
4) Dragging the box may be easier or harder. Express the condition that makes dragging easiest in terms of physics. Identify the parameter responsible for this condition using your free-body diagram.
5) Using the parameter found, give a numerical condition for the easiest way of moving the box by the student.

Solution
1) The student could compare the maximum force, required to apply to the box, and the total work to be done on the box. We can assume that the mass of the load was rather great and therefore the student compared the maximum force.

2) The maximum force to remove the box shows the advantage of the second method. The maximum force to apply to pick up the box in a vertical direction is its gravitational force, or F_V = Mg. Dragging the box at a constant speed with the help of a rope, inclined to the horizontal with the angle θ, the student applies the force $F_{incl} = \mu_k Mg/(\cos\theta + \mu_k \sin\theta)$ (1), where $\mu_k < 1$. Varying θ from 0 to 90°, the student may apply the force from $\mu_k Mg < Mg$ to Mg, which corresponds the vertical lifting. Therefore, any angle $\theta < 90°$ gives an advantage in the applied force compared to the method of lifting the box vertically upward.

3) The free-body diagram includes the applied force **F**, the gravitational force $\mathbf{F_g}$, the normal force **n**, and the force of kinetic friction $\mathbf{F_f}$.



4) In terms of Physics, "easier" means with the minimum applied force F. From the free-body diagram:

$n + F\sin\theta = Mg$

$F\cos\theta = \mu_k n = \mu_k (Mg - F\sin\theta)$

Solving this system, we obtain the formula (1):

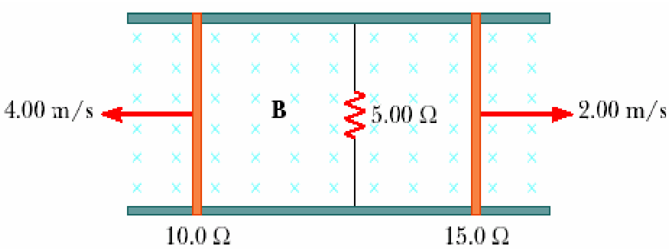
$F = \mu_k Mg/(\cos\theta + \mu_k \sin\theta)$. To find F_{min}, we may find the maximum for the expression $\cos\theta + \mu_k \sin\theta$. Thus, the parameter responsible for "easier" or "harder" removing the box is the angle θ.

5) The first derivative of the value $\cos\theta + \mu_k \sin\theta$ with respect to the angle θ is

$-\sin\theta + \mu_k \cos\theta = 0$, which gives the condition for the minimum force F as follows: $\tan\theta = \mu_k < 1$. We can conclude that depending on the coefficient of friction the angle $\theta < 45°$.

Problem 2

Two parallel rails with negligible resistance are 10.0 cm apart and are connected by a 5.00-Ω resistor. The circuit also contains two metal rods having resistances of 10.0 Ω and 15.0 Ω sliding along the rails as shown in the figure. The rods are pulled away from the resistor at constant speeds of 4.00 m/s and 2.00 m/s, respectively. A uniform magnetic field of magnitude 0.0100 T is applied perpendicular to the plane of the rails.



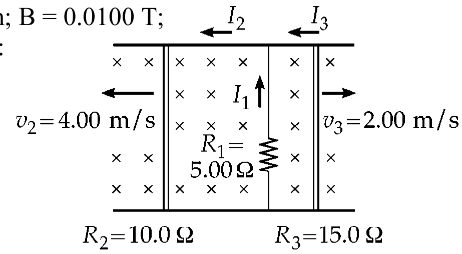
1) Determine the direction and the value of the current in the 5.00-Ω resistor.
2) Find forces, applied to the 10.0-Ω and the 15.0-Ω rods.
3) What is the force, applied to the segment of the circuit with the 5.00-Ω resistor?
4) If the segment with the 5.00-Ω rod could slide along the rails and was released at a given time, how would this segment move (qualitatively)?

Solution

Name the currents as shown in the diagram:

d = 10.0 cm; B = 0.0100 T;

1) Left loop:



$+Bdv_2 - I_2R_2 - I_1R_1 = 0$

Right loop: $+Bdv_3 - I_3R_3 + I_1R_1 = 0$

At the junction: $I_2 = I_1 + I_3$

Then, $Bdv_2 - I_1R_2 - I_3R_2 - I_1R_1 = 0$

$$I_3 = \frac{Bdv_3}{R_3} + \frac{I_1R_1}{R_3}.$$

So, $Bdv_2 - I_1(R_1 + R_2) - \frac{Bdv_3R_2}{R_3} - \frac{I_1R_1R_2}{R_3} = 0$

$$I_1 = Bd\left(\frac{v_2R_3 - v_3R_2}{R_1R_2 + R_1R_3 + R_2R_3}\right) = 145\ \mu\text{A (upwards)}$$

2) $I_2 = \frac{Bdv_2 - I_1R_1}{R_2} = 328\mu A$; $F_2 = I_2dB = 3.28\text{x}10^{-7}$N;

Magnetic force of the same magnitude opposes the external force and is directed to the right.

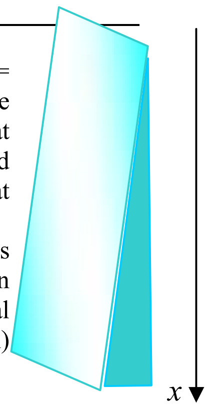
$I_3 = \frac{Bdv_3 + I_1R_1}{R_3} = 182\mu A$; $F_3 = I_3dB = 1.82\text{x}10^{-7}$N

3) $F_1 = I_1dB = 1.45\text{x}10^{-7}$N; F_1 is directed to the left (right-hand rule). After the bar with the 5.00-Ω resistor is released, it starts to accelerate to the left until the instant of time when its speed reaches 4 m/s. Since this moment, the current I_1 changes its direction to the opposite one and the bar starts to decelerate, then stops and starts moving to the right. Therefore, the bar oscillates, but the position of equilibrium is not stationary. The equilibrium position moves to the left, and oscillation of the bar is not harmonic.

Problem 3

A soap film with an index of refraction n = 1.33 is contained within a rectangular wire frame. The frame is held vertically so that the film drains downwards and approximates the shape of a wedge with flat faces near the top.

The thickness of the film at the very top is essentially zero. The film is viewed in reflected white light with near-normal incidence, and the first green (λ = 525 nm)



interference band is observed 4.00 cm from the top edge of the film.

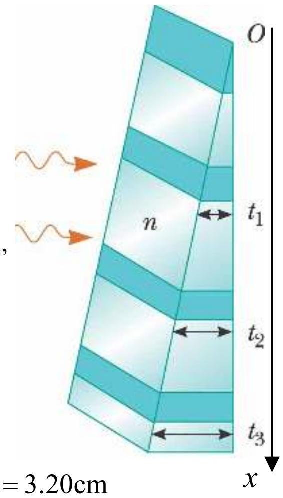
1) Locate the first violet ($\lambda = 420$ nm) interference band.
2) Determine the film thickness at the positions of the first violet and the first green bands.
3) What is the wedge angle of the film?

Solution

1) Bright bands are observed when $2nt = \left(m + \frac{1}{2}\right)\lambda$.

The first bright band ($m = 0$) corresponds to $nt = \frac{\lambda}{4}$. Let a subscript 1 identify the green band, and the subscript 2 identify the violet band.

Since $\frac{x_1}{x_2} = \frac{t_1}{t_2}$ (see the figure), we have



$$x_2 = x_1 \frac{t_2}{t_1} = x_1 \frac{\lambda_2}{\lambda_1} = 4.00\text{cm}\ \frac{420}{525} = 3.20\text{cm}$$

2) For the violet band:

$$t_2 = \frac{\lambda_2}{4n} = \frac{420\text{nm}}{4(1.33)} = 78.9\text{nm}$$

For the green band:

$$t_1 = \frac{\lambda_1}{4n} = \frac{525\text{nm}}{4(1.33)} = 98.7\text{nm}$$

3)

$$\theta \approx \tan\theta = \frac{t_1}{x_1} = \frac{98.7\text{nm}}{4.00\text{cm}} = 2.47 \times 10^{-6}\,\text{rad}$$

CAP High School Prize Exam

11 April 2008
9:00 – 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name:________________________________Given Name:_______________________

Home Address:____

____ Postal Code:______________

Telephone: (   )____________________ E-mail: __________________________________

School: ____ Grade:_____

Physics Teacher: ____

Date of Birth: ___________________________ Sex: Male ☐ Female ☐

Citizenship: ____ or

Immigration Status:____

For how many years have you studied in a Canadian school? ___________________________

Would you prefer the further correspondence in French or English? _______________________

Sponsored by:

Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists
2008 Prize Exam

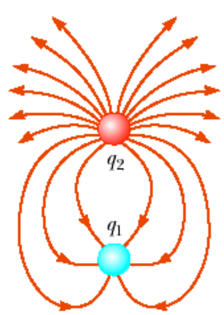
This is a three-hour exam. National ranking and prizes will be based on a student's performance on sections A, B, and C of the exam. Performance on the questions in part A will be used to determine whose written work in parts B and C will be marked for prize consideration by the CAP Exam National Committee. Part A consists of twenty multiple-choice questions; part B consists of three questions that require graphic solution. The problems in part C can also require graphing. The questions in part C have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.
Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

Speed of light $c = 3.00\text{x}10^8$ m/s
Gravitational constant $G = 6.67\text{x}10^{-11}$ N·m^2/kg^2
Acceleration due to gravity $g = 9.80$ m/s^2
Density of fresh water $\rho = 1.00\text{x}10^3$ kg/m^3
Specific heat capacity for water $c = 4.19\text{x}10^3$J/(kg·K)
The normal atmospheric pressure $P_0 = 1.01\text{x}10^5$Pa
Fundamental charge $e = 1.60\text{x}10^{-19}$ C
Mass of electron $m_e = 9.11\text{x}10^{-31}$ kg
Mass of proton $m_p = 1.67\text{x}\ 10^{-27}$ kg
Planck's constant $h = 6.63\text{x}10^{-34}$ J·s
Coulomb's constant $1/(4\pi\varepsilon_0) = 8.99\text{x}10^9$ N·m^2/C^2
Boltzmann constant $k = 1.38\text{x}10^{-23}$ J/K

Part A: Multiple Choice

Question 1



The Figure on the right shows the electric field lines for two point objects separated by a small distance. The charges q_1 and q_2 can be identified as

(a) q_1 is positive; q_2 is negative; $q_1/q_2 = -3$;
(b) q_1 is negative; q_2 is positive; $q_1/q_2 = -3$;
(c) q_1 is positive; q_2 is negative; $q_1/q_2 = -1/3$;
(d) q_1 is negative; q_2 is positive; $q_1/q_2 = -1/3$;
(e) q_1 is zero with charges redistributed over the surface of the object *1* due to electrostatic induction; q_2 is positive.

Question 2

As a simple pendulum swings back and forth, the forces acting on the suspended object may produce positive work, negative work or produce no work. Which of the forces if any from the listed three below does no work on the pendulum?

(a) Tension.
(b) Air resistance.
(c) Gravitational force.
(d) All the forces above do no work on the pendulum.
(e) All the forces above do work on the pendulum.

Question 3

A baseball bat is made of wood of uniform density. The bat is cut at the location of its centre of mass, as shown. Which of the following is true?

(a) The piece on the right has the smaller mass;
(b) The piece on the left has the smaller mass;
(c) Both pieces have the same mass;
(d) Impossible to determine without knowledge of the wood density.

Question 4

A section of a hollow pipe and a solid cylinder have the same radius, mass and length. They both rotate about their long central axes with the same angular speed. Which of the following is true?

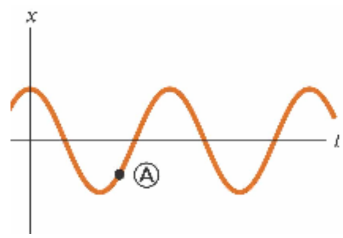
(a) The pipe has a higher rotational kinetic energy.
(b) The solid cylinder has a higher rotational kinetic energy.
(c) They have the same rotational kinetic energy.
(d) Impossible to determine without numerical data for radii, masses and lengths of the pipe and cylinder.

Question 5

In a comet-Sun system, the position of the comet closest to the Sun is called perihelion, and the position of the comet farthest from the Sun is called aphelion. The two quantities that both have their highest values when the comet is in perihelion are:

(a) the acceleration and the potential energy of the comet;
(b) the speed and the potential energy of the comet-Sun system;
(c) the speed and the acceleration of the comet;
(d) the acceleration and the total energy of the comet-Sun system.

Question 6
When an object is at point A on the graph of a simple harmonic motion below, what are respectively, its velocity and acceleration?
(a) both positive;
(b) both negative;
(c) positive and zero;
(d) positive and negative;
(e) negative and positive.



Question 7
Does the kinetic energy of an electron have an upper limit?
(a) Yes, m_ec^2.
(b) Yes, ½ m_ec^2.
(c) Yes, with another value.
(d) No.

Question 8
When you receive a chest x-ray at the hospital, the x-rays pass through a set of parallel ribs in your chest. Do your ribs act as a diffraction grating for the X-rays?
(a) Yes. They produce diffracted beams that can be observed separately.
(b) Not to a measurable extent. The ribs are too far apart.
(c) Essentially not. The ribs are too close together.
(d) Essentially not. The ribs are too few in number.
(e) Absolutely not. X-rays cannot diffract.

Question 9
A certain battery has some internal resistance. Can the potential difference across the terminals of the battery be equal to its *emf*?
(a) No.
(b) Yes, if the battery is absorbing energy by electrical transmission.
(c) Yes, if more than one wire is connected to each terminal.
(d)Yes, if the current in the battery is zero.

Question 10
A long solenoid with closely spaced turns carries electric current. Each turn of wire exerts:
(a) an attractive force on the next adjacent turn;
(b) a repulsive force on the next adjacent turn;
(c) zero force on the next adjacent turn; or
(d) either an attractive or a repulsive force on the next adjacent turn, depending on the direction of current in solenoid?

Question 11
An apple is held completely submerged just below the surface of a container of water. The apple is then moved to a deeper point in the water. Compared with the force needed to hold the apple just below the surface, what is the force needed to hold it at a deeper point?
(a) larger;
(b) essentially the same;
(c) smaller;
(d) impossible to determine.

Question 12
A small source radiates an electromagnetic wave with a single frequency into vacuum, equally in all directions. Indicate the pair of quantities that are decreasing as the wave moves away from the source:
(a) the frequency of the wave and an amplitude of electric field;
(b) the amplitude of the electric field and the intensity of the wave;
(c) the frequency and the intensity of the wave;
(d) the speed of propagation and the wavelength;

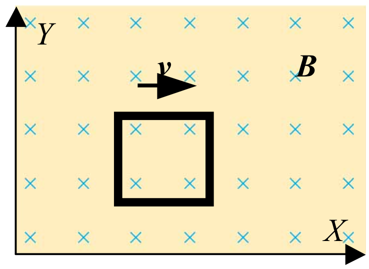
Question 13
Two identical balls with completely smooth surfaces are moving uniformly in free space without rotation. At some moment, they undergo a perfectly elastic glancing collision. After the collision, the angle between the two vectors of velocities is
(a) 30^o
(b) 45^o
(c) 90^o
(d) Impossible to answer without knowledge of the angle between the two velocities before the collision.

Question 14
Car batteries are often rated in ampere-hours. Does this information designate the amount of
(a) potential that the battery can supply;
(b) power;
(c) charge;
(d) energy?

Question 15
A laser beam travels from glass into air and strikes normally the smooth interface between the two media. In the air
(a) the light travels with a lower speed normally to the interface;
(b) the light travels with a higher speed normally to the interface;
(c) the light travels with unchanged speed but bends away from the normal;
(d) the light travels with a higher speed and bends away from the normal;
(e) the light travels with a lower speed and bends away from the normal.

Question 16
If only one external force acts on an object,
(a) it always changes the kinetic energy of the object;
(b) it always changes the speed of the object;
(c) it always changes the momentum of the object.

Question 17
A square conductive frame is moving in the vertical *YX* plane at a constant velocity ***v*** through a region of uniform magnetic field ***B*** directed perpendicular to the plane of the frame as shown in the figure. Does charge separation occur in the frame?



(a) Yes, with the top positive.
(b) Yes, with the top negative.
(c) No.
(d) Yes, with the left side negative.
(e) Yes, with the left side positive.

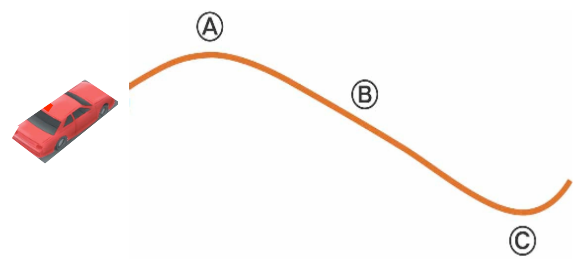
Question 18
A person spear-fishing from a boat sees a stationary fish a few meters away in a direction about 30^{o} below the horizontal. The index of refraction of the water is 1.34. Assume the dense spear does not change direction when it enters the water. To spear the fish, the person should
(a) aim above where he sees the fish;
(b) aim precisely at the fish; or
(c) aim below the fish?

Question 19
A spacecraft built in the shape of a sphere moves past an observer on the Earth with a speed of 0.5c. Approximately what shape does the observer measure for the spacecraft as it goes by?
(a) A sphere.
(b) A cigar shape, elongated along the direction of motion.
(c) A round "pillow" shape, flattened along the direction of motion.
(d) A conical shape, pointing in the direction of motion.

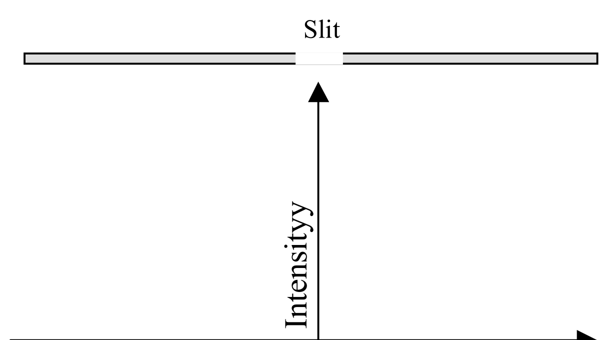
Question 20
One litre of water in a light thin-walled vessel is heated up under atmospheric pressure by an electric heater with unknown power rating. Initially the temperature of the water is 20^{o}C. After the temperature becomes 60^{o}C, it stops increasing, while the heater is still on. As the heater is unable to boil water, it is turned off. During the first 20 seconds the water becomes 2 degrees cooler.
Estimate the power output of the heater.
(a) 8.40 W.
(b) 420 W.
(c) 1260 W.
(d) 8400 W.

Part B: Questions that require graphical solutions

Question 1
A car accelerates uniformly along a curved horizontal road, moving from left toward right, as it is seen from a helicopter. Draw
(I) the vectors representing the forces exerted by the road on the car at points A and B;
(II) the vector of velocity at point C.



Question 2
Single-slit diffraction is observed with an interference pattern on a screen behind a slit for a red light source. The red light source is then replaced by a violet light source of same intensity, without any changes to distances and the slit width. Use the "Intensity/Position" system of coordinates as on the Figure below, to sketch the interference pattern on the screen for the two sources of light. If you think that the patterns are identical, show just one pattern. If you think that the two sources produce different patterns on the screen, make it clear which pattern corresponds to a given colour



Question 3
A skydiver jumps out of a plane at an altitude of 2000 m and begins her descent. At 1500 m, the skydiver reaches her terminal speed of 55 m/s. When the skydiver descends to a height of 500 m from the ground, she deploys her parachute which rapidly slows her down to 5 m/s to ensure a safe landing. Qualitatively, sketch the (vertical) speed of the skydiver as a function of height above ground starting just after she jumps out of the plane and finishing just before she lands.

Part C: Problems

Problem 1
A stepladder of negligible weight is constructed as shown in Figure 1C. A painter of mass 70.0 kg stands on the ladder 3.00 m from the bottom. Assuming the floor is frictionless, find

(a) the tension in the horizontal bar connecting the two halves of the ladder,
(b) the normal forces at *A* and *B*, and
(c) the components of the reaction force at the single hinge *C* that the left half of the ladder exerts on the right half.

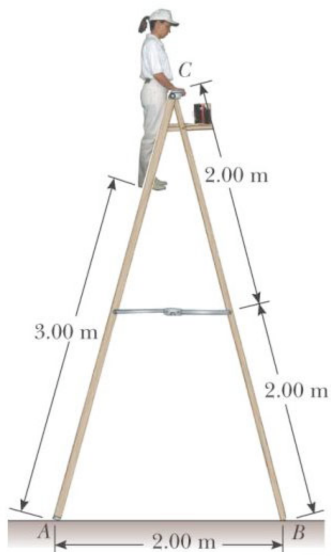


Fig.1C

Problem 2
Two horizontal metal plates, each 100 mm square, are aligned 10.0 mm apart, with one above the other. They are given equal-magnitude charges of opposite sign so that a uniform downward electric field of 2 000 N/C exists in the region between them. A particle of mass 2.00×10^{-16} kg and with a positive charge of 1.00×10^{-6} C leaves the centre of the bottom negative plate with an initial speed of 1.00×10^{5} m/s at an angle of 37.0° above the horizontal.
(a) Find the trajectory of the particle.
(b) Which plate does it strike?
(c) Where does it strike, relative to its starting point?

Problem 3
In the arrangement shown in Figure 3C, an object is hung from a string (with linear mass density μ =0.00200 kg/m) that passes over a light pulley. The string is connected to a vibrator (of constant frequency *f*), and the length of the string between point *P* and the pulley is *L* = 2.00 m. When

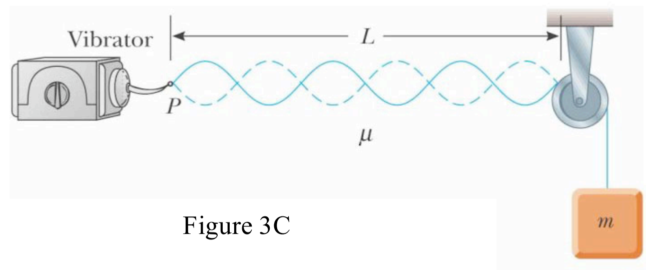


Figure 3C

the mass *m* of the object is either 16.0 kg or 25.0 kg, standing waves are observed; however, no standing waves are observed with any mass between these values. The speed of a transverse wave in a string experiencing the tension *T*, is given by: $v = \sqrt{T/\mu}$.

(a) What is the frequency of the vibrator?
(b) What is the total number of nodes observed along the compound string at this frequency, excluding the nodes at the vibrator and the pulley?
(c) What is the largest object mass for which standing waves could be observed?

CAP High School Prize Exam

11 April 2008
9:00 – 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name:________________________________Given Name:_______________________

Home Address:____

____ Postal Code:______________

Telephone: ( )____________________ E-mail: __________________________________

School: ____ Grade:_____

Physics Teacher: ____

Date of Birth: ___________________________ Sex: Male ☐ Female ☐

Citizenship: ____ or

Immigration Status:____

For how many years have you studied in a Canadian school? ____________________________

Would you prefer the further correspondence in French or English? _______________________

Sponsored by:

Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists 2008 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on a student's performance on sections A, B, and C of the exam. Performance on the questions in part A will be used to determine whose written work in parts B and C will be marked for prize consideration by the CAP Exam National Committee. Part A consists of twenty multiple-choice questions; part B consists of three questions that require graphic solution. The problems in part C can also require graphing. The questions in part C have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.

Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.

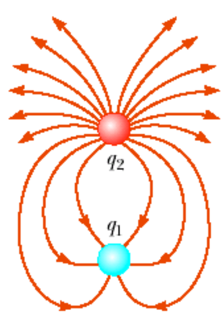
Data

Speed of light $c = 3.00\text{x}10^{8}$ m/s
Gravitational constant $G = 6.67\text{x}10^{-11}$ N·m²/kg²
Acceleration due to gravity $g = 9.80$ m/s²
Density of fresh water $\rho = 1.00\text{x}10^{3}$ kg/m³
Specific heat capacity for water $c = 4.19\text{x}10^{3}$J/(kg·K)
The normal atmospheric pressure $P_0 = 1.01\text{x}10^{5}$Pa
Fundamental charge $e = 1.60\text{x}10^{-19}$ C
Mass of electron $m_e = 9.11\text{x}10^{-31}$ kg
Mass of proton $m_p = 1.67\text{x}\ 10^{-27}$ kg
Planck's constant $h = 6.63\text{x}10^{-34}$ J·s
Coulomb's constant $1/(4\pi\varepsilon_0) = 8.99\text{x}10^{9}$ N·m²/C²
Boltzmann constant $k = 1.38\text{x}10^{-23}$ J/K

Part A: Multiple Choice

Question 1

The Figure on the right shows the electric field lines for two point objects separated by a small distance. The charges q_1 and q_2 can be identified as



(a) q_1 is positive; q_2 is negative; $q_1/q_2 = -3$;
(b) q_1 is negative; q_2 is positive; $q_1/q_2 = -3$;
(c) q_1 is positive; q_2 is negative; $q_1/q_2 = -1/3$;
(d) q_1 is negative; q_2 is positive; $q_1/q_2 = -1/3$;
(e) q_1 is zero with charges redistributed over the surface of the object *1* due to electrostatic induction; q_2 is positive.

Solution

Answer **(d)** is correct.

Question 2

As a simple pendulum swings back and forth, the forces acting on the suspended object may produce positive work, negative work or produce no work. Which of the forces if any from the listed three below does no work on the pendulum?

(a) Tension.
(b) Air resistance.
(c) Gravitational force.
(d) All the forces above do no work on the pendulum.
(e) All the forces above do work on the pendulum.

Solution

Answer **(a).**

Question 3

A baseball bat is made of wood of uniform density. The bat is cut at the location of its centre of mass, as shown. Which of the following is true?

(a) The piece on the right has the smaller mass;
(b) The piece on the left has the smaller mass;
(c) Both pieces have the same mass;
(d) Impossible to determine without knowledge of the wood density.

Solution

Answer **(b)** is correct

Question 4

A section of a hollow pipe and a solid cylinder have the same radius, mass and length. They both rotate about their long central axes with the same angular speed. Which of the following is true?

(a) The pipe has a higher rotational kinetic energy.
(b) The solid cylinder has a higher rotational kinetic energy.
(c) They have the same rotational kinetic energy.
(d) Impossible to determine without numerical data for radii, masses and lengths of the pipe and cylinder.

Solution

(a) is the correct answer

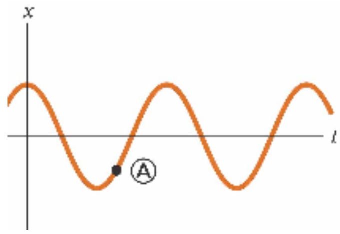
Question 5

In a comet-Sun system, the position of the comet closest to the Sun is called perihelion, and the position of the comet farthest from the Sun is called aphelion. The two quantities that both have their highest values when the comet is in perihelion are:

(a) the acceleration and the potential energy of the comet;
(b) the speed and the potential energy of the comet-Sun system;
(c) the speed and the acceleration of the comet;
(d) the acceleration and the total energy of the comet-Sun system.

Solution
Answer **(c)** is correct

Question 6
When an object is at point A on the graph of a simple harmonic motion below, what are respectively, its velocity and acceleration?
(a) both positive;
(b) both negative;
(c) positive and zero;
(d) positive and negative;
(e) negative and positive.



Answer **(a)** is correct

Question 7
Does the kinetic energy of an electron have an upper limit?
(a) Yes, m_ec^2.
(b) Yes, ½ m_ec^2.
(c) Yes, with another value.
(d) No.
Solution
(d) is the correct answer

Question 8
When you receive a chest x-ray at the hospital, the x-rays pass through a set of parallel ribs in your chest. Do your ribs act as a diffraction grating for the X-rays?
(a) Yes. They produce diffracted beams that can be observed separately.
(b) Not to a measurable extent. The ribs are too far apart.
(c) Essentially not. The ribs are too close together.
(d) Essentially not. The ribs are too few in number.
(e) Absolutely not. X-rays cannot diffract.
Solution
Answer **(b)** is correct

Question 9
A certain battery has some internal resistance. Can the potential difference across the terminals of the battery be equal to its *emf*?
(a) No.
(b) Yes, if the battery is absorbing energy by electrical transmission.
(c) Yes, if more than one wire is connected to each terminal.
(d)Yes, if the current in the battery is zero.
Solution
Answer **(d)** is correct

Question 10
A long solenoid with closely spaced turns carries electric current. Each turn of wire exerts:
(a) an attractive force on the next adjacent turn;
(b) a repulsive force on the next adjacent turn;
(c) zero force on the next adjacent turn; or
(d) either an attractive or a repulsive force on the next adjacent turn, depending on the direction of current in solenoid?
Solution
The correct answer is **(a)**

Question 11
An apple is held completely submerged just below the surface of a container of water. The apple is then moved to a deeper point in the water. Compared with the force needed to hold the apple just below the surface, what is the force needed to hold it at a deeper point?
(a) larger;
(b) essentially the same;
(c) smaller;
(d) impossible to determine.
Solution
The correct answer is **(b)**

Question 12
A small source radiates an electromagnetic wave with a single frequency into vacuum, equally in all directions. Indicate the pair of quantities that are decreasing as the wave moves away from the source:
(a) the frequency of the wave and an amplitude of electric field;
(b) the amplitude of the electric field and the intensity of the wave;
(c) the frequency and the intensity of the wave;
(d) the speed of propagation and the wavelength;
Solution
The correct answer is **(b)**

Question 13
Two identical balls with completely smooth surfaces are moving uniformly in free space without rotation. At some moment, they undergo a perfectly elastic glancing collision. After the collision, the angle between the two vectors of velocities is
(a) 30^o
(b) 45^o
(c) 90^o
(d) Impossible to answer without knowledge of the angle between the two velocities before the collision.
Solution
The correct answer is **(c)**

Question 14
Car batteries are often rated in ampere-hours. Does this information designate the amount of
(a) potential that the battery can supply;
(b) power;
(c) charge;
(d) energy?
Solution
Answer **(c)** is correct

Question 15
A laser beam travels from glass into air and strikes normally the smooth interface between the two media. In the air
(a) the light travels with a lower speed normally to the interface;
(b) the light travels with a higher speed normally to the interface;

(c) the light travels with unchanged speed but bends away from the normal;
(d) the light travels with a higher speed and bends away from the normal;
(e) the light travels with a lower speed and bends away from the normal.

Solution

The correct answer is **(b)**

Question 16

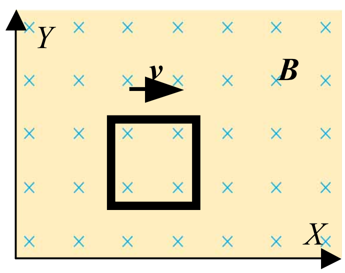
If only one external force acts on an object,
(a) it always changes the kinetic energy of the object;
(b) it always changes the speed of the object;
(c) it always changes the momentum of the object.

Solution

The correct answer is **(c)**

Question 17

A square conductive frame is moving in the vertical YX plane at a constant velocity $\boldsymbol{v}$ through a region of uniform magnetic field $\boldsymbol{B}$ directed perpendicular to the plane of the frame as shown in the figure. Does charge separation occur in the frame?



(a) Yes, with the top positive.
(b) Yes, with the top negative.
(c) No.
(d) Yes, with the left side negative.
(e) Yes, with the left side positive.

The correct answer is **(a)**

Question 18

A person spear-fishing from a boat sees a stationary fish a few meters away in a direction about 30^o below the horizontal. The index of refraction of the water is 1.34. Assume the dense spear does not change direction when it enters the water. To spear the fish, the person should
(a) aim above where he sees the fish;
(b) aim precisely at the fish; or
(c) aim below the fish?

Solution

The correct answer is **(c)**

Question 19

A spacecraft built in the shape of a sphere moves past an observer on the Earth with a speed of 0.5c. Approximately what shape does the observer measure for the spacecraft as it goes by?
(a) A sphere.
(b) A cigar shape, elongated along the direction of motion.
(c) A round "pillow" shape, flattened along the direction of motion.
(d) A conical shape, pointing in the direction of motion.

Solution

The correct answer is **(c)**

Question 20

One litre of water in a light thin-walled vessel is heated up under atmospheric pressure by an electric heater with unknown power rating. Initially the temperature of the water is 20^{o}C. After the temperature becomes 60^{o}C, it stops increasing, while the heater is still on. As the heater is unable to boil water, it is turned off. During the first 20 seconds the water becomes 2 degrees cooler.
Estimate the power output of the heater.
(a) 8.40 W.
(b) 420 W.
(c) 1260 W.
(d) 8400 W.

Solution

When the temperature of water reached its maximum, the system came to the state of dynamic equilibrium: the power loss became equal to the power, emitted by the heater. To simplify the problem, we can consider the temperature change of $\Delta T = 2^o$ small enough to not influence the power loss rate at the beginning of the cooling process during τ = 20s. Thus, the heater power

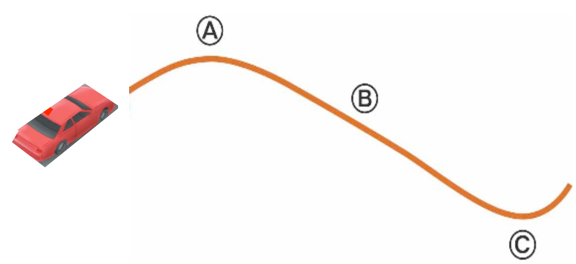
$$P = \frac{cm\Delta T}{\tau} = \frac{4200(J/kg\cdot K)\cdot 1kg\cdot 2K}{20s} = 420W$$

where c is the specific heat capacity of water; m is the mass of 1 litre of water. $\Delta T = 2^oC = 2$ K.
Correct answer is **(b)**

Part B: Questions that require graphical solutions

Question 1

A car accelerates uniformly along a curved horizontal road, moving from left toward right, as it is seen from a helicopter. Draw
(I) the vectors representing the forces exerted by the road on the car at points A and B;
(II) the vector of velocity at point C.



Solution

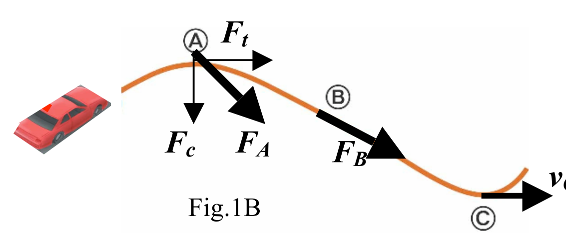
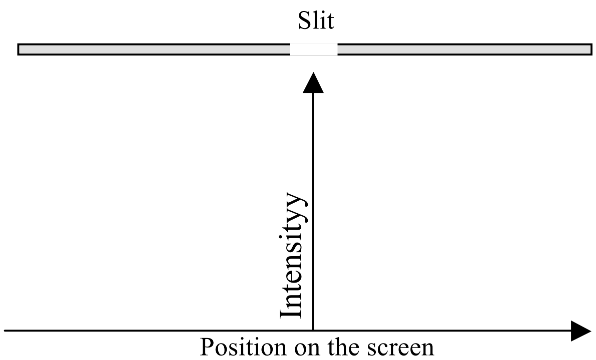


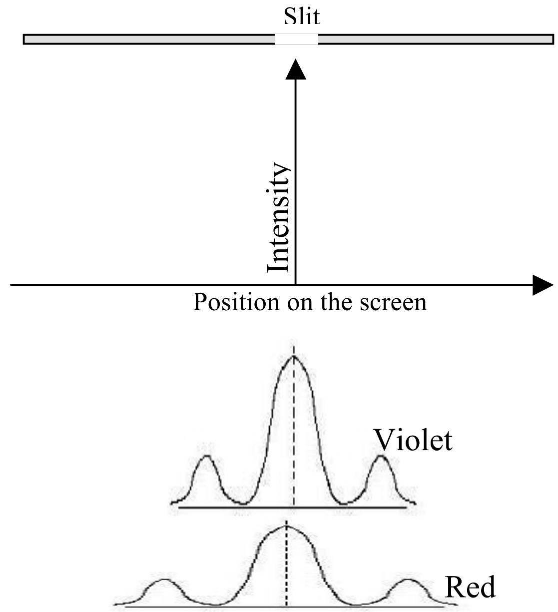
Fig.1B

$\boldsymbol{F_t}$ and $\boldsymbol{F_c}$ are the components of a force $\boldsymbol{F_A}$ of static friction between the tires of the car and the road at point A; vector of velocity $\boldsymbol{v_C}$ is tangential to the curve.

Question 2
Single-slit diffraction is observed with an interference pattern on a screen behind a slit for a red light source. The red light source is then replaced by a violet light source of same intensity, without any changes to distances and the slit width. Use the "Intensity/Position" system of coordinates as on the Figure below, to sketch the interference pattern on the screen for the two sources of light. If you think that the patterns are identical, show just one pattern. If you think that the two sources produce different patterns on the screen, make it clear which pattern corresponds to a given colour

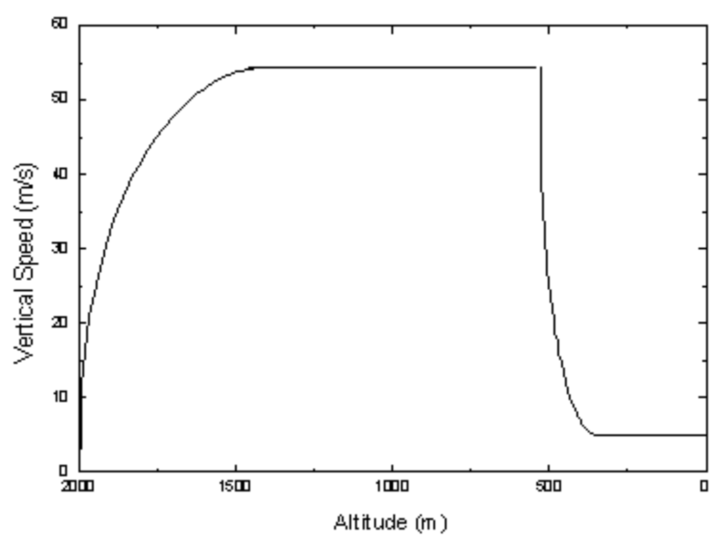


Solution
For both patterns, the central maximum of intensity on the screen must be opposite the slit and approximately twice as wide as the next couple of maxima. However, the patterns are different. Solution is based on the condition for observing the dark fringes in the interference pattern for sinle-slit diffraction: $w \sin\varphi = n\,\lambda$. The width w remains unchanged, while $\lambda_{red} > \lambda_{viol}$.
Positions of the red fringes must be farther from the origin of the system of coordinates than positions of the violet fringes for same order n.
As the intensities of the sources are equal, the amplitude of the red maximum is less than the amplitude of the violet maximum for the same order n, including the main maximum.



Question 3
A skydiver jumps out of a plane at an altitude of 2000 m and begins her descent. At 1500 m, the skydiver reaches her terminal speed of 55 m/s. When the skydiver descends to a height of 500 m from the ground, she deploys her parachute which rapidly slows her down to 5 m/s to ensure a safe landing. Qualitatively, sketch the (vertical) speed of the skydiver as a function of height above ground starting just after she jumps out of the plane and finishing just before she lands.

Solution



Part C: Problems

Problem 1

A stepladder of negligible weight is constructed as shown in Figure 1C. A painter of mass 70.0 kg stands on the ladder 3.00 m from the bottom. Assuming the floor is frictionless, find

(a) the tension in the horizontal bar connecting the two halves of the ladder,
(b) the normal forces at *A* and *B*, and
(c) the components of the reaction force at the single hinge *C* that the left half of the ladder exerts on the right half.

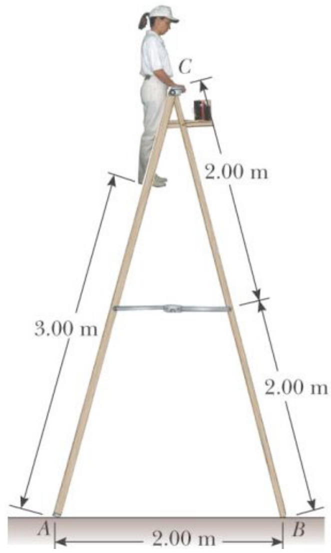


Fig.1C

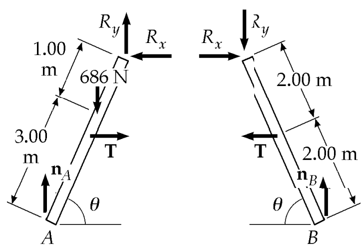
Solution

From geometry, observe that

$$\cos\theta = \frac{1}{4}$$

and

$$\theta = 75.5°$$



For the left half of the ladder, we have

$$\sum F_x = T - R_x = 0 \quad (1)$$

$$\sum F_y = R_y + n_A - 686 \text{ N} = 0 \quad (2)$$

$$\sum \tau_{\text{top}} = 686 \text{ N}(1.00\cos 75.5°) + T(2.00\sin 75.5°) - n_A(4.00\cos 75.5°) = 0 \quad (3)$$

For the right half of the ladder we have

$$\sum F_x = R_x - T = 0$$

$$\sum F_y = n_B - R_y = 0 \quad (4)$$

$$\sum \tau_{\text{top}} = n_B(4.00\cos 75.5°) - T(2.00\sin 75.5°) = 0 \quad (5)$$

Solving equations 1 through 5 simultaneously yields:

(a) $T = 133$ N

(b) $n_A = 429$ N and $n_B = 257$ N

(c) $R_X = 133$ N and $R_Y = 257$ N

The force exerted by the left half of the ladder on the right half is to the right and downward.

Problem 2

Two horizontal metal plates, each 100 mm square, are aligned 10.0 mm apart, with one above the other. They are given equal-magnitude charges of opposite sign so that a uniform downward electric field of 2 000 N/C exists in the region between them. A particle of mass 2.00×10^{-16} kg and with a positive charge of 1.00×10^{-6} C leaves the centre of the bottom negative plate with an initial speed of 1.00×10^5 m/s at an angle of 37.0° above the horizontal.

(a) Find the trajectory of the particle.
(b) Which plate does it strike?
(c) Where does it strike, relative to its starting point?

Solution

The particle feels a constant force:

$$\mathbf{F} = q\mathbf{E} = (1\times10^{-6}\text{ C})(2\,000\text{ N/C})(-\hat{\mathbf{j}}) = 2\times10^{-3}\text{ N}(-\hat{\mathbf{j}})$$

and moves with acceleration:

$$\mathbf{a} = \frac{\sum\mathbf{F}}{m} = \frac{(2\times10^{-3}\text{ kg}\cdot\text{m/s}^2)(-\hat{\mathbf{j}})}{2\times10^{-16}\text{ kg}} = (1\times10^{13}\text{ m/s}^2)(-\hat{\mathbf{j}})$$

.

Its *x*-component of velocity is constant at $(1.00\times10^5\text{ m/s})\cos 37° = 7.99\times10^4\text{ m/s}$. Thus it moves in a parabola opening downward. The maximum height it attains above the bottom plate is described by

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_{fi} - y)$$

$$0 = (6.02\times10^4\text{ m/s})^2 - (2\times10^{13}\text{ m/s}^2)(y_f - 0)$$

$$y_f = 1.81\times10^{-4}\text{ m}.$$

Since this is less than 10 mm, the particle does not strike the top plate, but moves in a symmetric parabola and strikes the bottom plate after a time given by

$$y_{fi} = y_{yi} + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = 0 + (6.02\times10^4\text{ m/s})t + \frac{1}{2}(-1\times10^{13}\text{ m/s}^2)t^2$$

since $t > 0$, $t = 1.20\times10^{-8}$ s.

The particle's range is $x_{fi} = x_{xi} + v_x t = 0 + (7.99\times10^4\text{ m/s})(1.20\times10^{-8}\text{ s}) = 9.61\times10^{-4}$ m

The particle strikes the negative plate after moving in a parabola with a height of 0.181 mm and a width of 0.961 mm.

Problem 3

In the arrangement shown in Figure 3C, an object is hung from a string (with linear mass density μ =0.00200 kg/m) that passes over a light pulley. The string is connected to a vibrator (of constant frequency f), and the length of the string between point P and the pulley is L = 2.00 m. When

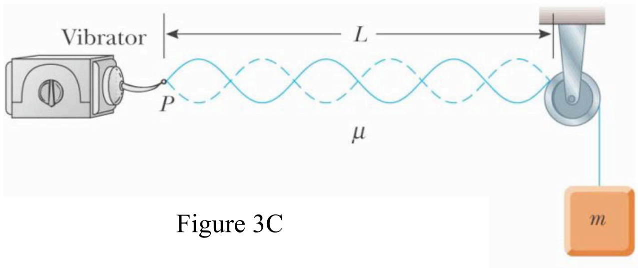


Figure 3C

the mass m of the object is either 16.0 kg or 25.0 kg, standing waves are observed; however, no standing waves are observed with any mass between these values. The speed of a transverse wave in a string experiencing the tension T, is given by: $v = \sqrt{T/\mu}$.

(a) What is the frequency of the vibrator?
(b) What is the total number of nodes observed along the compound string at this frequency, excluding the nodes at the vibrator and the pulley?
(c) What is the largest object mass for which standing waves could be observed?

Solution

(a) and (b)

Let n be the number of nodes in the standing wave resulting from the 25.0-kg mass. Then $n+1$ is the number of nodes for the standing wave resulting from the 16.0-kg mass. For standing waves, $\lambda = \frac{2L}{n}$, and the frequency is $f = \frac{v}{\lambda}$.

Thus, $f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}}$ and also $f = \frac{n+1}{2L}\sqrt{\frac{T_{n+1}}{\mu}}$

Thus,

$$\frac{n+1}{n} = \sqrt{\frac{T_n}{T_{n+1}}} = \sqrt{\frac{(25.0\text{ kg})g}{(16.0\text{ kg})g}} = \frac{5}{4}$$

Therefore, $4n + 4 = 5n$, or $n = 4$

Then,

$$f = \frac{4}{2(2.00\text{ m})}\sqrt{\frac{(25.0\text{ kg})(9.80\text{ m/s}^2)}{0.002\,00\text{ kg/m}}} = \boxed{350\text{ Hz}}$$

=

(c) The largest mass will correspond to a standing wave of 1 loop $(n = 1)$;

so $$350\text{ Hz} = \frac{1}{2(2.00\text{ m})}\sqrt{\frac{m(9.80\text{ m/s}^2)}{0.002\,00\text{ kg/m}}}$$

yielding m = 400 kg.

CAP High School Prize Exam

6 April 2009
9:00 – 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name:________________________________Given Name:_______________________

Home Address:____

____ Postal Code:______________

Telephone: (   )____________________ E-mail: __________________________________

School: ____ Grade:_____

Physics Teacher: ____

Date of Birth: ___________________________ Sex: Male ☐ Female ☐

Citizenship: ____ or

Immigration Status:____

For how many years have you studied in a Canadian school? _____________________________

Would you prefer the further correspondence in French or English? _______________________

Sponsored by:

Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists
2009 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on a student's performance on sections A, B, and C of the exam. Performance on the questions in part A will be used to determine whose written work in parts B and C will be marked for prize consideration by the CAP Exam National Committee. The marking scheme is: 40% for part A, 10% for part B, and 50% for part C. Part A consists of twenty multiple-choice questions; part B consists of five questions that require graphic solution. The problems in part C can also require graphing. The questions in part C have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.

Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.

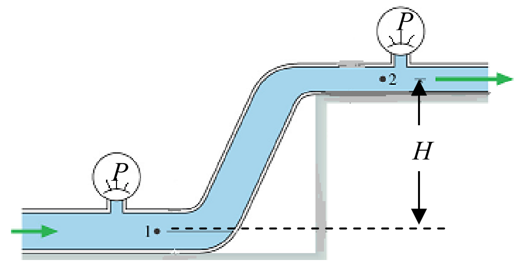
Data

Speed of light c = 3.00x10^8 m/s
Gravitational constant G = 6.67x10^{-11} N·m^2/kg^2
Acceleration due to gravity g = 9.80 m/s^2
Density of fresh water ρ = 1.00x10^3 kg/m^3
Specific heat capacity for water c = 4.19x10^3J/(kg·K)
The normal atmospheric pressure P_0 = 1.01x10^5Pa
Fundamental charge e =1.60x10^{-19} C
Mass of electron m_e = 9.11x10^{-31} kg
Mass of proton m_p= 1.67x 10^{-27} kg

Part A: Multiple Choice

Question 1
A ball falls to the earth from a height h and bounces to a height h'. Momentum is conserved in the ball-earth system:
(a) no matter what height h′ the ball reaches.
(b) only if h′ < h.
(c) only if h′ = h.
(d) only if h′ > h.
(e) only if h′ ≥ h.

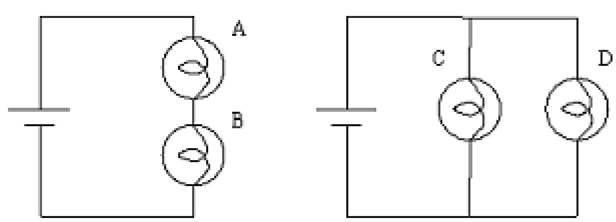
Question 2
A tap water pipe is bent as shown in the diagram. Sections 1 and 2 of the pipe are horizontal but section 2 is placed higher at a height of H. The diameter of the pipe of section 1 is 1.5 times larger than the diameter of section 2. Indicate which of the following is the most complete answer for the relationship between the hydrostatic pressures measured by the gauges in sections 1 and 2:



(a) $P_1 < P_2$ because water flows slower in section 1.
(b) $P_1 > P_2$ because water flows slower in section 1.
(c) $P_1 > P_2$ because section 1 is lower than section 2.
(d) $P_1 > P_2$ because water flows slower in section 1, and section 1 is lower than section 2.
(e) For some value of H, $P_1 = P_2$ because the lower speed of the flowing water in section 1 is compensated by the hydrostatic pressure due to the difference H in the height of the two sections.

Question 3
We know that a person emits about 500 W of radiation. We also know that a person sitting still uses about 100W of chemical energy. Where is the rest of the energy mainly coming from?
(a) Heat conducted from the air into our body.
(b) Convection of the heat.
(c) Radiation from objects around us.
(d) The energy, emitted by the body does not have to have a source.
(e) Burning fat that we have accumulated previously.

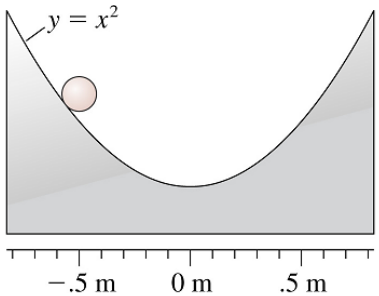
Question 4



In the two circuits shown above, the batteries are identical and maintain constant voltage. The light bulbs A, B, C and D, are identical and have resistance R. Assume that the bulbs are brighter when there is more current flowing through them. Which of the following relationships correctly describe the brightness of the bulbs?
(a) A = B > C = D
(b) C = D > A = B
(c) A = B = C = D
(d) A = C > B = D

Question 5

The figure shows an accelerometer: a device for measuring the horizontal acceleration of cars and airplanes. The device consists of a ball that is free to roll on a parabolic track.



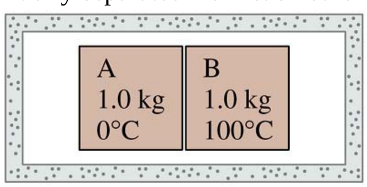
A scale along the bottom is used to measure the ball's horizontal position x.

What is the acceleration of the car in m/s^2 when the displacement of the ball in the accelerometer equals 0.20 m?

(a) 3.9
(b) 3.6
(c) 26.5
(d) 24.5
(e) 9.1

Question 6

Objects A and B that are initially separated from each other and well isolated from their surroundings are then brought into thermal contact. Initially T_A= 0C and T_B = 100C. The specific heat of A is less than the specific heat of B. After some time, the system comes to an equilibrium state. The final temperatures are:



(a) $T_A = T_B > 50^oC$
(b) $T_A > T_B > 50^oC$
(c) $T_A = T_B < 50^oC$
(d) $T_B > T_A > 50^oC$
(e) $T_A = T_B = 50^oC$

Question 7

An object is moving at a constant speed v_0 towards a source that is at rest and that is emitting sound waves of frequency f_0. The frequency of the echo that returns to the source after being reflected from the object is given by:

(a) $f_{echo} = f_0 \dfrac{v}{v - v_0}$

(b) $f_{echo} = f_0 \dfrac{v - v_0}{v + v_0}$

(c) $f_{echo} = f_0 \dfrac{v + v_0}{v - v_0}$

(d) $f_{echo} = f_0 \dfrac{v + v_0}{v}$

Question 8

An astronaut lifts off from planet Zuton in a spaceship. The free-fall acceleration on Zuton is four times less than on the Earth. At the moment of liftoff the acceleration of the spaceship is 2.45 m/s^2 (up). The weight of the astronaut at that instant is more than her weight on the surface of the earth by the factor of:

(a) 4 (b) 2
(c) 1 (d) 0.5
(e) 0.25

Question 9

A 0.50-kg mass attached to the end of a string swings in a vertical circle with a radius of 2.0 m. When the string is horizontal, the speed of the mass is 8.0 m/s. What is the magnitude of the force of the string on the mass at this position?

(a) 16 N
(b) 17 N
(c) 21 N
(d) 11 N
(e) 25 N

Question 10

What is the velocity of an electron that passes without being deviated through perpendicular electric and magnetic fields if E = 4.0 kV/m and B = 8.0 mT?

(a) 32 m/s
(b) 500 km/s
(c) 2x10^{-6} m/s
(d) 500 m/s
(e) 2 km/s

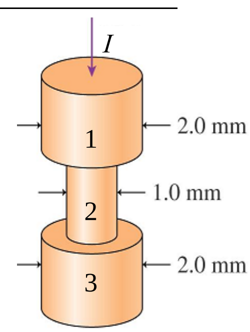
Question 11

A body that is oscillating harmonically in the vertical direction is suspended from two identical springs connected in series. The frequency of oscillation is f_1. After the springs are disconnected and attached to the body in parallel, the frequency of the vertical oscillation of the body is equal to:

(a) f_1
(b) $2 f_1$
(c) $f_1 /4$
(d) $4 f_1$
(e) $f_1 / 2$

Question 12

A piece of aluminum wire shown on the drawing is connected to a circuit with a source of constant current. Choose the correct conclusion on the values of the current (I), the amount of heat (P) emitted per second by a unit of length, and the electric field strength (E) inside the segments of the wire:



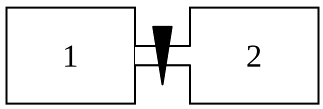
(a) $I_1 = I_2 = I_3$; $P_1 = P_2 = P_3$; $E_1 = E_2 = E_3$.
(b) $I_1 = I_3 = ½ I_2$; $P_1 = P_3 = 1/16\ P_2$; $E_1 = E_2 = E_3$.
(c) $I_1 = I_3 = ½ I_2$; $P_1 = P_3 = 1/16\ P_2$; $E_1 = E_3 = ½ E_2$.
(d) $I_1 = I_2 = I_3$; $P_1 = P_3 = ¼ P_2$; $E_1 = E_2 = E_3$.
(e) $I_1 = I_2 = I_3$; $P_1 = P_3 = ¼ P_2$; $E_1 = E_3 = ¼ E_2$.

Question 13

A spacecraft of mass m orbits a planet of mass M in a circular orbit of radius R. What is the minimum energy required to send this spacecraft to a distant point in space where the gravitational force of the planet on the spacecraft is negligible?

(a) $GmM/(4R)$
(b) GmM/R
(c) $GmM/(2R)$
(d) $GmM/(3R)$
(e) $2GmM/(5R)$

Question 14

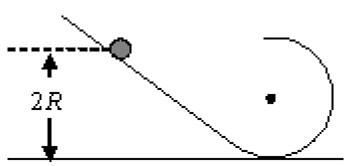


Two identical thermally isolated containers are separated by a valve. Initially, there is an ideal gas in container 1, and there is a vacuum in container 2. Some time after the valve is opened, the gas in the two containers comes to an equilibrium state. Which of the following statements about the gas during this process is true?

(a) The molar mass of the gas decreases.
(b) The work produced by the gas is zero.
(c) The temperature of the gas drops.
(d) The work produced by the gas is positive and is equal to the absolute value of the change of the internal energy of the gas.

Question 15

During the winter vacation, children use snow and water to build frictionless slides of different shapes in order to conduct various experiments. The side view of one of them is shown in the figure: the linear segment of the slide is smoothly transferred to a circle with a circumference of radius R. A puck starts sliding down from rest at an initial height of $2R$. The acceleration of the puck at the lowest point of its trajectory is:



(a) g
(b) $2g$
(c) $3g$
(d) $4g$
(e) 0

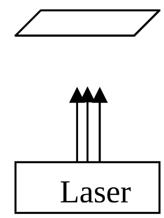
Question 16

A star undergoes a supernova explosion. Just after the explosion, the material left behind forms a uniform sphere of radius 8.0×10^6 m with a rotation period of 15 hours. This remaining material eventually collapses into a neutron star of radius 4.0 km with a period of rotation T of:

(a) 14 s
(b) 3.8 h
(c) 0.021 s
(d) 0.014 s
(e) 0.0075 h

Question 17

An impulse laser may be treated as a source of photons that are emitted during the time interval of the pulse which is followed by a time interval when no photons are produced. Pulses are periodically repeated. A laser beam of diameter d = 10 microns is directed upward and is perpendicular to the thin foil surface which has an index of reflection $\rho = 0.50$ (see the sketch of the experiment). The index of reflection of the surface is the ratio of the reflected energy to the impact energy. A pulse with duration of 0.13 ms has a total energy of 10 J. What is the mass of the piece of foil that can be supported in the air solely by the light pressure of the laser beam?



(a) < 39 g
(b) < 3.1×10^{-12} g
(c) < 39 mg
(d) < 3.7 g
(e) < 0.38 g

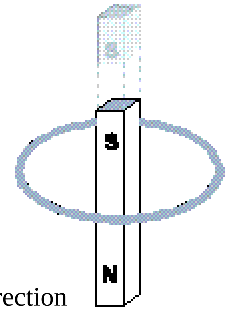
Question 18

Two sinusoidal waves traveling at the same speed in opposite directions interfere to produce a standing wave with the wave function $y = (1.50 \text{ m}) \sin(0.400x) \cos(200t)$, where x is in meters and t is in seconds. The speed of propagation of each of the interfering waves is

(a) 159 m/s
(b) 200 m/s
(c) 300 m/s
(d) 47.7 m/s
(e) 500 m/s

Question 19

A bar magnet is dropped from above and falls through a loop of wire as shown. A student measures the current in the loop between a time when the north pole of the magnet is above the plane of the loop and another time when the south pole of the magnet is below the plane of the loop. Which statement is correct about the result of the student's measurement?



(a) The current in the loop flows in one direction increasing steadily to its maximum value when the centre of the bar crosses the centre of the loop plane, after which the current begins to steadily decrease.
(b) The current in the loop undergoes harmonic oscillations because the magnetic flux through the loop is changing.
(c) The current in the loop flows first in one direction, then, after the centre of the bar crosses the centre of the loop plane, the current begins to flow in the opposite direction.
(d) No current flows in the loop because both ends of the magnet move through the loop.

Question 20

Unpolarized light goes through three successive Polaroid filters, each with its transmission axis at 45° relative to the preceding filter. What percentage of the light gets through?
(a) 0%
(b) 12.5%
(c) 25%
(d) 50%
(e) 33%

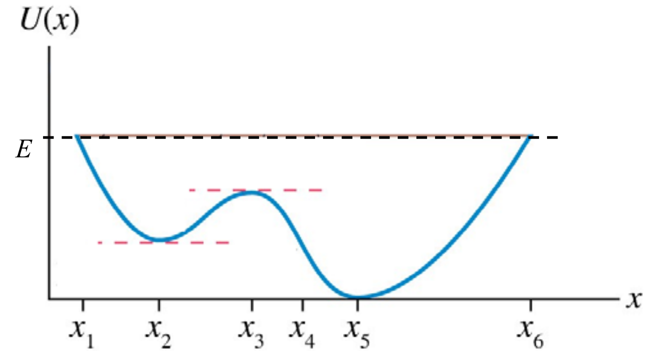
Part B: Questions that require graphical solutions

Question 1

The drag force due to air resistance on a falling object depends on the instantaneous velocity of the object as: $D = -1/4\ Av^2$. Sketch a diagram of height vs. distance for the trajectory of two projectile objects launched from the same point at ground level with the same angle to the horizontal: a) the projectile for which the air resistance is negligible, and b) the projectile that is experiencing a drag force in the air.

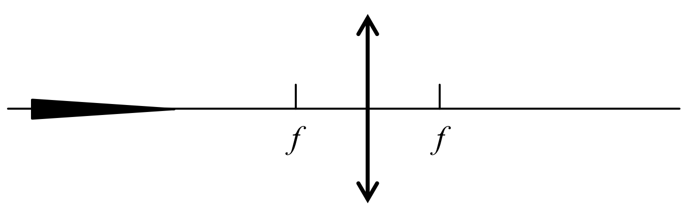
Question 2

A diagram below shows the potential energy of an object in an isolated mechanical system with total energy *E* and with conservative forces only. In the space below the given diagram, sketch a diagram for the *x*-component of the net force on the object as a function of x. Your diagram must show the correct trend of the function, zeros and vertex points.

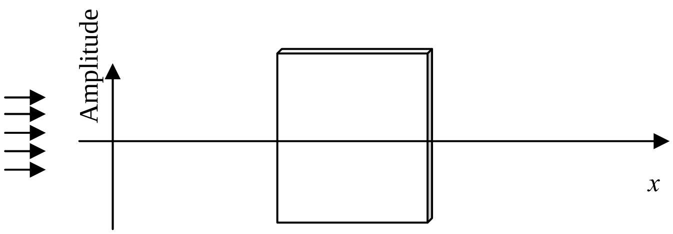


Question 3

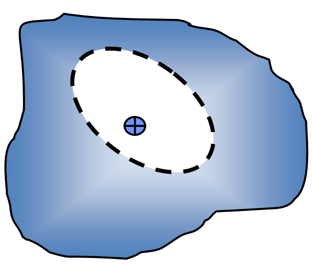
A very thin stick is placed on the optical axis of a thin convex lens as shown in the diagram. Draw the image of the object. Show all rays used for the image construction.



Question 4
Monochromatic light is produced by a laser and propagates through vacuum from left to right until it strikes normally the surface of a glass brick, as shown. The direction of the wave propagation coincides with the x-axis.
On the system of coordinates “amplitude vs. position”, sketch the wave before it enters the glass brick, inside the glass brick, and after it exits the brick.



Question 5
An object, shown in the figure below, is made out of conducting material and has a hole completely hidden inside its body. A point charge $+Q$ is kept motionless in the hole by some external force. Sketch the electric field lines inside the hole, inside the conductor, and outside the conductor as close to reality as it is necessary to be consistent with the laws of electromagnetism.



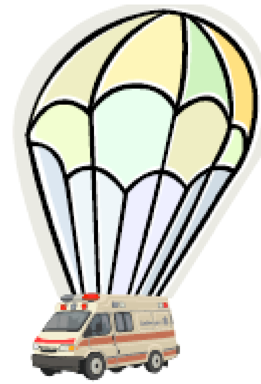
Part C: Open-Ended Problems

Problem 1
An ambulance needs to be delivered to a remote town devastated by a major earthquake. All roads leading into the town are blocked due to the earthquake and the ambulance can only be rushed to the area by airlift. The ambulance will be pushed out of a military cargo jet at 3000 m altitude and rescue staff need to find out what kind of parachute is needed for this mission. The drag force is given by the approximate formula: $F = ¼\,\rho A v^2$, where ρ is the density of air and $\rho = 1.2$ kg/m^3, A is the area of the cross-section of the parachute perpendicular to the motion and v is the velocity.

What should the diameter of the parachute be so that the ambulance can land safely?

Notice that you have to make assumptions on what is approximately a safe landing velocity and what is a weight of a typical ambulance.

Make sure you justify all your assumptions.



Problem 2
A very thin beam of protons is injected at non-relativistic velocities in a circular particle accelerator of radius R. The mass m and the charge e of the proton are known. The initial current in the accelerator is I and the total number of particles is n. The magnetic flux through the beam circuit changes at a rate of ρ Wb/s, while the radius of the beam track remains unaltered. What is the value of the current after one turn of the particles?

Problem 3
Very cold fresh water fills a vessel with a depth of 3.00 m and a diameter of 1.00 m. A vertically maintained ice cylinder with diameter d =30.0 cm and length l = 30.0 cm is carefully moved downwards and submerged in the water perpendicular to its surface and in the centre of the vessel. When ¾ of the cylinder is submerged, it is released. The density of the ice is ρ_i = 917 kg/m^3; the density of the water is ρ_w = 1.00x10^3 kg/m^3. For the following questions, you can neglect the melting of the ice.

1) Describe the behaviour of the cylinder after it is released.
2) Calculate the position of the centre of mass of the cylinder in equilibrium, taking the level of the water surface as zero and the vertical axis aimed upwards.
3) Write and solve the equation of motion for the cylinder valid during the first few seconds after it is released. The equation must have a solution in the form $y(t)$ where y is the displacement; and t is time.
4) Explain the physical significance of all parameters of the function $y(t)$ and give the numerical values for the parameters.

_______________ END OF EXAM__________________

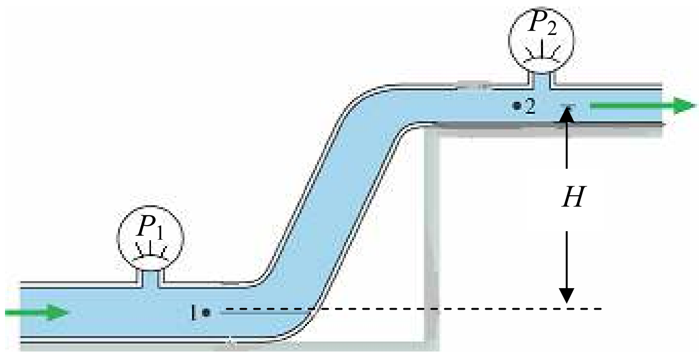
Part A. Multiple Choice.

1. A ball falls to the earth from a height *h* and bounces to a height *h′*. Momentum is conserved in the ball-earth system
   a. no matter what height h′ the ball reaches. *
   b. only if h′ < h.
   c. only if h′ = h.
   d. only if h′ > h.
   e. only if h′ ≥ h.

Solution

An attentive look at the name of the system permits to state that there is no external force in the direction of motion of the ball with respect to the ball-earth system. Therefore, the momentum is conserved as in answer (a).

2. A tap water pipe is bent as shown in the diagram. Sections 1 and 2 of the pipe are horizontal but section 2 is placed higher at a height of *H*. The diameter of the pipe in section 1 is 1.5 times larger than the diameter in section 2. Indicate which of the following is the most complete answer for the relationship between the hydrostatic pressure measured by the gauges in sections 1 and 2:



(a) $P_1 < P_2$ because water flows slower in section 1;
(b) $P_1 > P_2$ because water flows slower in section 1;
(c) $P_1 > P_2$ because section 1 is lower than section 2;
(d) $P_1 > P_2$ because water flows slower in section 1, and section 1 is lower than section 2; *
(e) For some value of *H*, $P_1 = P_2$ because the lower speed of the flowing water in section 1 is compensated by the hydrostatic pressure due to the difference *H* in the height of the two sections.

Solution

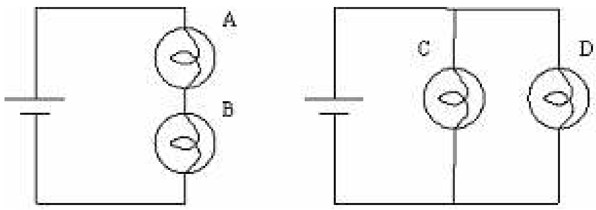
Bernoulli equation and hydrostatic pressure make the answer (d) correct.

3. We know that a person emits about 500 W of radiation. We also know that a person sitting still uses about 100W of chemical energy. Where is the rest of the energy mainly coming from?
(a) Heat conducted from the air into our body.
(b) Convection of the heat.
(c) Radiation from objects around us. *
(d) The energy, emitted by the body does not have to have a source.
(e) Burning fat, which we accumulate previously.

Solution

Answer (a) is wrong because the temperature of the air is lower. Answer (b) might be correct, but compared to radiation, the convection is less effective. Answer (e) actually refers to chemical energy.

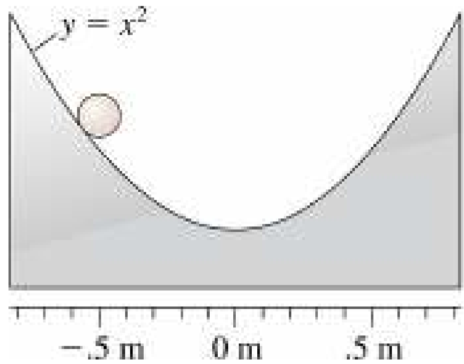
4. In two shown circuits, the two batteries are identical and maintain constant voltage. The bulbs with brightness A, B, C and D, are identical and have resistance R. Assume that the bulbs are brighter when there is more current through them. Which of the following relationships correctly describes the brightness of the bulb?
   (a) A = B > C = D
   (b) C = D > A = B *
   (c) A = B = C = D
   (d) A = C > B = D



Solution

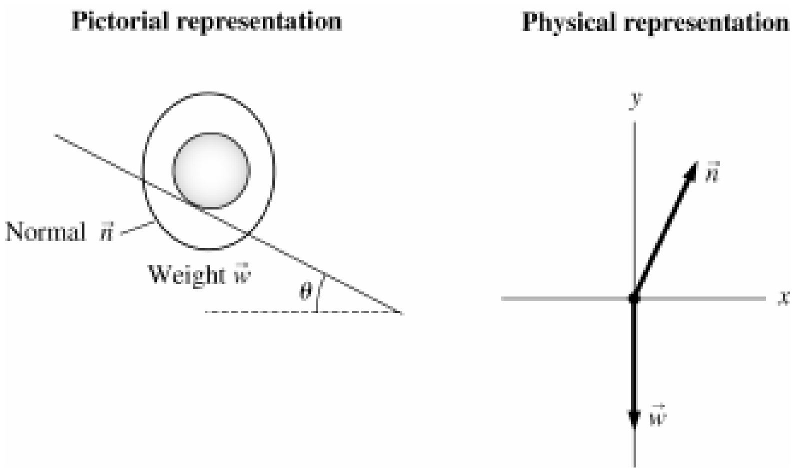
Current through the bulbs A and B is same and is equal to $\varepsilon/(2R)$. The currents through the bulbs C and D are equal to each other and can be obtained as: $\varepsilon/(R/2) = 2\varepsilon/R$, which is four times greater than the current in the first circuit The estimation does not consider the internal resistance of the battery.

5. The figure on the right shows an accelerometer: a device for measuring the horizontal acceleration of cars and airplanes. The device consists of a ball that is free to roll on a parabolic track. A scale along the bottom is used to measure the ball's horizontal position x. What is the acceleration of the car [in m/s^2] when the displacement of the ball in the accelerometer equals 0.20 m? $g = 9.8$ m/s^2.
   (a) 3.9 * (b) 3.6 (c) 26.5 (d) 24.5 (e) 9.1



Solution

The ball is a particle on a slope, and that the slope increases as the absolute value of x-displacement increases. Assume that there is no friction and that the ball is being accelerated to the right so that it remains at rest on the slope.
Although the ball is on a slope, it is accelerating to the right. Thus we'll use a coordinate system with horizontal and vertical axes.



Newton's second law is

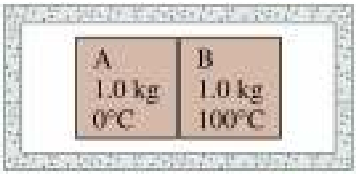
$$\Sigma F_x = n\sin\theta = ma_x \qquad \Sigma F_y = n\cos\theta - w = ma_y = 0 \text{ N}$$

Combining the two equations, we get

$$ma_x = \frac{w}{\cos\theta}\sin\theta = mg\tan\theta \Rightarrow a_x = g\tan\theta$$

The curve is described by $y = Ax^2$, where A = 1 m^{-1}. Its slope is $\tan\theta$ which is the derivative of the curve. Hence, $\frac{dy}{dx} = \tan\theta = 2Ax \Rightarrow a_x = g(2Ax)$ = 9.8x 0.4= 3.9 m/s^2. **Answer (a) is correct**

6. Objects A and B that are initially separated from each other and well isolated from their surroundings are then brought into thermal contact. Initially T_A= 0C and T_B = 100C. The specific heat of A is less than the specific heat of B. After some time, the system comes to an equilibrium state. The final temperatures are:



(a) $T_A = T_B > 50^oC$; *
(b) $T_A > T_B > 50^oC$;
(c) $T_A = T_B < 50^oC$;
(d) $T_B > T_A > 50^oC$;
(e) $T_A = T_B = 50^oC$.

Solution

Object A has a smaller specific heat and thus less thermal inertia. The temperature of A will change more than the temperature of B.

7. An object is moving at constant speed v_0 towards a source that is at rest and that is emitting sound waves of frequency f_0. The frequency of the echo that returns to the source after being reflected from the object is given by:

(a) $f_{echo} = f_0 \frac{v}{v - v_0}$

(b) $f_{echo} = f_0 \frac{v - v_0}{v + v_0}$

(c) $f_{echo} = f_0 \frac{v + v_0}{v - v_0}$ *

(d) $f_{echo} = f_0 \frac{v + v_0}{v}$

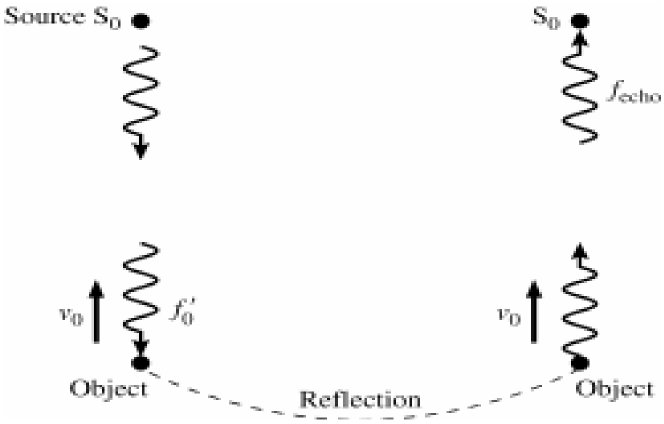
Solution

The frequency is Doppler-shifted to higher values for a detector moving toward the source. The frequency is also shifted to higher values for a source moving towards the detector. The formula is obtained in two steps. First, the object acts like a moving detector and "observes" a frequency that is given by

$$f_0' = f_0(1 + v_0 / v) \qquad (1)$$

Second, as this moving object reflects (or acts as a "source" of ultrasound waves), the frequency of echo as observed by the original source S_o is

$$f_{echo} = f_0'(1 - v_0 / v)^{-1} \qquad (2)$$



Combination of equations (1) and (2) gives

$$f_{\text{echo}} = \frac{f_0'}{1 - v_0/v} = \frac{f_0\left(1 + v_0/v\right)}{1 - v_0/v} = \frac{v + v_0}{v - v_0} f_0$$

8. An astronaut lifts off from planet Zuton in a spaceship. The free-fall acceleration on Zuton g_Z is four times less than on the Earth (g). At some moment of liftoff the acceleration of the spaceship is 2.45 m/s^2 (up). The weight of the astronaut at this instant of time W_Z is more than his weight on the surface of the earth W by the factor of

a. 4
b. 2
c. 1
d. 0.5 *
e. 0.25

Solution

The weight on Zuton (W_Z) is a downward normal force that the astronaut exerts on his seat. According to the 3rd Newton's law, the same force is applied to the astronaut from the seat. The equation of motion of the astronaut for the given instant of time is:

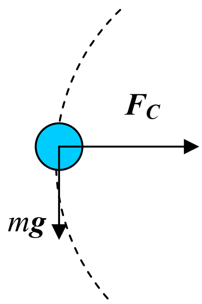
$$ma_{up} = W_Z - mg_Z \Rightarrow W_Z = m(a_{up} + g_Z) = mg\left(\frac{a_{up}}{g} + \frac{g_Z}{g}\right) = W\left(\frac{2.45}{9.8} + \frac{1}{4}\right) = W\left(\frac{1}{4} + \frac{1}{4}\right) = \frac{1}{2}W = 0.5W$$

9. A 0.50-kg mass attached to the end of a string swings in a vertical circle with a radius of 2.0 m. When the string is horizontal, the speed of the mass is 8.0 m/s. What is the magnitude of the force of the string on the mass at this position?

a. 16 N *
b. 17 N
c. 21 N
d. 11 N
e. 25 N

Solution

The free-body diagram is sketched in the figure. In the position shown the force of tension is the centripetal force. As there is no horizontal component of velocity at this instant of time, the relationship between the centripetal force F_C and the speed of the object is given by



$$F_C = \frac{mv^2}{R} = \frac{0.50 \cdot (8.0)^2}{2.0} = 16\,\text{N}$$

10. What is the velocity of an electron that passes undeviated through perpendicular electric and magnetic fields if E = 4.0 kV/m and B = 8.0 mT?
a. 32 m/s
b. 500 km/s *
c. $2x10^{-6}$ m/s
d. 500 m/s
e. 2 km/s

Solution

If the path of the electron in the crossed electric and magnetic fields is linear, as given, the motion of the electron can be only uniform, i.e. to take place without acceleration and hence,with zero net force.

$$eE = evB; \quad v = E / B; \quad v = \frac{E}{B} = \frac{(4.0\times10^{3})}{(8.0\times10^{-3})} = 5.0\times10^{5}\frac{m}{s} = 500\frac{km}{s}$$

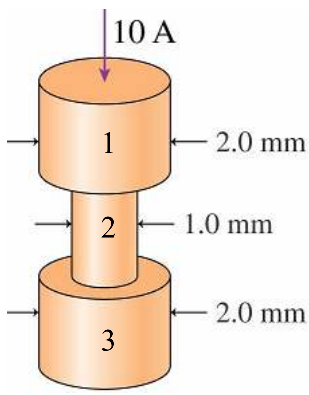
11. A body is oscillating harmonically in a vertical direction being suspended from two identical springs connected in series. The frequency of oscillation is f_1. After the springs are disconnected and attached to the body in parallel, the frequency of the vertical oscillation of the body is equal to
a. f_1.
b. $2f_1$. *
c. $f_1/4$
d. $4f_1$.
e. $f1/2$.

Solution

When the springs are connected in series, their elongations are equal and the total elongation is two times greater than each of them. In equilibrium mg = $k\Delta_1 = k\Delta_2 = k_{eff} 2\Delta$ The frequency is calculated as : $f_1 = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$. When the springs are connected in parallel, mg $= k\Delta_1 + k\Delta_2 = k_{eff}\,\Delta = 2k\Delta$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = 2f_1$$

12. A piece of aluminum wire shown on the drawing, is connected to a circuit with a source of constant current. The current flowing into segment 1 is I_1 = 10 A. Choose the correct conclusion on the values of current, amount of heat P emitted per second by a unit of length and electric field strength E inside the segments of the wire
a. $I_1 = I_2 = I_3$; $P_1 = P_2 = P_3$; $E_1 = E_2 = E_3$.
b. $I_1 = I_3 = ½ I_2$; $P_1 = P_3 = 1/16\ P_2$; $E_1 = E_2 = E_3$.
c. $I_1 = I_3 = ½ I_2$; $P_1 = P_3 = 1/16\ P_2$; $E_1 = E_3 = ½ E_2$.
d. $I_1 = I_2 = I_3$; $P_1 = P_3 = ¼ P_2$; $E_1 = E_2 = E_3$.
e. $I_1 = I_2 = I_3$; $P_1 = P_3 = ¼ P_2$; $E_1 = E_3 = ¼ E_2$. *



Solution

Segments 1 and 3 are identical from any point of view.
Current is everywhere same (law of conservation of electric charge).
The power, emitted by the unit of length is inversely proportional to the cross-sectionsl area of the wire; therefore, P_2 must be four times as great as P_1 or P_3.

Electric field lines must be confined within the wire. The number of lines penetrating the unit of surface area perpendicular to the lines is proportional to the value of *E*. The density of the electric field lines inside the segment 2 is four times greater than the densit of the lines in segments 1 or 3. So, $E_2 = 4E_1$.

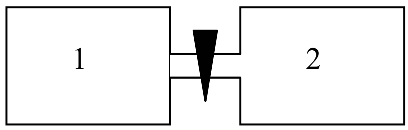
13. A spacecraft (mass = *m*) orbits a planet (mass = *M*) in a circular orbit (radius = *R*). What is the minimum energy required to send this spacecraft to a distant point in space where the gravitational force on the spacecraft by the planet is negligible?

a. $GmM/(4R)$
b. GmM/R
c. $GmM/(2R)$ *
d. $GmM/(3R)$
e. $2GmM/(5R)$

Solution

A spacecraft on a circular orbit of radius *R* has potential energy of $-GmM/R$ and kinetic energy of $GmM/2R$. So, the total mechanical energy of the spacecraft is equal to the sum of the potential and kinetic energies, i.e. $-GmM/2R$. The minimum work required to move the spacecraft from this orbit to the infinitely far place in the space is equal to the change in the energy of the object from the initial value to zero, which gives answer (c).

14. Two identical thermally isolated containers are separated by a valve. Initially, there is an ideal gas in container 1, and there is a vacuum in container 2. Some time after the valve is opened, the gas in the two containers comes to an equilibrium state. Which of the following statements about the gas in this process is true:



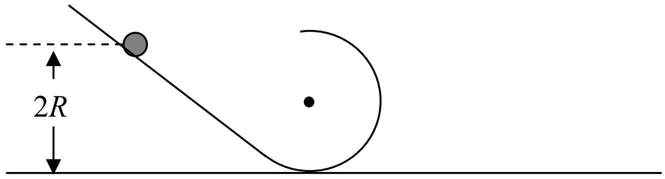
(a) The molar mass of the gas decreases;
(b) The work produced by the gas is zero; *
(c) The temperature of the gas drops;
(d) The work produced by the gas is positive and is equal to the absolute value of the change of the internal energy of the gas.

Solution

The molar mass cannot change if the gas is same. In this problem, the expansion into vacuum is adiabatic. The expansion into vacuum produces no work (no external pressure to produce the work against to). Therefore, answer (b) is correct. (c) cannot take place, because the internal energy of the gas does not change.

15. During the winter vacation children use snow and water to produce frictionless hillsides of different shapes for freestyle exercises. The side view of one of them is shown in the figure: a linear segment of the path is smoothly transferred into a piece of circumference with radius *R*. A puck starts sliding down from rest at initial height of $2R$. The acceleration of the puck in the lowest point of its trajectory is

(a) g.
(b) $2g$.
(c) $3g$.
(d) $4g$.*
(e) 0.



Solution

Law of conservation of energy gives: $mg2R = ½ mv^2; \rightarrow v^2 = 4Rg$.

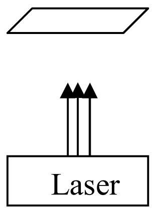
Centripetal force in the lowest point of the trajectory is the net force; hence, the acceleration of the puck is equal to $v^2/R = 4g$.

16. A star undergoes a supernova explosion. Just after the explosion, the material left behind forms a uniform sphere of radius 8.0×10^6 m with a rotation period of 15 hours. This remaining material eventually collapses into a neutron star of radius 4.0 km with a period of rotation T of:
(a) 14 s
(b) 3.8 h
(c) 0.021 s
(d) 0.014 s *
(e) 0.0075 h

Solution

The solution is based on the equation $I_1\omega_1 = I_2\omega_2$. Regardless the shape of the rigid object, its moment of inertia I is proportional to the product mR^2. In this problem, the mass of the object is constant, therefore, $\omega_2 = \omega_1\left(\frac{R_1}{R_2}\right)^2$. The period of rotation is $T = \left(\frac{4.0\times10^3\,m}{8.0\times10^6\,m}\right)^2(15h) = 3.75\times10^{-6}\,h = 0.014$ s.

17. An impulse laser may be treated as a source of photons that are emitted during the time interval of the pulse which is followed by a time interval when no photons are produced. Pulses are periodically repeated. The laser beam of diameter d = 10 microns is directed upward and is perpendicular to a surface of a thin foil with an index of reflection $\rho = 0.50$ (see the sketch of the experiment). The index of reflection of the surface is the ratio of the reflected energy to the impact energy. A pulse with duration of 0.13 ms has a total energy of 10 J. What must be the mass of the piece of foil that may be supported in the air solely by the light pressure of the laser beam?



(a) < 39 g.
(b) < 3.1×10^{-12} g.
(c) < 39 mg. *
(d) < 3.7 g.
(e) < 0.38 g.

Solution

The given value of the diameter of the beam is not used in the solution. Upon the definition of force, it is the rate of change of linear momentum. The linear momentum changes by its initial value $p_{initial}$ (total absorption) or by the product of $p_{initial}$ and $[1 + \rho]$ (reflection with coefficient ρ).

For a photon, the linear momentum $p = \frac{E}{c}$ (E is the energy of the photon, c is the speed of light). When the laser beam supports the piece of foil, the weight of the foil is equal to the normal force of light pressure. Summary:

$$mg = \frac{\Delta p}{\Delta t} = \frac{p_{initial}(1+\rho)}{\Delta t} = \frac{E}{c}\frac{1+\rho}{\Delta t}; \quad m = \frac{1}{g}\frac{E}{c}\frac{1+\rho}{\Delta t} = \frac{10\times1.5}{9.8\times3\times10^8\times0.13\times10^{-3}} = 3.92\times10^{-5}\,\text{kg}$$
$= 39$ mg

18. Two sinusoidal waves traveling at the same speed in opposite directions interfere to produce a standing wave with the wave function $y = (1.50\text{ m})\sin(0.400x)\cos(200t)$, where x is in meters and t is in seconds. The speed of propagation of each of the interfering waves is
(a) 159 m/s
(b) 200 m/s

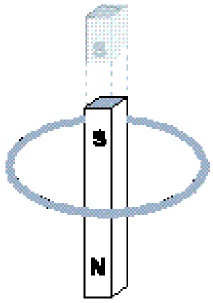
(c) 300 m/s
(d) 47.7 m/s
(e) 500 m/s *

Solution

$y = (1.50\text{ m})\sin(0.400x)\cos(200t) = 2A\sin(kx)\cos(\omega t)$.
Therefore, $k = 0.400$ rad/m; ω =200 rad/s. The speed of the traveling waves in the medium is

$$v = \lambda f = \frac{\lambda}{2\pi} 2\pi f = \frac{\omega}{k} = \frac{200}{0.400} = 500\frac{m}{s}$$

19. A bar magnet is dropped from above and falls through the loop of a wire as shown. A student measures the current in the loop between some instant when the north pole of the magnet is above the plane of the loop and the other instant when the south pole of the magnet is below the plane of the loop.
Which statement is correct about the result of the student's measurement?



a. The current in the loop flows in one direction increasing steadily to its maximum value when the center of the bar crosses the centre of the loop plane, after which the current begins to steadily decrease.
b. The current in the loop is harmonically oscillating because the magnetic flux through the loop is changing.
c. The current in the loop flows first in one direction, then, after the center of the bar crosses the centre of the loop plane, the current begins to flow in the opposite direction. *
e. No current flows in the loop because both ends of the magnet move through the loop.

Solution

The magnetic field lines are exit from the north pole of the bar magnet and enter the south pole of the magnet. Inside the bar magnet, the magnetic field lines are directed from the south pole to the north pole. When the number of the filed lines inside the wire loop is increasing (the magnetic flux is increasing), the induction produces a current of the direction that creates a magnetic field of the opposite direction. As the initial direction of the magnetic field lines is downwards, the current of induction produces the magnetic field directed upwards, so the current must flow counterclockwise. After the center of the magnet passes the plane of the loop, the number of lines, penetrating the loop, begins to decrease. According to the Lenz's law, the current of induction switches to the opposite direction to create a magnetic field parallel to the decreasing external magnetic field.

20. Unpolarized light is passed through three successive Polaroid filters, each with its transmission axis at 45° relative to the preceding filter. What percentage of the light gets through?

a. 0%
b. 12.5% *
c. 25%
d. 50%
e. 33%

Solution

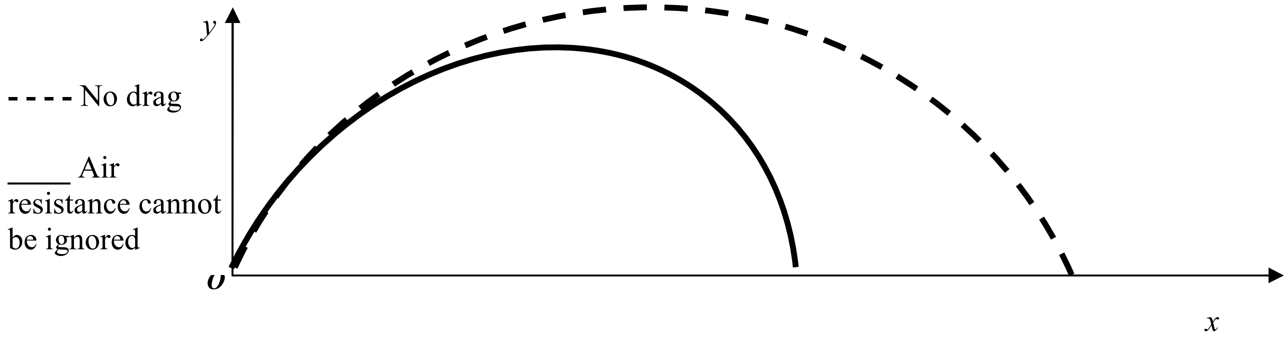
After the first filter, the intensity is $I_1 = ½\, I_0$; after the second filter the passed intensity becomes $I_2 = ½\, I_0 \cos^2(45^o) = ¼\, I_0$; after the third filter, the intensity is $I_3 = I_2 \cos^2(45^o) = 1/8\, I_0 = 0.125\, I_0$.

Part B.

1. The drag force due to air resistance on a falling object depends on the instantaneous velocity of the object as $D = -1/4\ Av^2$. Sketch a diagram of height vs. distance for the trajectory of two projectile objects launched from the same point at ground level with the same angle to the horizontal: a) – the projectile for which the air resistance is negligible, and b) - the projectile that is experiencing a drag force in the air.

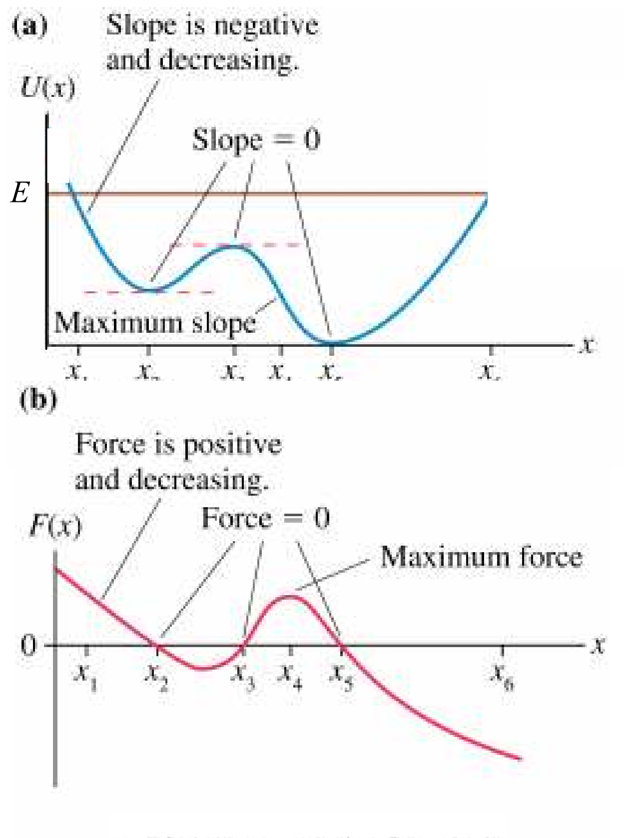
Solution

The drag will affect the *x*- and the *y*-component of the velocity of the projectile everywhere through its path. So, the lower height and the shorter horizontal distance are expected to be observed for an object that experiences the drag force due to the air resistance. The shape of the trajectory must change correspondingly.



2. The Figure below shows a diagram of the potential energy of an object in an isolated mechanical system with the total energy E and conservative forces only. On the space below the given diagram, sketch a diagram for the *x*-component of the net force on the object as a function of x. Your diagram must show the correct trend of the function, zeros and vertex points.

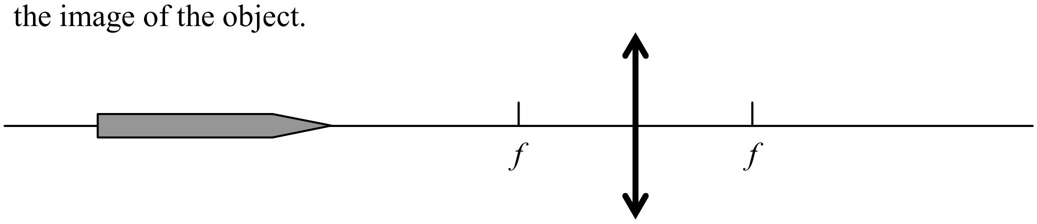
Solution



To draw a diagram, it is helpful to refer to some simple example of motion of a point mass in the field with maximum or minimum of the potential energy function. The simplest example is the motion of a point mass along a hill with a profile of the same shape as the shape of the given function of the potential energy. In this case, the potential energy of the object is the energy in the gravitational field of the earth, and the surface of the path is supposed to be frictionless. For this approach:

1) If the object is carefully placed in either of three points with zero slope of the potential energy function, it will remain at rest for the infinite time; so, it is reasonable to conclude that the net force is zero at the equilibrium positions; we expect either a maximum or a minimum value of the x-component of the force to appear between two zeros.
2) If the object is gently placed on the sections of the hill with positive slope, it will start moving in the negative x-direction; so, these sections must have negative values of force.
3) If the object is placed on the steep section of the hill, the acceleration would be greater in its absolute value; so, the steeper sections of the function of the potential energy must correspond to the greater magnitude of the force.

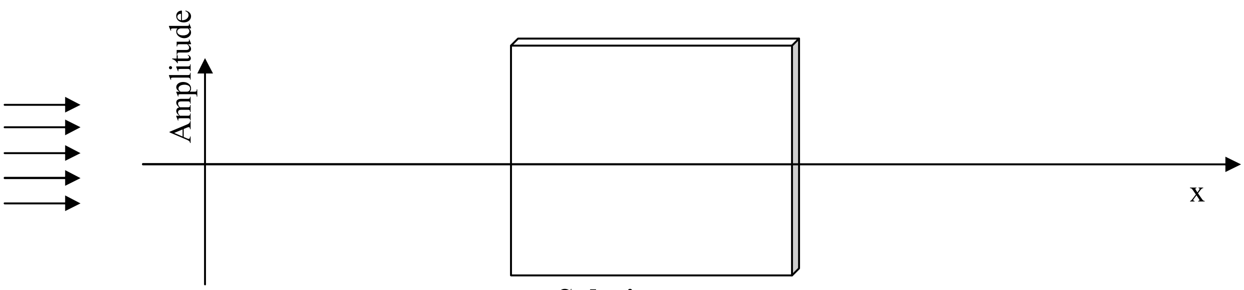
3. A very thin stick is placed on the optical axis of a thin convex lens as shown in the diagram. Draw the image of the object.



Solution

As a possible method of solving the problem, one can draw the vertical objects at the ends of the stick and then, construct the two images of these auxiliary objects to find the intersections of the images with the optical axis of the lens.

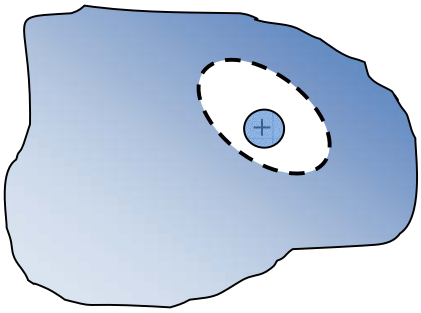
4. The monochromatic light is produced by a laser and propagates through vacuum from left to right until it strikes normally the surface of the glass brick, as shown. The direction of the wave propagation coincides with x-axis.
On the system of coordinates "amplitude vs. position", sketch the wave before it enters the glass brick, inside the glass brick and after it exits the brick.



Solution

The students are expected to draw a sinusoidal function with the wavelength shorter inside the brick, because of decrease in the speed of propagation in the medium with index of refraction >1. To the left and to the right from the brick, the wavelength must be same. However, the full mark must be given to those who will also show the decrease in the amplitude inside the brick and the smaller amplitude to the right from the brick due to the absorption in the glass.

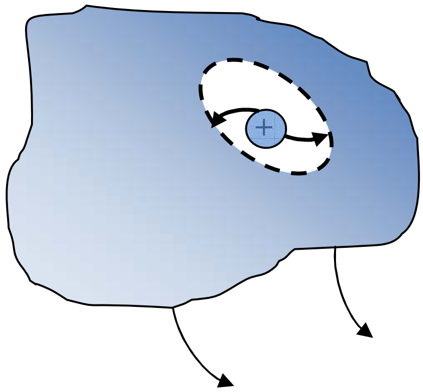
5. An object, shown in the figure, is made out of a conductive material and has a hole completely hidden inside its body. A point charge $+Q$ is kept motionless in the hole by some external force. Sketch the electric field lines inside the hole, inside the conductor and outside the conductor as close to reality as it is necessary to be consistent with the laws of electromagnetism.



Solution

Electric induction causes accumulation of negative charges on the inner surface of the hole with the total negative charge of $-Q$. The electric field lines go from the point charge into the surface of the hole and enter the surface of the hole normally. There must be no electric field lines inside the body of the conductor.

Due to the redistribution of electric charge in the conductor, the outer surface of the object becomes positively charged with the total charge of $+Q$. The electric field lines exit the outer surface of the object under the right angle everywhere and are directed into infinity.



Part C.

Problem 1.

An ambulance needs to be delivered to a remote town devastated by a major earthquake. All roads leading into the town are blocked due to the earthquake and the ambulance can only be rushed to the area by airlift. The ambulance will be pushed out of a military cargo jet at 3000 m altitude and rescue staff needs to find out what kind of parachute is needed for this mission.

The drag force is given by the approximate formula:

$F = \frac{1}{4}\rho A v^2$, where ρ is the density of air and $\rho = 1.2$ kg/m^3.

What should be the diameter of the parachute be so that the ambulance can land softly/safely?

A is the area of the cross-section of the parachute perpendicular to the motion and v is the velocity. Notice that you have to figure out what is approximately the safe landing velocity and what is a weight of a typical ambulance.

Solution

The estimated mass of the ambulance is 3000 kg; and the estimated landing velocity is 5 m/s.

Acceptable assumptions:

v will be anything from 1 to 10 m/s

mass of the ambulance - from 1000 to 10 000 kg

Drag force must be equal to force of gravity at given velocity to ensure constant velocity:

$m\,g = \frac{1}{4}\rho A v^2$

$A = 4mg/\rho v^2 = 4\cdot3000\text{kg}\cdot 10\text{m/s}^2/(1.3\ \text{kg/m}^3\cdot 25\text{m}^2/\text{s}^2) = 3700\text{m}^2$

Which results in a diameter value of 68 m

Still give the points when they end with the area of the parachute and don't show the radius or diameter.

Problem 2.

A very thin beam of protons is injected at non-relativistic velocities in a circular particle accelerator of radius *R*. The mass *m* and the charge *e* of the proton are well known. The initial current in the accelerator is *I* and the total number of particles is *n*. The magnetic flux through the beam circuit changes at a rate of ρ Wb/s, while the radius of the beam track remains unaltered.

What is the value of the current after one turn of the particles?

Solution

To simplify the problem let us assume that the bunch of protons is injected and then circulate in the accelerator chamber as a sole particle with charge *ne*. Upon the definition a current is a rate at which electric charge flows through the cross section of conductor. If the motion of protons was uniform the relationship between the value of current I_U and speed v_U in the chamber could be expressed as:

$$I_U = \frac{en}{T} = \frac{env_U}{2\pi R}$$

Factor $\frac{en}{2\pi R}$ does not depend on time. That is why just the same formula can be used for the instantaneous current *I(t)* and speed *v(t)* of protons:

$$I(t) = \frac{en}{T} = \frac{env(t)}{2\pi R} \qquad (1)$$

To calculate current after one turn it is necessary to determine the speed of protons after one turn. It can be done using the law of conservation of energy and the law of electromagnetic induction. The magnitude of electromotive force of induction is equal to φ, its work produced to accelerate a proton is $e\varphi$. This work results in increasing of the kinetic energy of a proton. After one turn a speed of a proton can be calculated from the law of conservation of energy as:

$$v_1 = \sqrt{v_0^2 + \frac{2e\varphi}{m}} \qquad (2)$$

where v_0 is initial speed of a proton, and can be obtained from Eq.(1) with given initial current *I*:

$$v_0 = \frac{2\pi RI}{en} \qquad (3)$$

Combining Eqs.(1), (2), and (3) we can calculate current I_1 after one turn as:

$$I_1 = \frac{en}{2\pi R}v_1 = \frac{en}{2\pi R}\sqrt{\left(\frac{2\pi RI}{en}\right)^2 + \frac{2e\varphi}{m}} = I\sqrt{1 + \frac{n^2e^3\varphi}{2\pi^2R^2mI^2}}$$

==

Problem 3.

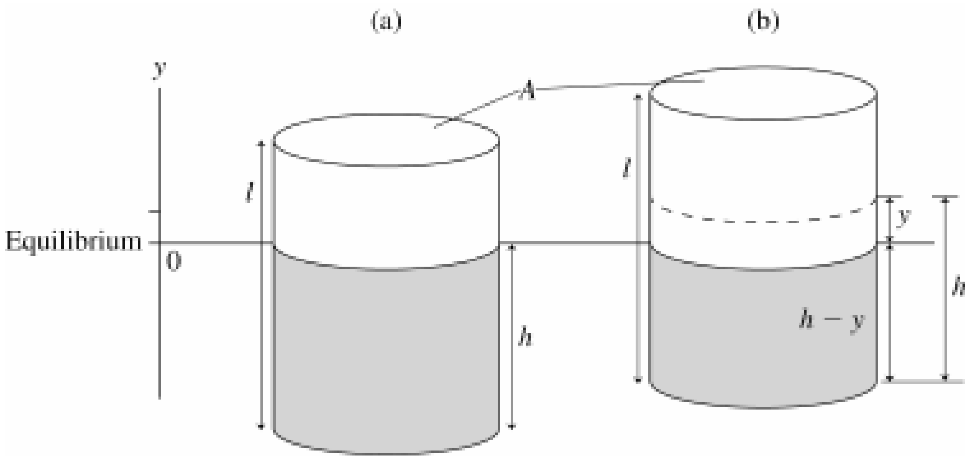
Very cold fresh water fills a vessel with a depth of 3.00 m and a diameter of 1.00 m. A vertically maintained ice cylinder with diameter *d* =30.0 cm and length *l* = 30.0 cm is carefully moved downwards and submerged in the water perpendicular to its surface and in the centre of the surface of the vessel. When ¾ of the length of the cylinder is submerged, it is released. The density of the ice is ρ_i = 917 kg/m^3; the density of the water is ρ_w = 1.00x10^3 kg/m^3.

1) Describe the behavior of the cylinder after it is released.

2) Ignoring the melting of the cylinder during the first few seconds after it has been released, calculate the position of the center of mass of the cylinder in equilibrium, taking the level of the water surface as zero and the vertical axis aimed upwards.
3) Write and solve the equation of motion for the cylinder valid during the first few seconds after it is released. The equation must have a solution in the form *y(t)* where *y* is the displacement; and *t* is time.
4) Explain the physical significance of all parameters of the function *y(t)* and give the numerical values for the parameters.

Solution

The diameter of the cylinder plays no role in the phenomenon.
The buoyant force on the cylinder is given by Archimedes' principle.



1) Because the density of the ice is less than the density of the water, the center of mass of the cylinder must be under the surface in equilibrium. Just after the cylinder is released, it starts to move downwards and will pass its equilibrium position due to inertia. When the center of mass of the cylinder appears below its position of equilibrium, the buoyant force being directed against the displacement of the center of mass starts to push the cylinder up. This results in oscillatory motion of the cylinder.

2) The equilibrium position of the center of mass gives for the displacement Δy of the center of mass the following relationship: $\rho_i\, l = \rho_w\, (l/2 + \Delta y) \rightarrow \Delta y = l\left(\frac{\rho_i}{\rho_w} - \frac{1}{2}\right) = 0.3\left(\frac{917}{1000} - 0.5\right)m = 0.125$ m = 12.5 cm

The position of the center of mass of the cylinder in equilibrium is $y = -12.5$ cm. It is 5 cm lower than the position of the cylinder at the instant it was released.

3) A is cross-sectional area of the cylinder. Variables y and h are shown in the figure.
In static equilibrium, and we can write $h = (\rho_i / \rho_w)\, l$ (1)
and $h = (15 + 12.5)$ cm = 27.5 cm
The volume of the displaced liquid is $A(h - y)$. Applying Newton's second law in the y-direction,
$\sum F_y = -\rho_i\, lgA + \rho_w\, (h - y)gA$
Substitution of result (1) gives $\sum F_y = -\rho_w\, ygA = a_y m$ (2)
The result in (2) is $F = -ky$, where $k = \rho_w\, gA$. This is Hooke's law, and equation (2) is the equation of a simple harmonic motion.

The angular frequency for the resulting simple harmonic motion is $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\rho_w gA}{m}} = \sqrt{\frac{\rho_w gA}{\rho_i lA}} = \sqrt{\frac{g}{h}}$,

and the period is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{h}{g}} = 1.10$ s

$$y(t) = (12.5 - 7.5)\cos\left(\sqrt{\frac{g}{h}}t\right)cm = 5\cos 5.97t \text{ cm} = 5.00\text{x}10^{-2}\cos(5.97t) \text{ m}$$

CAP High School Prize Exam

12 April 2010

9:00 – 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name:________________________________Given Name:_______________________

Home Address:____

____ Postal Code:______________

Telephone: (   )____________________ E-mail: __________________________________

School: ____ Grade:_____

Physics Teacher: ____

Date of Birth: _____________________________ Sex: Male ☐ Female ☐

Citizenship: ____ or

Immigration Status:____

For how many years have you studied in a Canadian school? ______________________________

Would you prefer the further correspondence in French or English? ________________________

Sponsored by:

Canadian Association of Physicists

Canadian Chemistry and Physics Olympiads

University of Toronto, Department of Physics

Canadian Association of Physicists
2010 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on a student's performance on sections A and B of the exam. Performance on the questions in parts A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. Part A consists of twenty-two multiple-choice questions. The questions in part C span a range of difficulties, and may require graphing. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.

Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

Speed of light $c = 3.00\text{x}10^8$ m/s
Gravitational constant $G = 6.67\text{x}10^{-11}$ N·m²/kg²
Acceleration due to gravity $g = 9.80$ m/s²
Density of fresh water $\rho = 1.00\text{x}10^3$ kg/m³
The normal atmospheric pressure $P_0 = 1.01\text{x}10^5$Pa
The specific heat of water $c = 4{,}186\text{x}10^3$J/(kg·K)
Fundamental charge $e = 1.60\text{x}10^{-19}$ C
Mass of electron $m_e = 9.11\text{x}10^{-31}$ kg
Mass of proton $m_p = 1.67\text{x}\ 10^{-27}$ kg
Planck's constant $h = 6.63\text{x}10^{-34}$ J·s
Coulomb's constant $1/(4\pi\varepsilon_0) = 8.99\text{x}10^9$ N·m²/C²
Boltzmann constant $k = 1.38\text{x}10^{-23}$ J/K

Part A: Multiple Choice

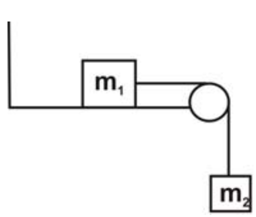
Question 1

A ball is dropped from a height h. It hits the ground and bounces back up with a momentum loss of 10% due to the impact. The maximum height it will reach is

a) 0.90 h; b) 0.81 h; c) 0.949 h; d) 0.3 h.

Question 2

A mass m_1 rests on a frictionless surface. It is attached to mass m_2 by a light string which passes over a massless, frictionless pulley, as shown in the figure. The system is released from rest. The initial acceleration a of m_2 is given by

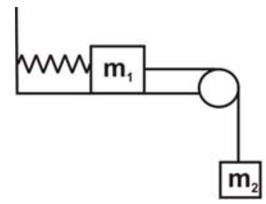


a) $a = g$; b) $a = \frac{m_2}{(m_1 + m_2)} g$;

c) $a = (m_2 - m_1)g$; d) $a = \frac{m_1}{(m_1 + m_2)} g$.

Question 3

The system in question 2 is attached to a spring of spring constant k which in turn is attached to a vertical wall, as shown to the right. The spring is neither stretched nor compressed. The system is released from rest. The mass m_2 falls and then oscillates about a mean displacement x from its original position before release given by

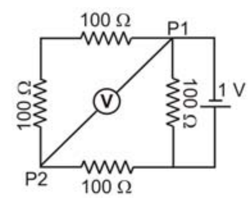


a) $x = \frac{m_2 g}{k}$; b) $x = \frac{m_1 g}{k}$;

c) $x = \frac{(m_1 + m_2)g}{k}$; d) $x = \frac{k}{m_2 g}$.

Question 4

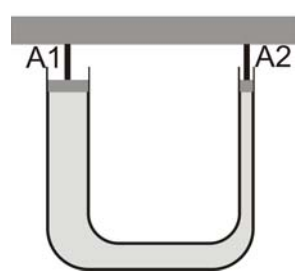
Four 100 ohm resistors and a 1 V battery are connected as shown in the circuit diagram. The potential difference between points P1 and P2 is given by



a) 0.250 V; b) 0.333 V; c) .500 V; d) 0.667 V.

Question 5

A heavy plate is held horizontal resting on two frictionless pistons, each of which is placed in the opening of an asymmetric U tube filled with an incompressible fluid, as shown in the figure. Once the restraints on the plate are released, then



a) piston A1 will rise and A2 will fall;
b) piston A2 will rise and A1 will fall;
c) the plate will remain level and not move;
d) the plate will remain level and both pistons will sink a similar small distance.

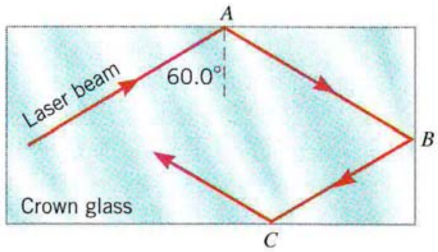
Question 6

Two identical glasses, one completely filled with water, and the other filled with vodka, are viewed directly from above. The refractive index of vodka is slightly greater than the refractive index of water. Which glass appears to have a greater depth of fluid?

a) the water glass;
b) the vodka glass;
c) neither glass because the apparent depth is the same as the real depth in both glasses;
d) neither glass, because the effect of the clear fluid is to reduce the apparent depth by the same factor in both glasses.

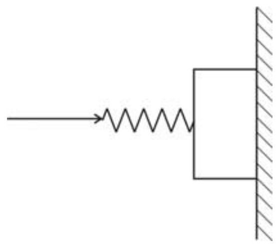
Question 7

The drawing shows a crown glass slab of refractive index 1.5 with a rectangular cross section. As illustrated, a laser beam strikes the upper surface at an angle of 60°. After reflecting from the upper surface, the beam reflects from the side and bottom surfaces. If the glass is surrounded by air, determine at which point part of the beam first exits the glass.



a) A; b) B; c) C; d) none of the above.

Question 8
A block of mass m is sandwiched between a rough vertical wall and a horizontally oriented spring with spring constant k. If the spring is compressed a distance x beyond its uncompressed length, then the minimum coefficient of static friction for which the block does not fall is



a) $\frac{mg}{kx}$; b) $\frac{kx}{mg}$; c) $\frac{kx}{mg+kx}$; d) $\frac{mg}{mg+kx}$.

Question 9
A light wave has a high enough frequency to ionize an atom. If the amplitude of the light wave is tripled, the chance of an atom being ionized by the light wave increases by a factor of

a) 1; b) $\sqrt{3}$; c) 3; d) 9.

Question 10
A 100-kg student eats a 200-calorie doughnut. To 'burn off the sugar high', he decides to climb the steps of a tall building. How high would he have to climb to expend an equivalent amount of work? (1 food calorie = 10^3 calories ~ 4200 J).

a) 273 m; b) 428 m; c) 857 m; d) 8400 m.

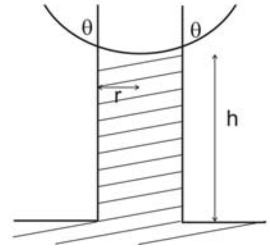
Question 11
In an insulated vessel, 250 g of ice at 0°C is added to 600 g of water at 18°C. The latent heat of fusion is ~ 3.33×10^5 J/kg. The amount of ice remaining when the system reaches thermal equilibrium is

a) 0 g; b) 114 g; c) 136 g; d) 45.2 g.

Question 12
A liquid will rise in a narrow tube (capillary action) if the adhesive forces between the molecules of the liquid and the molecules of the tube are greater than the cohesive forces between the molecules of the liquid. The height of a column of liquid of density ρ drawn up a tube of radius r is given by

$$h = \frac{2\gamma}{\rho g r}\cos\theta$$

where θ is the contact angle between the surface of the liquid and the tube, g is the acceleration due to gravity, and γ is a constant that depends on a number of parameters. The dimensions of γ are given by

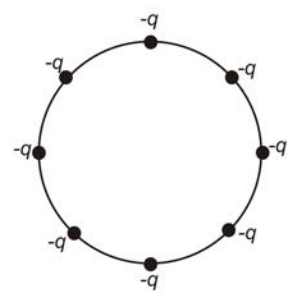


a) ML^{-2}; b) MLT^{-2}; c) $M^{-1}LT^{-2}$
d) MT^{-2}; e) none of the above.

Question 13
If the Earth's polar land-based glaciers melted, water would be redistributed on the planet, and the depression in the earth's crust made by the weight of the ice would rebound, as the land rises when the weight of the glaciers is removed (an effect known as Post-Glacial Rebound or Glacial Isostatic Adjustment). Following post-glacial rebound, the length of day would

a) increase;
b) decrease;
c) remain the same as the present length of day;
d) at first increase but then shortly return to the present length of day;
e) at first decrease but then shortly return to the present length of day.

Question 14
Eight identical negative charges $-q$ are arranged symmetrically on a circle of radius r, each equidistant from the next. Assume the point charges have zero potential at infinity. If the constant of proportionality in Coulomb's law is k, then the magnitude of the electric field and the electrostatic potential at the centre of the circle are given by

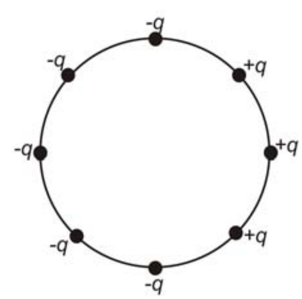


a) $E=\frac{8kq}{r^2}$, $V=\frac{-8kq}{r}$; b) $E=\frac{8kq}{(2\pi r)^2}$, $V=\frac{-8kq}{(2\pi r)}$;

c) $E=\frac{8kq}{r^2}$, $V=0$; d) $E=0$, $V=0$;

e) $E=0$, $V=\frac{-8kq}{r}$.

Question 15
Three of the negative charges in the previous questions are replaced with positive charges with the same magnitude, and are arranged as in the figure. The magnitude of the electric field and the electrostatic potential at the centre of the circle are given by



a) $E = \frac{2kq}{r^2}\left(1 + 2\sqrt{2}\right), V = 0$;

b) $E = \frac{2kq}{r^2}\left(1 + \sqrt{2}\right), V = \frac{-2qk}{r}$;

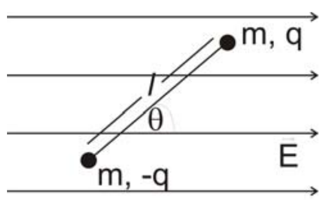
c) $E = \frac{2kq}{r^2}\left(1 + 2\sqrt{2}\right), V = \frac{-2qk}{r}$;

d) $E = 0, V = \frac{-2qk}{r}$.

e) None of the above.

Question 16

An electric dipole consists of two small particles. Each has mass m but opposite electric charge $\pm q$. They are separated by a light, non-conducting rod of length l. The dipole is placed in a uniform electric field $\vec{E}$ such that it forms an angle θ to the field, then is released. Its initial angular acceleration about its centre of mass is given by



a) $\frac{2qE\cos\theta}{ml}$; b) $\frac{2qE\sin\theta}{ml}$; c) $\frac{qEl\cos\theta}{m}$;
d) $\frac{qEl\sin\theta}{m}$; e) $\frac{qE\sin\theta}{ml^2}$.

Question 17

At time $t = 0$ a block of mass m moves with momentum p along a rough horizontal surface. If it comes to a complete stop in time t, then the coefficient of kinetic friction μ_k is given by

a) $\frac{p}{mgt}$; b) $\frac{pg}{mt}$; c) $\frac{ptg}{m}$; d) $\frac{pt}{mg}$;
e) None of the above.

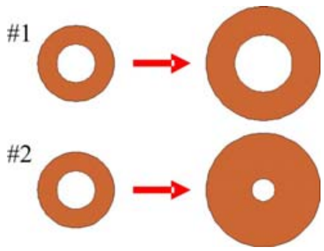
Question 18

A fish swimming in water with constant velocity v_0 experiences a viscous drag force that is proportional to the square of its velocity (i.e. $F_{drag} \propto v^2$). If the fish doubles its velocity, then the thrust force F that it must generate to maintain this new velocity is related to the fish's original thrust force F_0 via

a) $F = 2F_0$; b) $F = \sqrt{2}F_0$; c) $F = 4F_0$; d) $F = F_0^{\,2}$;
e) None of the above.

Question 19

A solid object has a hole in it. Which of these illustrations more correctly shows how the size of the object and the hole change as the temperature increases?



a) illustration #1;
b) illustration #2;
c) The answer depends on the material of which the object is made;
d) The answer depends on how much the temperature increases;
e) Both c) and d) are correct.

Question 20

Two gear wheels of the same thickness, but with one twice the diameter of the other, are mounted on parallel light axles far enough apart not to mesh the teeth of the wheels. The larger wheel is spun with angular velocity Ω and the wheels are then moved together so they mesh. What is the subsequent angular velocity of the larger wheel?

a) $\frac{\Omega}{2}$; b) $\frac{\Omega}{5}$; c) $\frac{2\Omega}{5}$; d) $\frac{4\Omega}{5}$.

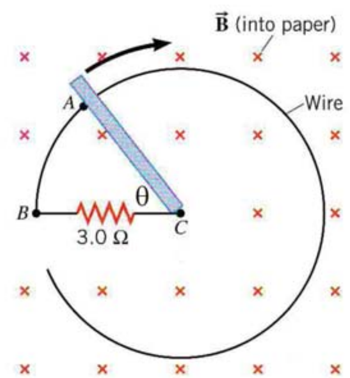
Question 21

For the meshed rotating gears of the previous question, what fraction of the original energy is lost?

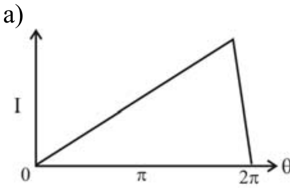
a) 1/4; b) 1/8; c) 1/5; d) 2/5.

Question 22

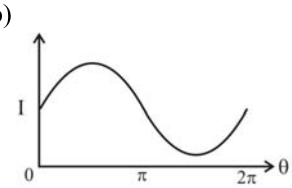
The drawing shows a copper wire (of negligible resistance) bent into a circular shape with a radius of r. The radial section BC is fixed in place, while the copper bar AC sweeps around at an angular speed of ω. The bar makes electrical contact with the wire at all times. The wire and the bar have negligible resistance. A uniform magnetic field exists everywhere, is perpendicular to the plane of the circle, and has a magnitude of B. Which of the following shows the magnitude of the current as a function of θ?



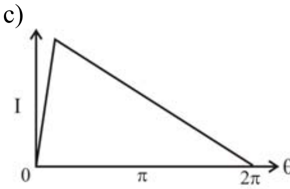
a)



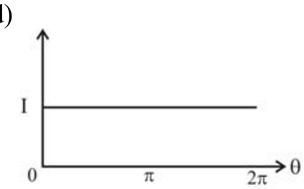
b)



c)



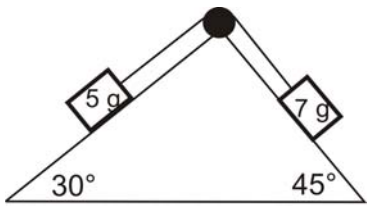
d)



Part B: Problems

Problem 1

A double inclined plane is in the form of a wedge of weight 10 g, held on the ground with its base horizontal and rough faces inclined at 30° and 45° respectively to the horizontal. A 5 g particle on the face inclined at 30° is connected by a mass less, flexible, inextensible string over a smooth mass less pulley at the ridge of the wedge to a 7 g particle on the other face. If the coefficient of friction between each of the particles and the wedge is 1/5, find the accelerations of the particles and the magnitude of the vertical force between the wedge and the ground.



Problem 2

Oceanographers have deployed an array of instruments off the west coast of Vancouver Island, connected together by cables carrying power and communication. The power is supplied through a single DC line. The return path is through the sea water. The grounds at each end of the line are large metal spheres.

Consider two copper spheres, radii a, buried in an infinite medium of resistivity ρ. Current flows out of one sphere and is collected by the second sphere. The current flow in vicinity of each sphere can be considered radially uniform as the distance between the spheres is large compared with a.

a) By considering the series resistance of many thin spherical shells around a sphere show graphically (or by direct integration) that much of the resistance between the spheres is due to material within a radius $2a$ of each sphere and thereby estimate the total resistance between the spheres.
b) If the current flow between the spheres is 1 A, calculate the charge in coulombs on each one.

Problem 3

Sir Ernest Rutherford, a very famous New Zealander who worked at McGill University in Montreal and the Cavendish Laboratory in Cambridge, won the Nobel Prize for his studies of the disintegration of nuclei. He established an important relationship for a nuclear reaction, in which a parent atom disintegrates into a daughter atom, which is called the Rutherford – Soddy law of radioactive disintegration. This law states that if N is the number of radioactive nuclei present at some instant, the number of nuclei ΔN that decay in a time Δt is given by:

$$\Delta N = -\lambda N \Delta t$$

where λ is called the decay constant. We can integrate this expression to get the relationship

$$N(t) = N_0 e^{-\lambda t}$$

where N_0 represents the number of radioactive nuclei at time t = 0.

a) Show that the time taken for N(t) = $N_0/2$ (known as the half-life), is given by $T_{1/2} = (\ln 2)/\lambda$.

Potassium-argon (K-Ar) dating is used in earth science to determine the age of a rock sample. The potassium isotope $^{40}_{19}\mathrm{K}$ is radioactive and decays over time to the argon isotope $^{40}_{18}\mathrm{Ar}$. When a rock is molten the argon gas is released into the atmosphere but as the rock cools and crystallizes, the daughter argon atoms are trapped within the rock matrix. Time since crystallization is calculated by measuring the ratio of the amount of $^{40}_{18}\mathrm{Ar}$ accumulated to the amount of $^{40}_{19}\mathrm{K}$ remaining.

The two possible nuclear reaction for $^{40}_{19}\mathrm{K}$ to change to $^{40}_{18}\mathrm{Ar}$ are:

$$^{40}_{19}\mathrm{K} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{40}_{18}\mathrm{Ar} + \text{ a neutrino}$$

$$^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{18}\mathrm{Ar} + {}^{0}_{+1}\mathrm{e} + \text{ a neutrino}$$

b) Describe the physical process that is occurring in these reactions.
c) Which of the two is the more likely to occur?
d) Most of the potassium $^{40}_{19}\mathrm{K}$ (89.1%) decays to $^{40}_{20}\mathrm{Ca}$. What is the equation for this reaction?

The *approximate* ratio of the amount of $^{40}_{18}\mathrm{Ar}$ to that of $^{40}_{19}\mathrm{K}$ is directly related to the time elapsed since the rock was cool enough to trap the argon by the following equation:

$$t = \frac{t_{1/2}}{\log_e(2)} \log_e\left(\frac{K_f + \frac{Ar_f}{0.109}}{K_f}\right)$$

Where $T_{1/2}$ is the half-life of $^{40}_{19}\mathrm{K}$, K_f is the amount of $^{40}_{19}\mathrm{K}$ remaining in the sample, and Ar_f is the amount of $^{40}_{18}\mathrm{Ar}$ found in the sample.

e) If a rock sample has remained undisturbed for 50 million years and the half-life of $^{40}_{19}\mathrm{K}$ is 1.248 x10^9 years, what ratio of Ar/K should be measured in the sample?

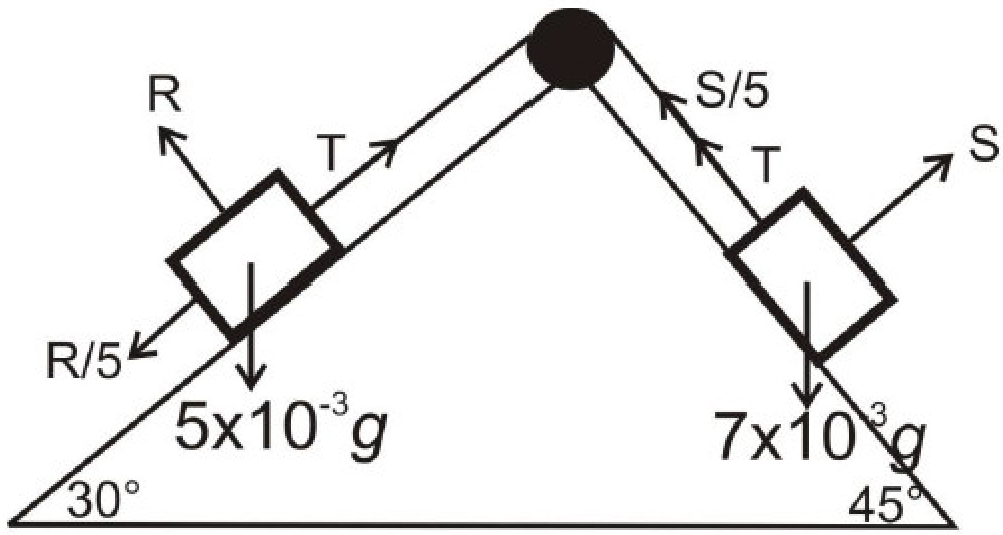
CAP exam 2010: Solutions:

Multiple Choice:

1: b	7: b	13: b	19: a
2: d	8: a	14: e	20: d
3: a	9: a	15: b	21: c
4: d	10: c	16: b	22: d
5: a	11: b	17: a	
6: a	12: d	18: c	

Long Answer:

1. Let T be the tension in the string, and R and S bet the reactions between the masses and their respective planes.



Then

$$R = 5\times10^{-3} g\cos 30^\circ$$
$$S = 7\times10^{-3} g\cos 45^\circ$$

$g = 9.8$ m/s^2 and the frictional forces are $\frac{R}{5}$ and $\frac{S}{5}$.

Let f be the acceleration of the 7 gram mass down its plane and of the 5 gram mass up its plane. The equations of motion of the masses are:

$$7\times10^{-3} g\sin 45^\circ - \frac{1}{5}7\times10^{-3} g\cos 45^\circ - T = 7\times10^{-3} f$$
$$T - 5\times10^{-3} g\sin 30^\circ - 1\times10^{-3} g\cos 30^\circ = 5\times10^{-3} f$$

or

$$\frac{7\times10^{-3} g}{\sqrt{2}} - \frac{1}{5\sqrt{2}} 7\times10^{-3} g - T = 7\times10^{-3} f$$
$$T - \frac{5\times10^{-3} g}{2} - \frac{\sqrt{3}\times10^{-3} g}{2} = 3.27\times10^{-3} f$$

Adding these together gives

$$\frac{g}{120}\left(28\sqrt{2}-25-5\sqrt{3}\right)=f$$
$$f=0.049g=0.481m/s^2$$

Thus the acceleration of the blocks is to the right (ie. the 7 gm mass moves downwards) at 0.481 m/s^2. From this we can calculate T, which is $T=0.0032N$.

The forces exerted by the 5 gm mass on the wedge are $5\times10^{-3}g\cos30^\circ$ perpendicular to the plane, and $1\times10^{-3}g\cos30^\circ$ up the plane. The component vertically downwards of these forces is

$$5\times10^{-3}g\cos^2 30^\circ-1\times10^{-3}g\cos30^\circ\sin30^\circ=\left(\frac{15-\sqrt{3}}{4}\right)\times10^{-3}g$$

Similarly, the component vertically downwards of the forces exerted by the 7 gram mass on the wedge are

$$7\times10^{-3}g\cos^2 45^\circ-\frac{7}{5}\times10^{-3}g\cos45^\circ\sin45^\circ=\frac{21}{5}\times10^{-3}g$$

The downward component of the force exerted by the string on the pulley is

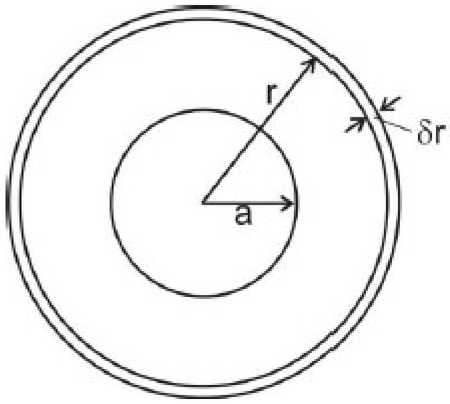
$$T\cos60^\circ+T\cos45^\circ=4.36\times10^{-3}g.$$

Adding up all the downward forces, we find

$$10\times10^{-3}g+4.36\times10^{-3}g+\frac{21}{5}\times10^{-3}g+\frac{15-\sqrt{3}}{4}\times10^{-3}g=21.88\times10^{-3}g=0.214N$$

Or the equivalent of weight of 21.88 gram (slightly less than the mass of the three blocks if they were not moving).

2. Consider a sphere surrounded by a spherical shell as shown in the figure. The thickness of the shell is δr. The current I flows perpendicular to the shell.



The resistance dR of the shell is

$$\delta R = \rho \frac{\text{length parallel to the current flow}}{\text{area perpendicular to the current flow}}$$

The resistance from *a* to *2a* is

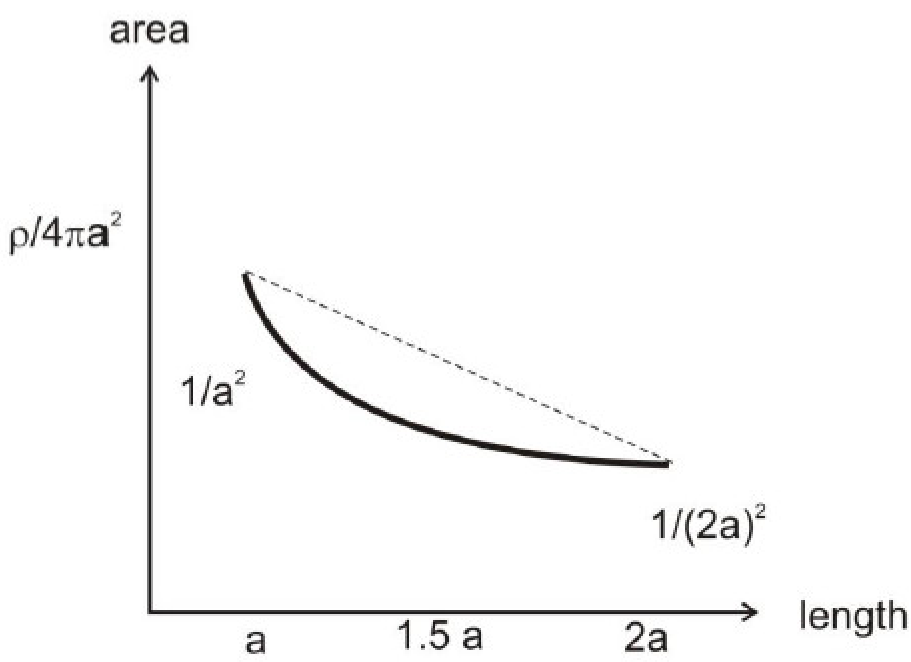
$$R\Big|_a^{2a} = \rho \int_a^{2a} \frac{dr}{4\pi r^2} = \frac{\rho}{4\pi}\left[\frac{1}{a} - \frac{1}{2a}\right] = \frac{\rho}{8\pi a}$$

$$R\Big|_a^{\infty} = \rho \int_a^{\infty} \frac{dr}{4\pi r^2} = \frac{\rho}{4\pi a} = 2 \times R\Big|_a^{2a}$$

For two spheres,

$$R_{Total} = 2 \times R\Big|_a^{\infty} = \frac{\rho}{2\pi a}$$

Graphically, this can be done in a number of ways. For instance, here is a typical way this can be approached:



One could represent the integral as a trapezium and get for the area under the curve (i.e. the approximation for the resistance between a and 2a)

$$\frac{1}{2}\left[\frac{1}{a^2}+\frac{1}{(2a)^2}\right]=\frac{5}{8a}\times\frac{\rho}{4\pi}$$

which is a little larger than the integral value. For the value between a and infinity, students can obtain a table of partial sums by estimating the area under the curve for different successive intervals of a, and demonstrate that the solution converges to a finite value. Please note that the exact sum of the inverse square series was solved by Euler and is quite beyond the brilliant highschool student (we think!). The subtleties of this result are one of the great advertisements for integral calculus.

b) At the surface of the sphere, the current is

$$I = 4\pi a^2 J_r$$

where J_r is the radial component of the current density. Thus

$$J_r = \frac{I}{4\pi a^2}$$

From Ohm's law, the radial component of the electric field is then

$$E_r = \frac{\rho I}{4\pi a^2}$$

If Q is the charge on the sphere, then the electric field is given by

$$E_r = \frac{1}{4\pi\varepsilon_o}\frac{Q}{a^2}$$

By comparison, then

$$Q = \rho I \varepsilon_0$$

How big is Q? If I is 1 A, ρ is of the order of 1 Ohm, and ε_0 is roughly 10^{-11} F/m, so Q is of order 10^{-11} c.

3. a) This comes down to showing that $N_0 e^{-\lambda T_{1/2}} = \frac{N_0}{2}$.

$$e^{-\lambda T_{1/2}} = \frac{1}{2}$$

$$\lambda T_{1/2} = \ln 2$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

b) In the first reaction, the potassium nucleus captures an electron from the inner shell (since the electron has a finite probability of being inside the nucleus), and mutates into Argon. In the second reaction, a positron is emitted by the potassium radioactive nucleus (by standard beta+ decay), and the potassium mutates into Argon.

c) The reaction most likely to occur is that which requires the least change in total energy (or equivalently, total mass). Therefore, let us look at the energy balances. The total energy of the reaction is determined by the total mass of the particles before the decay minus that of the particles after the decay:

Electron capture:

$$Q_{ec} = (m_{potassium} + m_{electron} - m_{\text{argon}} - \nu)c^2$$

Positron emission:

$$Q_{pe} = (m_{potassium} - m_{electron} - m_{\text{argon}} - \nu)c^2$$

Subtracting the two we find that $Q_{ec} = Q_{pe} + 2m_{electron}c^2$. Therefore electron capture is less energetically demanding and is more likely to occur.

d) ${}^{40}_{19}\text{K} \rightarrow {}^{40}_{20}Ca + {}^{0}_{-1}\text{e} +$ an antineutrino

which is a standard β^- decay

e) Rearranging this equation to solve for $\frac{K_f}{Ar_f}$:

$$\frac{t \ln 2}{T_{1/2}} = \ln(1 + \frac{Ar_f}{.109 K_f})$$

$$0.109(e^{\frac{t \ln 2}{T_{1/2}}} - 1) = \frac{Ar_f}{K_f}$$

Plugging in the given values for the half-life and the time since the formation of the rock gives $\frac{K_f}{Ar_f} = 0.003069$.

CAP High School Prize Exam

10 March 2011
9:00 - 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS

Family Name: ______________________ Given Name: ______________

Home Address: ____

______________________ Postal Code: ______________

Telephone: ( ) ______________ Email: ______________________

School: ______________________________ Grade: ______________

Physics Teacher: ____

Date of Birth: ______________________ Sex: Male ☐ Female ☐

Citizenship: ______________________________ or

Immigration Status: _______________________________________

For how many years have you studied in a Canadian school? ______________

Would you prefer the further correspondence in French or English? ______________

Sponsored by:

Canadian Association of Physicists,
Canadian Physics Olympiad,

University of British Columbia,
Department of Physics and Astronomy.

Canadian Association of Physicists 2011 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on students performance on sections A and B of the exam. Performance on the questions in parts A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. Part A consists of twenty-three multiple-choice questions. The questions in part B span a range of difficulties, and may require graphing. Be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway.

Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Notice: Full marks will be given to a student who provides any full correct solution to the long problems. Partial marks will be given for partial solutions. There are no penalties for incorrect answers. The questions are not of equal difficulty. Remember we are challenging the best physics students in Canada; it is possible that even the best papers may not achieve an overall score of 80%. This is meant to be tough!

Data

Speed of light $c = 3.00 \times 10^8\,\mathrm{m/s}$
Gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$
Acceleration due to gravity $g = 9.80\,\mathrm{m/s^2}$
Density of fresh water $\rho = 1.00 \times 10^3\,\mathrm{kg/m^3}$
The normal atmospheric pressure $P_0 = 1.01 \times 10^5\,\mathrm{Pa}$
The specific heat of water $c = 4.186 \times 10^3\,\mathrm{J/kgK}$
Fundamental charge $e = 1.60 \times 10^{-19}\,\mathrm{C}$
Mass of electron $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of proton $m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$
Planck's constant $h = 6.63 \times 10^{-34}\,\mathrm{Js}$
Coulomb's constant $1/(4\pi\epsilon_0) = 8.99 \times 10^9\,\mathrm{Nm^2/C^2}$
Boltzmann's constant $k = 1.38 \times 10^{-23}\,\mathrm{J/K}$
A.U. Astronomical Unit $= 1.49598 \times 10^{11}\,\mathrm{m}$ The approximate distance from the Sun to the Earth.
Radius of the Earth $= 6.371 \times 10^6\,\mathrm{m}$
Linear expansion coefficient of iron
$\alpha_{\mathrm{Fe}} = 1.1 \times 10^{-5}\,\mathrm{K^{-1}}$
Linear expansion coefficient of zinc
$\alpha_{\mathrm{Zn}} = 3.0 \times 10^{-5}\,\mathrm{K^{-1}}$
1 US mile = 1609 meters
1 US gallon = 3.785 liters

Part A: Multiple Choice

Question 1

Incandescent light bulbs are notorious for being relatively inefficient in producing visible light. The tungsten wire inside such a bulb is at a temperature of approximately 3000 K and the emission spectrum is very similar to that of a blackbody. The efficiency is so low because

a) Most of the electrons are absorbed in the tungsten wire.
b) Most of the power is lost due to the resistance of the bulb.
c) The electric power actually is efficiently transformed into radiation but at 3000 K, most of it is infrared.
d) A blackbody absorbs more light than it emits, hence it appears black.

Question 2

A solar panel installed on a spaceship has a maximum energy output of 5 kW near the Earth. What is the maximum energy output of the solar panel when the spaceship is near Mars? (The distance from the Earth to the Sun is 1 A.U. and from Mars to the Sun is 1.5 A.U.)

(a) 3.3 kW; (b) 2.2 kW; (c) 1.0 kW;
(d) 0.55 kW; (e) 0.20 kW.

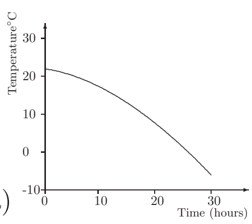
Question 3

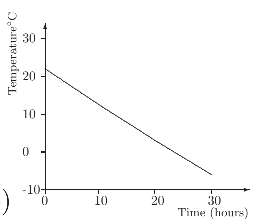
A scientist measured a $1\mu A$ current in an area of a human brain. Unlike current in metals which is carried by free electrons, the current in the brain is mainly carried by potassium ions. Each potassium ion has one unit of charge e. This current corresponds to the flow of how many potassium ions per second?

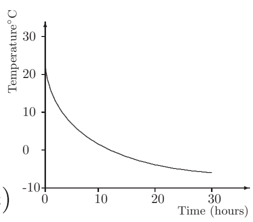
(a) 6×10^5; (b) 6×10^6; (c) 6×10^8;
(d) 6×10^{12}; (e) 6×10^{18}.

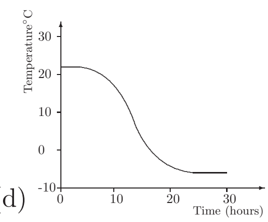
Question 4

The gas supply to your physics professor's house suddenly stops due to a gas line failure. It is winter and the temperature outside is -5°C and constant. Assuming all the doors and windows remain closed, which of these graphs best describes how the temperature in the house changes with time after the gas supply stops?

(a)


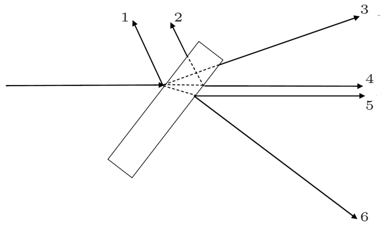
(b)


(c)


(d)


Question 5
The ray of light travelling in air hits the parallel glass plate as shown. Which are the possible continuations of the light ray?

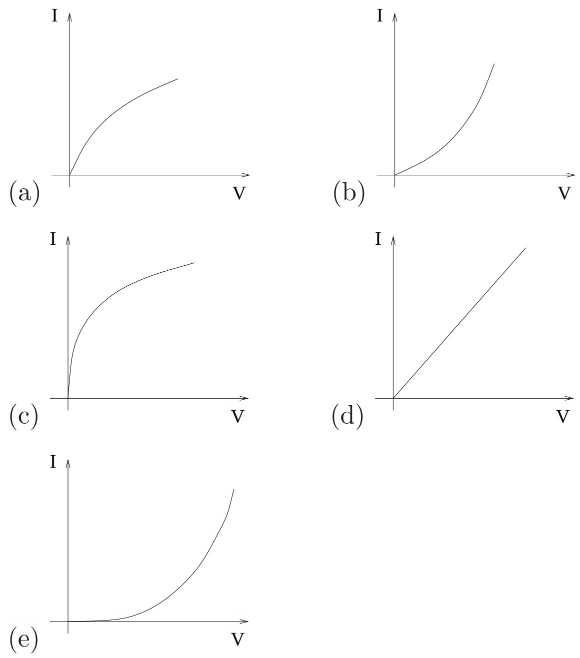
a) 4 only.
b) both 2 and 4.
c) both 3 and 6.
d) both 1 and 5.



Question 6
Approximately, what fraction of the energy emitted by the Sun reaches to the Earth?

(a) 10^{-7}; (b) 10^{-10}; (c) 10^{-13}; (d) 10^{-16}.

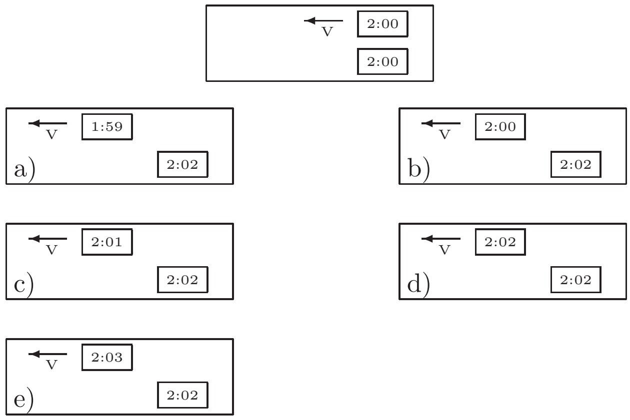
Question 7
The resistance of the tungsten in a light bulb increases with temperature. Which of the following graphs shows the current as a function of voltage for the light bulb?



Question 8
A fast rocket travels directly upwards at $0.5c$ (half the speed of light). There is an explosion directly below the rocket. If astronauts manage to measure how fast the light produced by the explosion is moving relative to their spacecraft, they will find the light to be moving:

a) Upwards at speed $0.5c$.
b) Upwards at speed c.
c) Upwards at speed $1.5c$.
d) Downwards at speed $0.5c$.
e) Downwards at speed c.

Question 9
Two identical clocks are set to the same time as one passes the other at very high (relativistic) velocity as shown in the top figure. Which of the other figures represents a possible observation of the clocks at some later time in the reference frame of the stationary clock?



Question 10
For the statements:
1) Mass can be directly converted into kinetic energy.
2) Kinetic energy can be directly converted into mass.

a) Only first is true.
b) Only second is true.
c) Both are true.
d) Neither is true.

Question 11
A stable Helium-4 nucleus has two protons and two neutrons. If the mass of Helium-4 nucleus, proton and neutron are denoted by m_{He}, m_{p} and m_{n} respectively, we can conclude that:

a) $m_{\mathrm{He}} = 2m_{\mathrm{p}} + 2m_{\mathrm{n}}$.
b) $m_{\mathrm{He}} > 2m_{\mathrm{p}} + 2m_{\mathrm{n}}$.
c) $m_{\mathrm{He}} < 2m_{\mathrm{p}} + 2m_{\mathrm{n}}$.
d) Any of the above may be true.

Question 12
A clean metal surface is placed in a vacuum. The surface is irradiated with monochromatic light of variable intensity I(number of photons per unit area) and frequency f. We measure the maximum kinetic energy K of electrons emitted from the metal due to the photoelectric effect. How does K behave when I increases?

a) K increases.
b) K is constant.
c) K decreases.
d) Impossible to determine.

Question 13
A point mass is moving in the xy plane. Its acceleration is a constant vector perpendicular to the x axis, then:

a) only v_y is constant.
b) only v_x is constant.
c) only the acceleration is constant.
d) the acceleration and v_x are constant.
e) the position and a_y are constant.

Question 14

A huge case, attached to a cable, is descending at a constant velocity. The tension in the cable is (neglecting the air resistance):

a) greater than the weight of the case.
b) smaller than the weight of the case.
c) equal to the weight of the case.
d) we cannot tell since we dont know the weight of the case.

Question 15

Objects A and B, isolated from the environment, are initially separated from each other and are then placed in thermal contact. Their initial temperatures are $T_A = 0°C$ and $T_B = 100°C$. The heat capacity of B is twice the one of A. After a certain time, the system reaches equilibrium. The final temperatures are:

a) $T_A = T_B = 50°C$.
b) $T_A = T_B > 50°C$.
c) $T_A = T_B < 50°C$.
d) $T_A > T_B > 50°C$.
e) $T_A > 50°C > T_B$.

A	B
1.0Kg	2.0Kg
0°C	100°C

Question 16

When a car is starting, its driving wheels experience:

a) The force of kinetic friction directed backward.
b) The force of static friction directed backward.
c) The force of kinetic friction directed forward.
d) The force of static friction directed forward.

Question 17

An aquarium partly filled with water accelerates down an incline which is at an angle θ with respect to the horizon. The surface of water in the aquarium:

a) is horizontal.
b) is parallel to the plane of the incline.
c) forms an angle α with the horizon, where $0° < \alpha < \theta$
d) forms an angle α with horizon, where $\theta < \alpha < 90°$.

Question 18

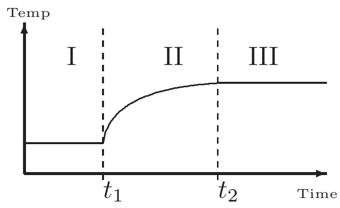
Objects around us have different colours. This is because

a) They are at different temperatures.
b) They are non-thermal radiation sources.
c) Different materials or paints reflect light at different speeds.
d) Different materials or paints reflect different wavelengths.

Question 19

This graph shows the average temperature inside a room. At time t_1 the heater is turned on. We want to compare the power input to the room (P_{in}) and the power output from the room (P_{out}). For which region(s) on the graph is $P_{\text{in}} \neq P_{\text{out}}$?

a) Only region I.
b) Only region II.
c) Only region III.
d) Only regions I & III.
e) Regions I, II & III.



Question 20

A spherical asteroid with a radius of 1 km is illuminated by sunlight. In order to calculate the solar power absorbed by the asteroid, what area should be used?

a) $1\,\text{km}^2$.
b) $3.14\,\text{km}^2$.
c) $12.6\,\text{km}^2$.
d) Answer cannot be determined from the available data.

Question 21

If you toss a ball up, at the highest point

a) The velocity changes direction.
b) The acceleration changes direction.
c) The acceleration is zero.
d) Both velocity and acceleration are zero.
e) More than one of the above is correct.

Question 22

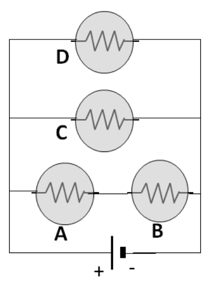
Many cars are now equipped with anti-lock brakes (ABS), which prevents locking of the wheels during emergency braking. What is the main advantage?

a) This saves the tires. Otherwise too much rubber is left on the road.
b) Provides more control over the car but stopping distance increases slightly.
c) This leads to a shorter stopping distance because tires exert rolling friction which is larger than kinetic friction.
d) This leads to a shorter stopping distance because tires exert rolling friction which is larger than static friction.
e) This leads to a shorter stopping distance because tires exert static friction which is larger than kinetic friction.

Question 23

Rank in order, from brightest to dimmest, the identical bulbs A to D.

a) $A = B = C = D$.
b) $A = B > C = D$.
c) $A > C > B > D$.
d) $A > C = D > B$.
e) $C = D > B = A$.



Part B: Problems

Problem 1

Last year, a customer tried to compare two cars. A manufacturer of the USA car claims fuel efficiency of 30 mpg (miles per gallon). The manufacturer of the European car states the fuel efficiency of 7.8 l/100km (liters per hundred kilometers).

(a) Which car is more efficient and by how much?

This year, still undecided, this customer noticed that both manufacturers improved their fuel efficiency numbers by 20%.

(b) Which car is more efficient now, and by how much?

Fig. 1 shows the velocity as a function of time graph for a trip you took with your car.

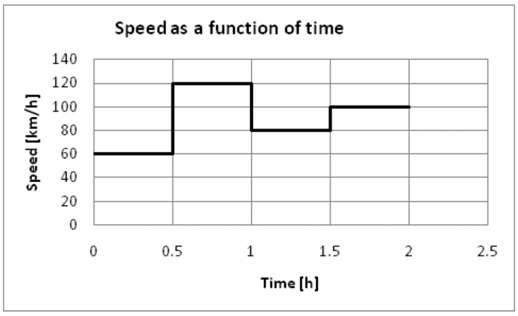


Figure 1

(c) Determine the total distance travelled during this two hour trip.

(d) Fig. 2 displays the air drag force as a function of velocity. What is the total work done by the drag force on the car during this two hour trip?

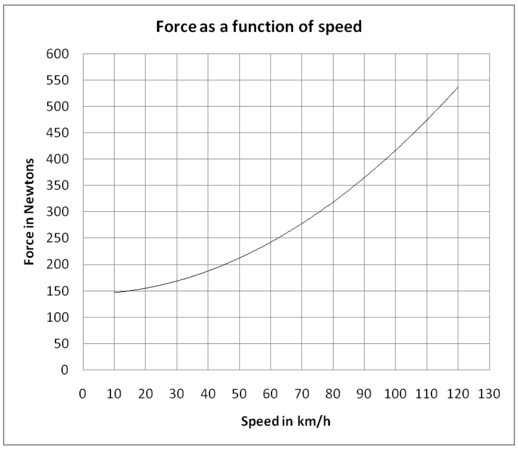


Figure 2

(e) Each liter of gasoline has 35 MJ of energy. The efficiency of the motor as a function of the car's speed is shown on Fig. 3. How much gasoline is used to overcome the air drag during this two hour trip?

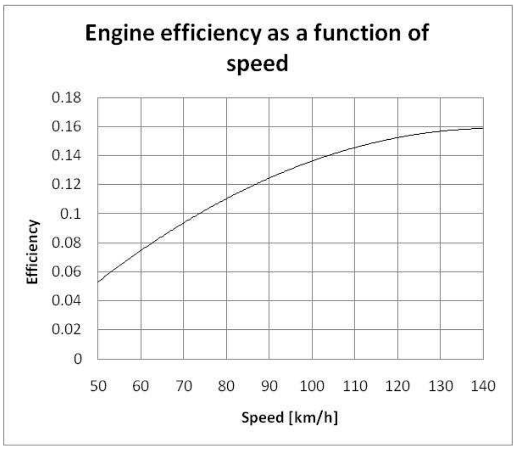


Figure 3

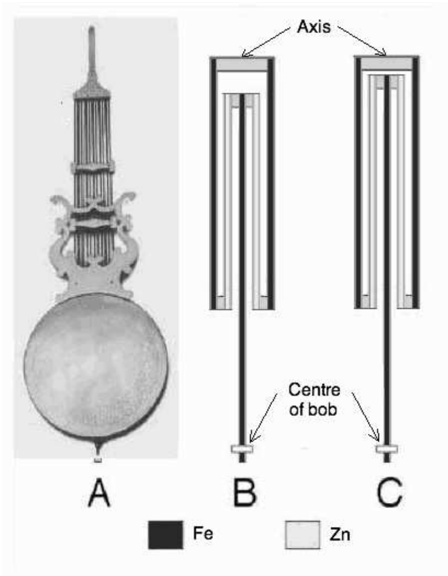
(f) In which part of the trip the fuel was used most efficiently (least gasoline used per kilometer travelled)?

Problem 2

Photo A shows a pendulum for a grandfather clock. It consists of very thin sets of iron and zinc rods and the pendulum bob. At room temperature there is one iron rod of length $L_{0\mathrm{Fe}} = 68.00\,\mathrm{cm}$, two iron rods of length $L_{1\mathrm{Fe}}$ and two zinc rods of length L_{Zn} connected as in diagram B. Both sets of rods are of negligible weight compared to the pendulum bob. The thickness of the connecting pieces is negligible compared to the length of the rods. The bob is attached to the iron rod. The pendulum length, then is $L = L_{0\mathrm{Fe}} + (L_{1\mathrm{Fe}} - L_{\mathrm{Zn}})$, $L = 70.00\,\mathrm{cm}$.

(a) Does the position of the rods in diagram B correspond to a warmer day or a colder day compared to their position in diagram C?

(b) Find $L_{1\mathrm{Fe}}$ and L_{Zn} if the pendulum period does not change when the temperature changes.



Problem 3

Two radio antennas are 100 m apart on a north-south line. They broadcast identical radio waves at a frequency of 3.0 MHz. Your job is to monitor the signal strength with a handheld receiver. To get to your first measuring point, you walk 800 m east from the midpoint between the antennas, then 600 m north.

(a) What is the phase difference between the waves at this point?

(b) Is the interference at this point maximally constructive, perfectly destructive, or somewhere in between? Explain.

(c) If you now begin to walk further north, does the signal strength increase, decrease, or stay the same? Sketch the amplitude of the signal as a function of the distance.

CAP High School Prize Exam

March 2011

Solutions

Part A: Multiple Choice

Question 1-(c)

Question 2-(b)

Question 3-(d)

Question 4-(c)

Question 5-(d)

Question 6-(b)

Question 7-(a)

Question 8-(b)

Question 9-(c)

Question 10-(c)

Question 11-(c)

Question 12-(b)

Question 13-(d)

Question 14-(c)

Question 15-(b)

Question 16-(d)

Question 17-(c)

Question 18-(d)

Question 19-(b)

Question 20-(b)

Question 21-(a)

Question 22-(e)

Question 23-(e)

Part B: Problems

Problem 1

(a) To compare the efficiencies, we need to convert them to the same unit:

$$\text{USA car efficiency} = 30\,mpg \Rightarrow \frac{3.785\,L/gal}{(30\,mpg)\times(1.609\,km/mile)} = 7.84\,L/100km$$
$$\text{European car efficiency} = 7.80\,L/100km$$

The European car is the most efficient.

(b)

$$0.2 \times 7.8\,L/100km = 1.56\,L/100km \Rightarrow \text{European car new efficiency} = 6.24\,L/100km$$
$$0.2 \times 30\,mpg = 6\,mpg \Rightarrow \text{USA car new efficiency} = 36\,mpg$$
$$\text{USA car new efficiency} = 36\,mpg \Rightarrow \frac{3.785\,L/gal}{(36\,mpg)\times(1.609\,km/mile)} = 6.53\,L/100km$$

Therefore the European car is still the most efficient, by $\frac{6.53-6.24}{6.24}\times 100\% = 4.65\%$. Note that a 20% improvement in efficiency measured in $L/100km$ is more important than a 20% improvement in efficiency measured in mpg.

(c) The distance traveled in each part of the trip is given by the velocity during this part, multiplied by the travel time, so the total distance traveled is:

$$\Delta X = 0.5h \times 60km/h + 0.5h \times 120km/h + 0.5h \times 80km/h + 0.5h \times 100km/h = 180km$$

(d) The work done by the the drag force is the sum of the work in each part of the motion:

$$W = \sum_{i=1}^{4} F_i \Delta X_i = F_i V_i \Delta t_i =$$
$$240N \times 60km/h \times 0.5h + 540N \times 120km/h \times 0.5h + 320N \times 80km/h \times 0.5h + 420N \times 100km/h \times 0.5h = 73.4\,MJ$$

(e) The total gasoline used is given by the sum of the amounts used in each part of the motion:

$$l = \sum_{i=1}^{4} \frac{F_i V_i \Delta t_i}{35\,MJ/L \times \text{Efficiency}_i} =$$
$$0.5h \times \left(\frac{240N\times 60km/h}{0.075} + \frac{540N\times 120km/h}{0.15} + \frac{320N\times 80km/h}{0.11} + \frac{420N\times 100km/h}{0.135}\right)/(35\,MJ/L) = 16.7\,L$$

(f) The amount of gasoline used per kilometer travelled (denoted by l_{km}) for each part of the motion is then given by:

$$l_{km} = \frac{FV\Delta t}{35MJ/L \times \text{Efficiency} \times V\Delta t} = \frac{F}{\text{Efficiency}} \times \frac{1}{35}\frac{J/m}{MJ/L} = \frac{F}{\text{Efficiency}} \times \frac{1}{35} \times 10^{-3}\frac{L}{km}$$

For the different speeds of the car the l_{km} can be calculated:

$$V = 60km/h \Rightarrow l_{km} = 9.14\,L/100km$$
$$V = 120km/h \Rightarrow l_{km} = 10.3\,L/100km$$
$$V = 80km/h \Rightarrow l_{km} = 8.31\,L/100km$$
$$V = 100km/h \Rightarrow l_{km} = 8.89\,L/100km$$

Therefore gasoline is used most efficiently when the car is traveling at $80km/h$.

Problem 2

(a) The position of the rods in diagram B is on a colder day compared to their position in diagram C.

(b) The period of the oscillations of the pendulum is given by:

$$\tau = 2\pi\sqrt{\frac{L}{g}}$$

Thus, to keep the period constant, the total length of pendulum needs to be constant as the temperature varies. If the temperature increases by ΔT, the new length of the pendulum is:

$$L' = L_{0Fe}(1 + \alpha_{Fe}\Delta T) + L_{1Fe}(1 + \alpha_{Fe}\Delta T) - L_{Zn}(1 + \alpha_{Zb}\Delta T)$$
$$\Rightarrow L' = L + \Delta T \times (L_{0Fe}\alpha_{Fe} + L_{1Fe}\alpha_{Fe} - L_{Zn}\alpha_{Zn})$$

If we want the length to be constant, $L' = L$ and:

$$L_{0Fe}\alpha_{Fe} + L_{1Fe}\alpha_{Fe} - L_{Zn}\alpha_{Zn} = 0 \Rightarrow \frac{L_{0Fe}+L_{1Fe}}{L_{Zn}} = \frac{\alpha_{Zn}}{\alpha_{Fe}} = 2.73$$

$$L_{0Fe} + L_{1Fe} - L_{Zn} = 70cm, \qquad L_{0Fe} = 68cm$$

$$\boxed{\Rightarrow \quad L_{1Fe} = 42.5cm \quad , \quad L_{Zn} = 40.5cm}$$

Problem 3

(a) The phase difference between the waves at the point of interest is given by:

$$\Delta\phi = \frac{2\pi\Delta r}{\lambda}$$

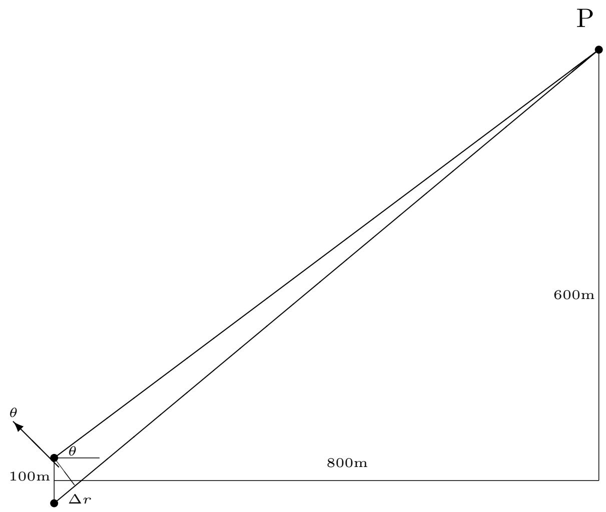
$$\Delta r = \sqrt{(600\,m + 50\,m)^2 + (800\,m)^2} - \sqrt{(600\,m - 50\,m)^2 + (800\,m)^2} = 60\,m$$

$$\lambda = \frac{c}{f} = \frac{3.0\times10^8\,m/s}{3.0\times10^6\,Hz} = 100\,m$$

$$\Rightarrow \Delta\phi = 2\pi\frac{60\,m}{100\,m} = 1.2\pi$$

Alternatively, since the distance between the two sources is small compared to the distance to the receiver, one can use the approximation shown in the figure, and Δr can be computed as follows:

$$\Delta r = d\sin(\theta) = 100\,m \times \frac{600\,m}{\sqrt{(600\,m)^2+(800\,m)^2}} = 60\,m$$



(b) Maximally constructive interference happens when the phase difference is $\Delta\phi = 2\pi n$, and perfectly destructive interference happens when $\Delta\phi = 2\pi(n + \frac{1}{2})$, where $n \in \mathbb{Z}$. Therefore, at the point of interest it is neither perfectly constructive or destructive: something in between.

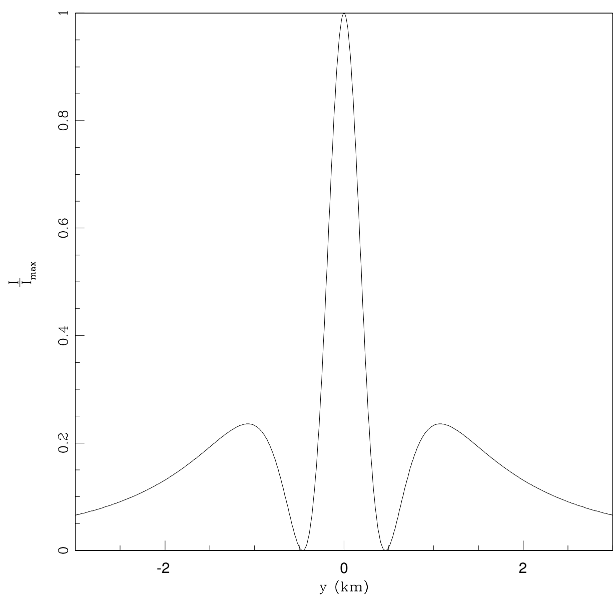
(c) Walking north from the first point, the value Δr increases, so the phase difference increases. Starting from a value of 1.2π, as we walk toward north the signal will increase (the minimum being at π phase difference). On the north-south line where $x = 800\,m$, the phase difference is given by:

$$\Delta\phi = 2\pi\Delta r/\lambda = \frac{2\pi d\sin(\theta)}{\lambda} = 2\pi\sin(\theta)\ ;\ \text{for}\ \ d = 100\,m\ \ \text{and}\ \ \lambda = 100\,m$$

Since $\sin(\theta) = \frac{y}{\sqrt{y^2+(800\,m)^2}}$ varies between $(-1, 1)$, there are one peak at the center and two minima at $\sin(\theta) = \pm 0.5$.

Finally, the intensity will eventually decrease as we walk away from the sources.

We therefore obtain the following graph:



$\frac{I}{I_{max}}$
1
0.8
0.6
0.4
0.2
0
-2
0
2
y (km)

CAP High School Prize Exam

April 5th, 2012

9:00 - 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS

Family Name: ______________________ Given Name: ______________________

Home Address: ____

______________________ Postal Code: ______________________

Telephone: (   ) ______________________ Email: ______________________

School: ______________________ Grade: ______________________

Physics Teacher: ____

Date of Birth: ______________________ Sex: Male ☐ Female ☐

Citizenship: ____

or Immigration Status: ______________________________________

For how many years have you studied in a Canadian school? ______________________

Would you prefer further correspondence in French or English? ______________________

Sponsored by:

Canadian Association of Physicists,
Canadian Physics Olympiad,

The University of British Columbia,
Department of Physics and Astronomy.

Canadian Association of Physicists
2012 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on students' performance on sections A and B of the exam. Performance on the questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. Part A consists of twenty-five multiple-choice questions. The questions in part B span a range of difficulties, and may require graphing. Be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway.
Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.
Notice: Full marks will be given to a student who provides any full correct solution to the long problems. Partial marks will be given for partial solutions. There are no penalties for incorrect answers. The questions are not of equal difficulty. Remember we are challenging the best physics students in Canada; it is possible that even the best papers may not achieve an overall score of 80%. This is meant to be tough!

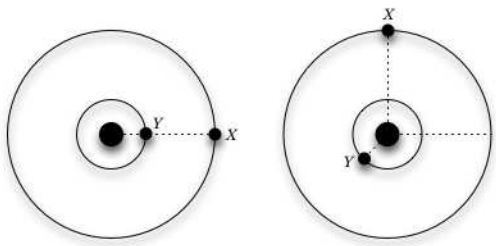
Data

Speed of light $c = 3.00 \times 10^8\,\mathrm{m/s}$
Gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$
Acceleration due to gravity $g = 9.80\,\mathrm{m/s^2}$
Normal atmospheric pressure $P_0 = 1.01 \times 10^5\,\mathrm{Pa}$
Density of fresh water $\rho = 1.00 \times 10^3\,\mathrm{kg/m^3}$
Specific heat of water $C_w = 4186\,\mathrm{J/kgK}$
Specific heat of ice $C_i = 2050\,\mathrm{J/kgK}$
Latent heat of water $L_w = 2260\,\mathrm{kJ/kg}$
Latent heat of ice $L_i = 334\,\mathrm{kJ/kg}$
Density of ice $\rho_i = 916\,\mathrm{kg/m^3}$
Fundamental charge $e = 1.60 \times 10^{-19}\,\mathrm{C}$
Mass of electron $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of proton $m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$
Planck's constant $h = 6.63 \times 10^{-34}\,\mathrm{Js}$
Coulomb's constant $1/(4\pi\epsilon_0) = 8.99 \times 10^9\,\mathrm{Nm^2/C^2}$
Boltzmann's constant $k = 1.38 \times 10^{-23}\,\mathrm{J/K}$
A.U. Astronomical Unit $1.49598 \times 10^{11}\,\mathrm{m}$: The approximate distance from the Sun to the Earth.
Radius of the Earth $R_e = 6.371 \times 10^6\,\mathrm{m}$
Radius of the Sun $R_s = 6.96 \times 10^8\,\mathrm{m}$

Part A: Multiple Choice

Each multiple choice question is worth 1 point.

Question 1
Two planets X and Y travel counterclockwise in circular orbits around a star as shown in the figure below:



The radii of their orbits are in the ratio 3 : 1. At some time, they are aligned as in Fig. 1, making a straight line with the star. After five Earth years, the angular displacement of planet X is 90.0°, as in Fig. 2. What is the angular displacement of planet Y at this time?

a) 35°
b) 90°
c) 180°
d) 360°
e) 468°

Question 2
You are holding a bottle of sparkling water inside a car moving forward. When the driver applies the brakes:

a) Bubbles in the middle of the liquid will start to move forward with respect to the bottle.
b) Bubbles will start to move backward with respect the bottle.
c) Bubbles will stay at the same horizontal location in the water.
d) Depending on the speed of the car, bubbles might move forward or backward.

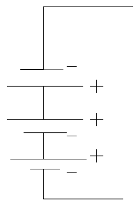
Question 3
An Earth satellite revolves in a circular orbit at a height h from the surface of the Earth. If R is the Earth's radius and g is the acceleration due to gravity at the surface of the Earth, then the speed of the satellite is given by:

a) $\sqrt{gR}$
b) $\sqrt{g(R+h)}$
c) $\sqrt{g\frac{R^2}{R+h}}$
d) $\sqrt{g\frac{(R+h)^2}{R}}$

Question 4
Assume that sodium produces monochromatic light with a wavelength of 5.89×10^{-7}m. At what approximate rate would a 10 watt sodium-vapour light be emitting photons? Assume that the efficiency of the light bulb is about 30%.

a) 8.9×10^{18} photons/sec.
b) 3.0×10^{19} photons/sec.
c) 9.9×10^{19} photons/sec.
d) 2.0×10^{20} photons/sec.

Question 5
Three batteries are placed in series. Each battery has an internal resistance r. If one of the batteries is placed the wrong way around as shown in the picture, what will be the total resistance of the three cells now?



a) $r/3$
b) $r/2$
c) r
d) $2r$
e) $3r$

Question 6
At the designed intensity, the two beams circulating in the Large Hadron Collider (LHC) at CERN consist of 5616 bunches (2808 in each direction) of approximately 1.15×10^{11} protons per bunch. A small commercial hydrogen cylinder contains 40 L of gas at a pressure of 10 MPa and a temperature of 25°C. Assuming an injection efficiency of 70%, how many times could the LHC be filled at the designed intensity using a single, perfectly hermetic cylinder?

a) 1.1×10^{11}
b) 1.5×10^{11}
c) 2.1×10^{11}
d) 1.1×10^{14}
e) 1.5×10^{14}

Question 7
The Earth is constantly receiving energy from the Sun. To stay at approximately the same temperature, the Earth loses energy by:

a) Conduction
b) Radiation
c) Convection
d) Evaporation
e) The Earth does not lose energy, this is why we have global warming.

Question 8
A car starts to move at time $t = 0$. If the engine of the car is able to provide constant power, which of the following statements is correct about the speed of car at the beginning of the motion?

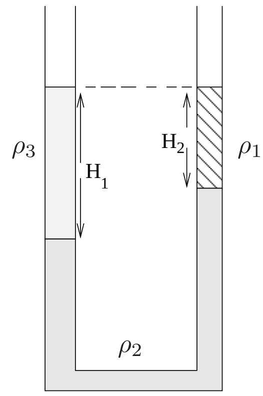
a) The speed is constant.
b) The speed increases proportionally to time passed ($v \propto t$).
c) The speed increases as the square root of time ($v \propto \sqrt{t}$).
d) The speed increases as time squared ($v \propto t^2$).

Question 9
A detector far away from the source of a wave is detecting pulses of that wave every 0.2 second. If the detector starts to move towards the source at a speed of 6.0 km/h, then it would detect a total of 18200 pulses per hour. What is the speed of the wave?

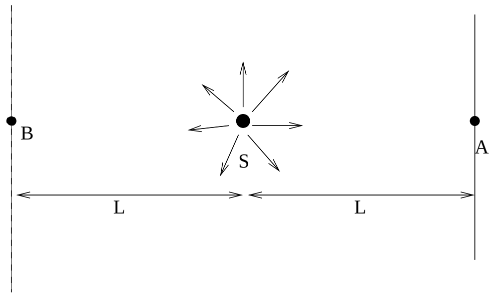
a) 100 m/s
b) 150 m/s
c) 200 m/s
d) 300 m/s

Question 10
There are three different liquids, with densities ρ_1, ρ_2 and ρ_3, in a U-shaped container as shown in the picture. The lengths shown are $H_1 = 15$ cm and $H_2 = 10$ cm. Which of the following equations gives the correct relation between the densities of the fluids in the container?

a) $3\rho_3 = 2\rho_1 + \rho_2$
b) $\rho_3 = 2\rho_1 + 3\rho_2$
c) $2\rho_3 = 3\rho_1 + \rho_2$
d) $\rho_3 = 3\rho_1 + 2\rho_2$

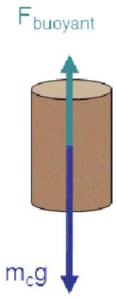


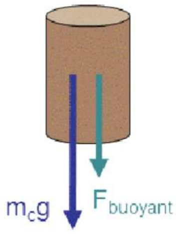
Question 11
The intensity of light from a source decreases with the distance x from the source, as $\frac{1}{x^2}$. In the following picture, the intensity of light from the source S at point A on a curtain is 8.1U where U is some unit. We then add a big mirror parallel to the curtain at point B. The distances AS and SB are equal, and the points A, S and B are in line perpendicular to the curtain and the mirror. What is the intensity of light reaching point A now?

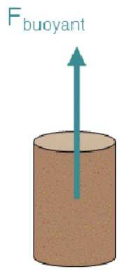


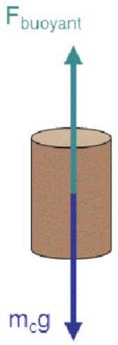
a) 9.0 U
b) 10.15 U
c) 10.8 U
d) 16.2 U

Question 12
Which of the following is the correct free-body diagram for a cylinder floating in water?

a) 

b) 

c) 

d) 

Question 13
Vancouver's latitude is about 49°, and Earth's axial tilt is about 23°. The power delivered by the Sun reaching a horizontal surface in Vancouver at noon differs in winter and summer. What is the approximate ratio of this quantity measured in winter, compared to that measured in summer?

a) 1 : 2
b) 1 : 3
c) 1 : 4
d) 1 : 5

Question 14
Consider a metal rod firmly attached to a wall. When you strike the rod with a hammer, which kind of wave will you excite?

a) A longitudinal wave.
b) A transverse wave.
c) Either kind, or both, depending on where and how you hit the rod.
d) No wave will be excited, the metal is too strong.

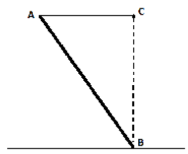
Question 15

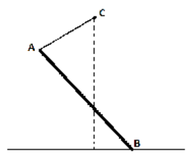
Assume that you videotaped the fall of a ball due to gravity in vacuum, and are now playing the video in reverse at the same speed. Which of the following statements is correct about the acceleration of the ball seen in these conditions, compared to that of the actual falling ball?

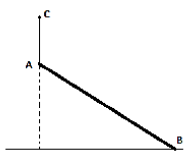
a) The accelerations are the same in both cases.
b) They have the same value but opposite directions.
c) They have different values but the same direction.
d) Both the values and directions differ.

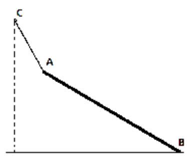
Question 16

A rod (AB) is attached to a fixed point (C) using a light rope (AC). The other end of the rod (B) is sitting on ice with negligible friction and the system is in stationary position. Which of the following can be the equilibrium configuration of this system?

a) 

b) 

c) 

d) 

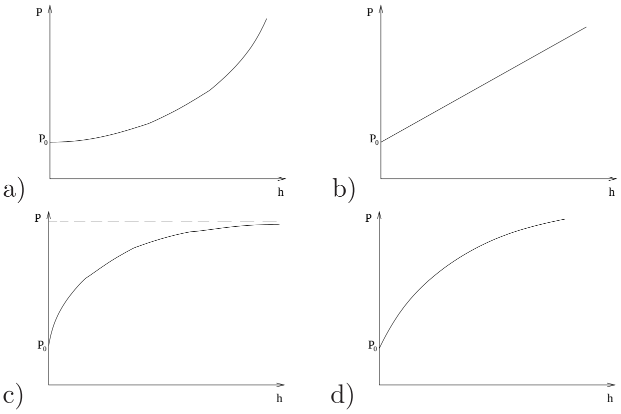
Question 17

Which of the following is the closest to the total mass of the atmosphere around the Earth?

a) 10^{13} kg
b) 10^{16} kg
c) 10^{19} kg
d) 10^{22} kg

Question 18

At shallow depth, h, the pressure in the ocean is simply given by $P = P_0 + \rho g h$, in which ρ is the density of water and P_0 is the air pressure. As we go deeper, the high pressure causes the water to compress and become denser. Which of the following sketches illustrates the correct dependence of the pressure on the depth h?



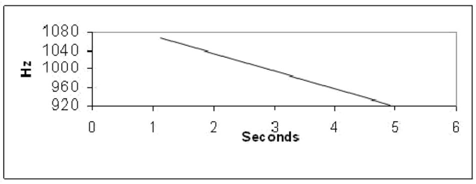
Question 19

An aluminium plate and a glass plate are left in a room for a long time. Putting one ice cube on each plate, you notice that the ice melts faster on the aluminium plate. Why?

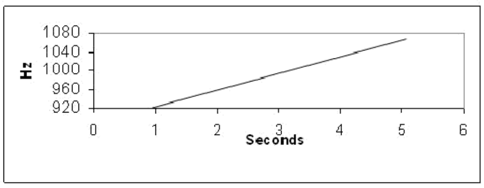
a) The ice is in thermal equilibrium with the glass plate, but not with the aluminium plate.
b) Aluminium conducts heat to the ice more rapidly than glass.
c) The aluminium plate holds more heat.
d) The aluminium plate is warmer.

Question 20

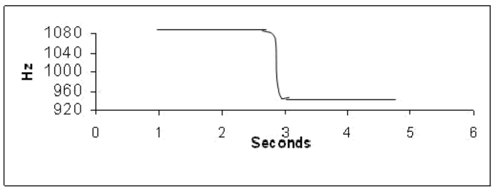
You are on the side of the highway, listening to the siren of a fast-approaching ambulance. When the ambulance is not moving the frequency of its siren is 1000 Hz. Which of the following graphs best describes the frequency that you hear as the ambulance approaches and then passes you?



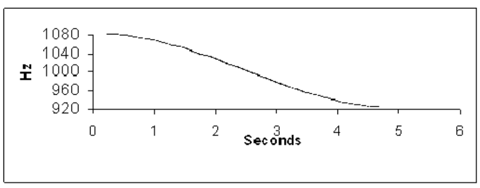
a)



b)



c)



d)

Question 21

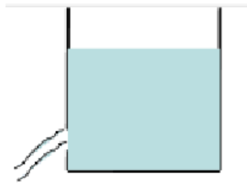
Which of the following radiation types has the longest wavelength?

a) Radio waves
b) Visible light
c) X rays
d) Infrared light
e) They all have at the same wavelength

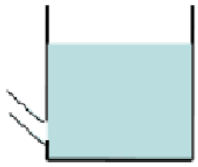
Question 22

A cup filled with water has a hole in the side, through which the liquid is flowing out. If the cup is dropped from a height, what will happen to the water flowing from the cup?

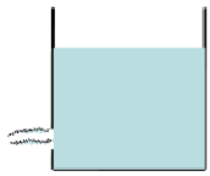
a) It will keep coming out, flowing at the same rate as before:



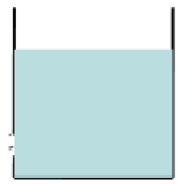
b) It will keep coming out, but it will flow a bit slower than before.
c) It will keep coming out, but start to flow upwards relative to the cup:



d) It will keep coming out, flowing horizontally with respect to the falling cup:



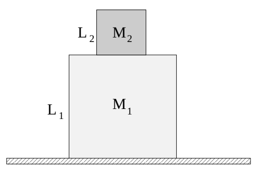
e) It will stop flowing.



Question 23

Two cubes, with respective masses M_1 and M_2 and side lengths L_1 and L_2, are lying on a smooth table as shown. What is the pressure exerted by the cubes on the table?

a) $\frac{M_1 g}{{L_1}^2} + \frac{M_2 g}{{L_2}^2}$
b) $\frac{M_1 g}{{L_1}^2} + \frac{M_2 g}{{L_1}^2}$
c) $\frac{M_1 g}{{L_2}^2} + \frac{M_2 g}{{L_2}^2}$
d) None of the above.



Question 24
Consider an object that floats in water but sinks in oil. When the object floats in water, half of it is submerged. If we slowly pour oil on top of the water so it completely covers the object, the object:

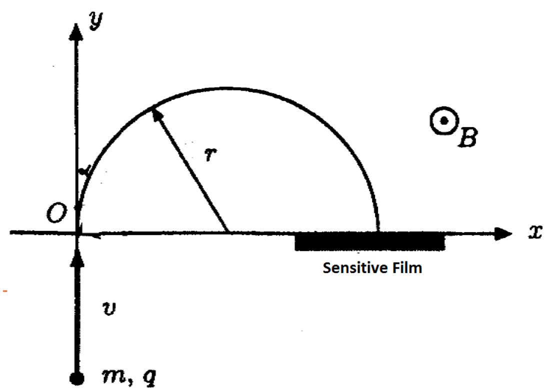
a) moves up
b) stays in the same place
c) moves down

Question 25
The pressure in a stream of water flowing out of a faucet is:

a) largest at the bottom
b) largest near the faucet
c) the same everywhere in the stream

Part B: Problems

Problem 1 (10 points)
Mass spectrometry is a way of measuring the mass-to-charge ratio of charged particles as well as distinguishing different components of a beam of particles. As an example of such an experiment, consider ions of two isotopes of Potassium, K^{39}_{19} and K^{41}_{19}, with positive charge $+e$ entering as a beam of particles into a region with magnetic field B in the z direction, as shown in the picture. The beam is injected at point O. Due to the magnetic field, all the ions travel in a circle with radius given by $r = \frac{mv}{qB}$, in which v is the velocity of ions, m is the mass of ions and q is the charge. After these two types of ions travel through the magnetic field, they reach a sensitive film and record bright spots on it as show in the picture.
Consider the distance from point O to the bright spot produced by K^{39}_{19} ion to be L_1, and the distance to the spot recorded by K^{41}_{19} ion to be L_2. We use L for the average of L_1 and L_2.



(a) Assuming that the energy of both isotopes is the same, compute the ratio $\frac{L_2-L_1}{L}$

(b) Since these ions have different masses, one can distinguish them by observing the two bright spots on the film. However, some experimental uncertainties can complicate the situation. For example, tuning the energy of the beam to a specific value is impossible. If the energy of particles in the beam varies from 7.9×10^{-17}J to 8.1×10^{-17}J, is it possible to distinguish these two types of ions?

(c) Furthermore, adjusting the ions to enter exactly in the y direction, as shown in the picture, is also impossible. Assuming that the beam enters at a small angle α from the y direction, compute L as a function of α.

(d) Now assume that the particles in the incident beam can have any angle α between 0 and 3° from the y direction, due to the uncertainty in the angle. Is it possible to distinguish the two types of ions in this case?

Problem 2 (10 points)
Andrzej's electric bike is equipped with a motor built into the front wheel hub. The stator of the electric motor (stationary part with the coils) is fixed solidly to the axle and the magnets are attached to the wheel. He made the following measurements on the bike:

1. If he runs it on a horizontal road at 20 km/h and then switches the motor off, it takes the bike 50 m to stop.
2. At full power, it can accelerate up a 5° slope from 5 to 20 km/h in 6 seconds.
3. The total mass of the bike plus the rider, is 106 kg.
4. The diameter of the wheels is 66 cm.
5. The diameter of the front wheel axle is 11 mm.
6. The bike uses a 48 V battery with a capacity of 8 Ah (ampere-hour).
7. The motor is 85% efficient.

Find out:
(a) What is the current flowing through the motor when the bike is driven at 20 km/h on a horizontal road without pedaling?
(b) How far can it go on one battery charge at 20 km/h?
(c) What is the net torque acting on the front axle?
(d) What is the current flowing through the motor when the bike is accelerating up a 5° slope from 0 to 20 km/h in 20 seconds?
(e) What is the full mechanical power of this motor?
(f) What is the maximum slope one can bike up at constant speed using the motor only (not pedaling)?
(g) What is the current flowing through the motor at maximum power?
(h) What was the most important simplifying assumption that you had to make in order to solve this problem?

Problem 3

This problem has two independent parts.

Problem 3A (5 points)

A Russian nuclear icebreaker named *50 Let Pobedy* has a length of 160 m, a width of 30 m and two nuclear reactors, each delivering about 30 MW of power. It can clear a path 35 m wide in 2 m thick ice (at $-10°C$) at a speed of 5 knots (2.5 m/s). Somebody suggested that instead of breaking the ice, one should melt it with a gigantic heater mounted at the front of the ship.

(a) For this to work, what power should this heater have, in order to achieve the same performance as the icebreaker?

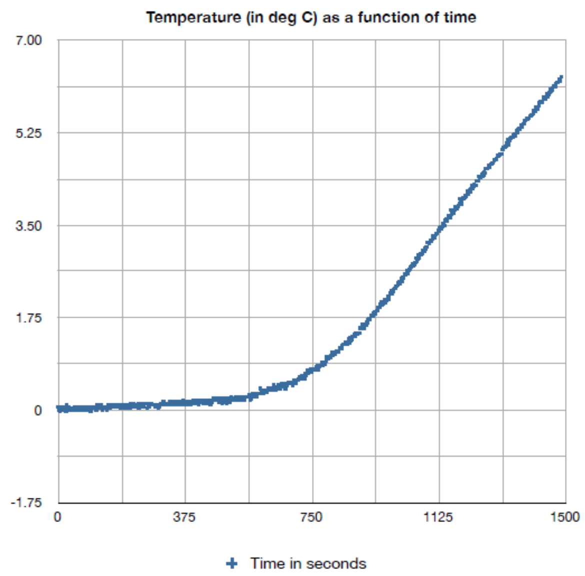
(b) Explain in one sentence whether such a heater would work or not on this icebreaker.

Problem 3B (5 points)

You are given a graph of temperature as a function of time of a container with a mixture of ice and water slowly heated at constant rate. When all the ice melts, there is 850 mL of water in the container. In the following questions, neglect the evaporation of water.

(a) At which rate is the container heated?

(b) How much ice was initially present?



Question 1	a	b	c	d	e
Question 2	a	b	c	d	
Question 3	a	b	c	d	
Question 4	a	b	c	d	
Question 5	a	b	c	d	e
Question 6	a	b	c	d	e
Question 7	a	b	c	d	e
Question 8	a	b	c	d	
Question 9	a	b	c	d	
Question 10	a	b	c	d	
Question 11	a	b	c	d	
Question 12	a	b	c	d	
Question 13	a	b	c	d	
Question 14	a	b	c	d	
Question 15	a	b	c	d	
Question 16	a	b	c	d	
Question 17	a	b	c	d	
Question 18	a	b	c	d	
Question 19	a	b	c	d	
Question 20	a	b	c	d	
Question 21	a	b	c	d	e
Question 22	a	b	c	d	e
Question 23	a	b	c	d	
Question 24	a	b	c		
Question 25	a	b	c		

CAP High School Prize Exam

April 2012

Solutions

Part A: Multiple Choice

Question 1-(e)

Question 2-(b)

Question 3-(c)

Question 4-(a)

Question 5-(e)

Question 6-(c)

Question 7-(b)

Question 8-(c)

Question 9-(b)

Question 10-(a)

Question 11-(a)

Question 12-(d)

Question 13-(b)

Question 14-(c)

Question 15-(a)

Question 16-(c)

Question 17-(c)

Question 18-(a)

Question 19-(b)

Question 20-(c)

Question 21-(a)

Question 22-(e)

Question 23-(b)

Question 24-(a)

Question 25-(c)

Part B: Problems

Problem 1

(a)

Using the formula given for the radius of the beam inside the magnetic field:

$$L_2 = 2r_2 = 2\frac{m_2 V_2}{qB} = 2\frac{\sqrt{2Em_2}}{qB} \tag{1}$$

$$L_1 = 2r_1 = 2\frac{m_1 V_1}{qB} = 2\frac{\sqrt{2Em_1}}{qB} \tag{2}$$

$$\Rightarrow \frac{L_2 - L_1}{L} = \frac{r_2 - r_1}{r_2 + r_1} = 2\frac{\sqrt{m_2} - \sqrt{m_1}}{\sqrt{m_2} + \sqrt{m_1}} = 2\frac{\sqrt{41} - \sqrt{39}}{\sqrt{41} + \sqrt{39}} = 0.025 \tag{3}$$

In above, we used the fact that energies of the beams are the same.

(b)

Since the position of collision of the beam is given by:

$$L = 2\frac{\sqrt{2Em}}{qB} \tag{4}$$

Then if the energy varies by ΔE the position varies by:

$$\frac{\Delta L}{L} = \frac{L_{max} - L_{min}}{(L_{max} + L_{min})/2} = 2\frac{\sqrt{E_{max}} - \sqrt{E_{min}}}{\sqrt{E_{max}} + \sqrt{E_{min}}} \tag{5}$$

$$\Rightarrow \frac{\Delta L}{L} = 2\frac{\sqrt{8.1} - \sqrt{7.9}}{\sqrt{8.1} + \sqrt{7.9}} = 0.0125 \tag{6}$$

The variation (error) created by the energy variation is of the same order of the difference in the position of the particles calculated in previous part, this would make distinguishing the particles impossible.

(c)

If the beams enter at the angle α the position is given by:

$$L = 2r\cos\alpha = 2\frac{mv}{qB}\cos\alpha \tag{7}$$

(d)

The uncertainity created by the angle variation is:

$$\frac{\Delta L}{L} = (2r - 2r\cos\alpha)/2r = 1 - \cos(3^o) = 1.37 \times 10^{-3} \tag{8}$$

Since the uncertainty due to angle variation is much less than the seperation in part (a), distinguishing particles is possible.

Problem 2

(a) From the first statement, we can calculate the combined force of friction and air resistance, F_f. We have to assume that it is independent of speed, so we basically neglect the air resistance. The bike's kinetic energy is converted to work of the force of friction on the distance $d = 50$ m.

$$F_f d = mv^2/2$$
$$v = 20 \text{ km/h} = 5.56 \text{ m/s}$$
$$F = mv^2/2d = 32.7 \text{ N}$$

So the corresponding coefficient of friction is

$$\mu = F/mg = 0.0315.$$

To bike at 20 km/h, we need mechanical power

$$P_1 = Fv = 182 \text{ W},$$

which at efficiency $\epsilon = 85\%$ gives us electrical power

$$P_{1\epsilon} = P_1/\epsilon = 283 \text{ W}.$$

Electrical power is a product of the voltage V times the current I, so $P_{1\epsilon} = VI_1$.

We know the voltage $V = 48$ V, so finally

$$I_1 = P_{1\epsilon}/V = Fv/\epsilon V = 4.45 \text{ A}.$$

(b) The battery capacity is 8 Ah, so is will supply 4.45 A for a time equal to:

$$8 \text{ Ah} \ / \ 4.45 \text{ A} = 1.8 \text{ h}.$$

During this time, at 20 km/h, the cyclist will bike for 36 km.

(c) At this constant speed, the net force on the bike and net torque must be zero, as there is no acceleration or angular acceleration of the wheel. [We also gave points to the students who calculated the torque related to the force driving the bike: $\tau = 32.7$ N * 0.33 m = 10.8 Nm.]

(d) Now we have to calculate the total force acting on the bike. There are three forces:

- The force driving the bike, F
- The force of friction, $-\mu mgcos(\theta)$
- The component of the force of gravity, $-mgsin(\theta)$

We assume constant acceleration $a = v/t_1 =$ (5.56 m/s) / 20 s.

From Newton's Law

$$ma = F - \mu mgcos(\theta) - mgsin(\theta),$$

so we need the force F:

$$F = (mv/t_1 + \mu mgcos(\theta) + mgsin(\theta)).$$

The average velocity during this time is $v/2$, so the average mechanical power of the motor is in this situation is:

$$P_2 = v(mv/t_1 + \mu mgcos(\theta) + mgsin(\theta))/2 = 424 \text{ W}.$$

The average electrical power needed is

$$P_{2\epsilon} = P_2/\epsilon = 499 \text{ W},$$

so the average current is $I_2 = P_{2\epsilon}/V = 10.4$ A.

(e) We again assume constant acceleration. Maximum power is needed at the end, to reach the maximum speed of 20 km/h. From statement 2 we have the average mechanical power like in (d), but now the acceleration is $(v_2 - v_1)/t_2$ where $v_2 = 20$ km/h, $v_1 = 5$ km/h and $t_2 = 6$ s. Therefore the acceleration is $a = 0.69$ m/s. At the top speed the power needed is

$$P_3 = v_2(m(v_2 - v_1)/t_2 + \mu mgcos(\theta) - mgsin(\theta))/2 = 1093 \text{ W}.$$

(f) Assuming speed v going up the slope, we have

$$P_3 = v(\mu mgcos(\theta) + mgsin(\theta))$$

One can substitute $cos(\theta)$ with $\sqrt{1 - sin^2(\theta)}$ and get a quadratic equation for $sin(\theta)$, or solve the problem graphically for a given velocity. $v = 20$ km/h gives $\theta = 9°$. At 5 km/h one gets about 45°, which is probably impossible - the wheels will slide.

(g) $I_3 = P_3/V = 26.8$ A

(h) We assumed constant friction force and constant acceleration.

Problem 3A

(a)

The volume of the ice that needs to be melted per second is given by:

$$Q = w \times d \times V = 35m \times 2m \times 2.5m/s = 175m^3/s \tag{9}$$

Therefore the total mass melted per second:

$$\dot{M} = Q \times \rho_i = 175 \times 916 = 160300kg/s \tag{10}$$

The enegy required to melt this amount of ice is given by:

$$W = \dot{M}(C_i \Delta T + L_w) = 160300 \times (2050 \times 10 + 2260) = 3.65 \times 10^9 KJ/sec \tag{11}$$

Therefore the power should be $3.65 \times 10^9 kJ/sec$ to provied the same performance.

(b)

It will not work, since having such a big power in the icebreaker is impossible.

Problem 3B

(a) From the graph, one notices that starting at $t = 750$ s, temperature rose steadily as a function of time, which corresponds to the heating of liquid water after all the ice melted. From $t = 750$ s to $t = 1500$ s, the temperature rose by 5.25 K. The heating power was therefore

(5.25 K * 4186 J/kg K * 0.85 kg) / 750 s = 25 W

(b) The complete melting of ice took approximately 750 s. The heat delivered during this time was

750 s * 25 W = 18 750 J

corresponding to

18 750 J / (334 J/g) = 56 grams of ice initially.

CAP High School Prize Exam

April 9th, 2013

9:00 - 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS

Family Name: ______________________ Given Name: ______________________

Home Address: ____

______________________ Postal Code: ______________________

Telephone: (   ) ______________________ Email: ______________________

School: ______________________ Grade: ______________________

Physics Teacher: ____

Date of Birth: ______________________ Sex: Male ☐ Female ☐

Citizenship: ____

If not a Canadian citizen, what is your Immigration Status? ______________________

For how many years have you studied in a Canadian school? ______________________

Would you prefer further correspondence in French or English? ______________________

Sponsored by:

Canadian Association of Physicists,
Canadian Physics Olympiad,

The University of British Columbia,
Department of Physics and Astronomy.

Canadian Association of Physicists 2013 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on students' performance on sections A and B of the exam. Performance on the questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. Part A consists of twenty-five multiple-choice questions. The questions in part B span a range of difficulties, and may require graphing. Be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway.

Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions ***on the answer sheet*** provided; most importantly, write your solutions to the three long problems on ***three separate*** sheets as they will be marked by people in different parts of Canada. Good luck.

Notice: Full marks will be given to a student who provides any full correct solution to the long problems. Partial marks will be given for partial solutions. There are no penalties for incorrect answers. The questions are not of equal difficulty. Remember we are challenging the best physics students in Canada; it is possible that even the best papers may not achieve an overall score of 80%. This is meant to be tough!

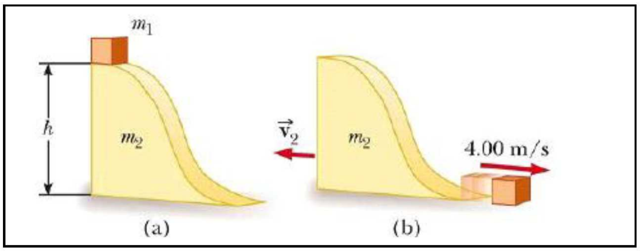
Data

Speed of light $c = 3.00 \times 10^8\,\mathrm{m/s}$
Gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$
Acceleration due to gravity $g = 9.80\,\mathrm{m/s^2}$
Normal atmospheric pressure $P_0 = 1.01 \times 10^5\,\mathrm{Pa}$
Density of fresh water $\rho = 1.00 \times 10^3\,\mathrm{kg/m^3}$
Specific heat of water $C_w = 4186\,\mathrm{J/(kg \cdot K)}$
Specific heat of ice $C_i = 2050\,\mathrm{J/(kg \cdot K)}$
Latent heat of water $L_w = 2260\,\mathrm{kJ/kg}$
Latent heat of ice $L_i = 334\,\mathrm{kJ/kg}$
Density of ice $\rho_i = 916\,\mathrm{kg/m^3}$
Fundamental charge $e = 1.60 \times 10^{-19}\,\mathrm{C}$
Mass of electron $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of proton $m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$
Planck's constant $h = 6.63 \times 10^{-34}\,\mathrm{Js}$
Coulomb's constant $1/(4\pi\epsilon_0) = 8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2}$
Boltzmann's constant $k = 1.38 \times 10^{-23}\,\mathrm{J/K}$
A.U. Astronomical Unit $= 1.49598 \times 10^{11}\,\mathrm{m}$: The approximate distance from the Sun to the Earth.
Radius of the Earth $R_E = 6.371 \times 10^6\,\mathrm{m}$
Radius of the Sun $R_S = 6.96 \times 10^8\,\mathrm{m}$

Part A: Multiple Choice

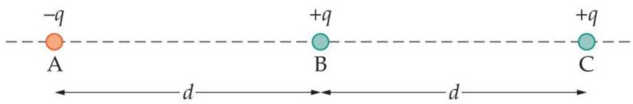
Each multiple choice question is worth 1 point.

Question 1
A small block of mass m_1 is released from rest at the top of a curve-shaped, frictionless wedge of mass m_2 which sits on a frictionless horizontal surface as shown. When the block leaves the wedge its velocity is measured to be 4.00 m/s to the right as shown in the figure. If the mass of the block is doubled to become $2m_1$, what can be said about the speed with which it leaves the wedge?



a) Its speed is less than 4.00 m/s
b) Its speed is equal to 4.00 m/s
c) Its speed is greater than 4.00 m/s
d) Not enough information is given.

Question 2
Consider the electric charges A, B, C shown in the figure below, where q is a positive number. Which answer correctly describes the magnitude of the net force experienced by the charges?



a) $F_A > F_B > F_C$
b) $F_A > F_C > F_B$
c) $F_B > F_A > F_C$
d) $F_A = F_B = F_C$
e) $F_A > F_B = F_C$

Question 3
A long, straight wire carries a current that decreases linearly with time. What is the direction of the induced electric field outside the wire?

a) Parallel to the current in the wire, in the same direction.
b) Parallel to the current in the wire, in the opposite direction.
c) Pointing radially outward from the wire.
d) Pointing radially inward toward the wire.
e) There is no induced electric field outside the wire.

Question 4
A car driven at a constant speed turns left. Which force makes the car turn?

a) The force of friction, directed towards the left.
b) The force of friction, directed towards the right.
c) The force with which the driver is turning the steering wheel, directed towards the left.
d) The force with which the driver is turning the steering wheel, directed towards the right.

Question 5
A cyclist stops pedaling at a velocity $v = 36$ km/h and notices that her bike keeps moving for $d = 500$ m before it stops. The total mass of the bike, the biker and her camping equipment is $m = 100$ kg. What is the average combined force on the bicycle due to friction and drag?

a) 10 N
b) 20 N
c) 130 N
d) 260 N

Question 6
A child is swinging to and fro on a playground swing. At the instant the chains of the swing are vertical, what is the direction of the child's acceleration?

a) Downwards.
b) Upwards.
c) In the direction of motion.
d) Opposite to the direction of motion.
e) At that moment, the acceleration is zero.

Question 7
A bullet of mass 5 g is accelerated in a rifle barrel with an approximately constant force of 2500 N. The mass of the rifle is 5 kg. What is the force pushing the rifle back?

a) 2.5 N
b) 2 500 N
c) 2 500 000 N
d) 0 N

Question 8
Two balls of different masses collide head-on. After the collision, the balls remain at rest, and no external forces act on them. Which statement is true about the speeds of the balls before the collision?

a) They must have been different, and the collision was inelastic.
b) They must have been different, and the collision was elastic.
c) They must have been identical, and the collision was inelastic.
d) They must have been identical, and the collision was elastic.
e) No combination of speed values could have caused both balls to stop after the collision.

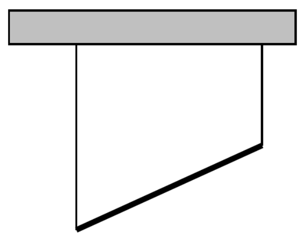
Question 9

A Ferris wheel is turning about a horizontal axis through its center. The linear speed of a passenger sitting on the rim is constant. Which one of the following sentences about the passenger's acceleration is correct?

a) Its magnitude at the highest point is higher than at the lowest point, and it is downwards at both points.
b) Both its magnitude and direction are the same at the highest and at the lowest points.
c) Its magnitude is the same at the highest and at the lowest points, but the directions are opposite.
d) Its magnitude at the highest point is smaller than at the lowest point, and it is downwards at both points.

Question 10

A uniform rod is suspended on two thin strings as shown below. Which string has a larger tension force?



a) The right one
b) The left one
c) The tension force is the same on both
d) It depends on the angle

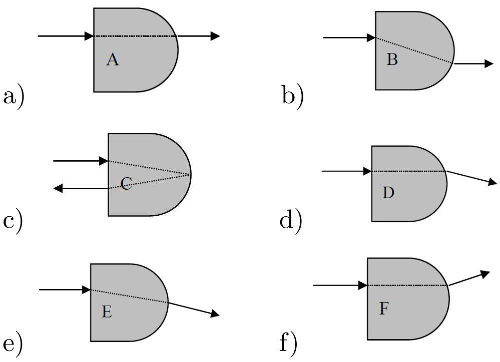
Question 11

Two point charges $-2Q$ and $+Q$ are placed on the x-axis with $-2Q$ at $x = 0$ and $+Q$ at $x = a$. Which of the following statements is true?

a) There is no point on the x-axis where the electric field is zero.
b) There is a point on the x-axis, at $x < 0$, where the electric field is zero.
c) There is a point on the x-axis, between the charges ($0 < x < a$), where the electric field is zero.
d) There is a point on the x-axis, at $x > a$, where the electric field is zero.

Question 12

In a lab experiment, a laser beam hits a semicircular glass object off the centre axis. The ray enters perpendicularly to the surface, and the environment is air. Which ray diagram is correct?



Question 13

You put two identical ice cubes on plates of different materials. One cube is put on an aluminum plate, and the other on a glass plate. Both plates have been in the room for a long time prior to the experiment. You notice that the ice melts faster on the metal plate. Why?

a) The ice is in thermal equilibrium with the plastic plate, but not with the metal plate.
b) The metal plate conducts heat to the ice more rapidly than the plastic plate.
c) The metal plate holds more heat than the plastic plate.
d) The metal plate was warmer than the plastic plate initially.

Question 14

Two black objects of the same diameter, a sphere and a flat disk, are placed in a parallel beam of light. The plane of the disk is perpendicular to the beam. What can be said about the force acting on them?

a) The force is zero.
b) The force is larger on the disk than on the sphere.
c) The force is larger on the sphere than on the disk.
d) The force is the same on both objects.

Question 15

Adrianne has a device that emits photons with a wavelength of $\lambda = 1.498$ km. Kwan has a similar device that emits photons at a frequency of $f = 201$ kHz. Whose device emits photons in air at the highest frequency?

a) Adrianne's
b) Kwan's
c) The frequencies are the same.
d) There is not enough information to compare.

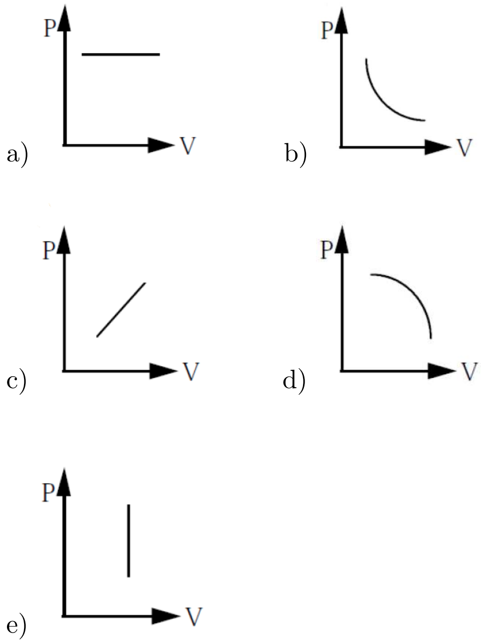
Question 16

A girl sitting on a floating mattress in a small pool holds a model of a motorboat in her hands. When she puts it in the water to start driving it with her remote control, what happens to the water level in the pool?

a) It rises by a small amount.
b) It drops by a small amount.
c) It stays the same.
d) The qualitative result depends on the weight of the model boat.

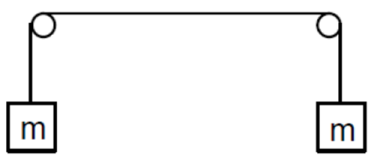
Question 17

Which of the accompanying $P-V$ diagrams best represents an isothermal process, that is a process occurring at constant temperature?



Question 18

Two identical objects of mass m are connected to a massless string which is hung over two frictionless pulleys as shown below. If everything is at rest, what is the tension in the cord?



a) Less than mg.
b) Exactly mg.
c) More than mg, but less than $2mg$.
d) Exactly $2mg$.
e) More than $2mg$.

Question 19

Car A and car B are both traveling down a straight highway at 90 km/h. Car A is only 6.0 meters behind car B. The driver of car B brakes, slowing down with a constant deceleration of $2.0\,\mathrm{m/s^2}$. After 1.2 seconds, the driver of car A begins to brake, also at $2.0\,\mathrm{m/s^2}$, but this is insufficient and the cars eventually collide, both still moving forward. What is the relative velocity of the two cars at the moment of the collision?

a) 2.4 m/s
b) 5.0 m/s
c) 9.5 m/s
d) 21 m/s
e) 24 m/s

Question 20

Two interfering waves have the same wavelength, frequency, and amplitude. They are traveling in the same direction but are 90 degrees out of phase. Compared to the individual waves, what can be said about the resultant wave?

a) It will have the same amplitude and velocity, but a different wavelength.
b) It will have the same amplitude and wavelength, but a different velocity.
c) It will have the same wavelength and velocity, but a different amplitude.
d) It will have the same amplitude and frequency, but a different velocity.
e) It will have the same frequency and velocity, but a different wavelength.

Question 21

A spinning ice skater has an initial kinetic energy $\frac{1}{2}I\omega^2$. She pulls in her outstretched arms, decreasing her moment of inertia to $\frac{1}{4}I$. What is her new angular speed?

a) $\omega/4$
b) $\omega/2$
c) ω
d) 2ω
e) 4ω

Question 22

A ball of mass m is thrown vertically upward. Instead of neglecting air resistance, assume that the force of air resistance has a magnitude proportional to the ball's velocity, but pointing in the opposite direction. What is the ball's acceleration at the highest point?

a) 0
b) Less than g.
c) g
d) Greater than g.

Question 23

A hypothetical planet has density ρ, radius R, and surface gravitational acceleration g. What would be the acceleration due to gravity at the surface of a planet with double the radius, and the same planetary density?

a) $4g$
b) $2g$
c) g
d) $g/2$
e) $g/4$

Question 24

You are given two lenses placed close together: a converging lens with focal length +10 cm, and a diverging lens with focal length −20 cm. Which of the following would produce a virtual image that is larger than the object?

a) Placing the object 5 cm from the converging lens.
b) Placing the object 15 cm from the converging lens.
c) Placing the object 25 cm from the converging lens.
d) Placing the object 15 cm from the diverging lens.
e) Placing the object 25 cm from the diverging lens.

Question 25

An electromagnetic wave is propagating in vacuum in the positive z direction. At an instant when its magnetic field B at a certain position is in the positive x direction and getting stronger, what happens to its electric field E at that same position?

a) E is in the positive x direction and getting weaker.
b) E is in the negative x direction and getting stronger.
c) E is in the positive y direction and getting weaker.
d) E is in the negative y direction and getting stronger.
e) E is zero.

Part B: Problems

Problem 1

A light emitting diode (LED) is connected in series with a super capacitor of capacity 3000 F and internal resistance $\sim$ 1 mΩ, and a microprocessor-controlled resistor. The capacitor is charged to 5 V. The LED requires constant current of 800 mA. At this current, the voltage drop over the LED is 2.7 V.

a) What should be the dependence of the variable resistor on time for the current to remain constant?

b) What should be the initial value of the resistor? Does the value of the resistor stabilize after some time, and if so, to which resistance?

c) How long can the LED operate at full brightness before the capacitor needs to be recharged?

Problem 2

A light spring with a force constant of 3.85 N/m is compressed by 8.00 cm and held between two blocks. The block on the left has a mass of 250 g, while the block on the right has a mass of 500 g. Both blocks are initially held at rest on a horizontal surface, and are then released simultaneously so that the spring pushes them apart.
What is the maximum velocity each block attains if the coefficient of kinetic friction between each block and the surface is 0.100 and the coefficient of static friction is 0.120?

Problem 3

The luminosity of a star is defined as the energy it emits per unit time. We know from astronomical observations that the luminosity L of the stars that have a mass $M < 10M_S$ is approximately related to their mass as:

$$\frac{L}{L_S} = (\frac{M}{M_S})^{3.5} \tag{1}$$

where L_S and M_S are the luminosity and mass of the Sun.
Furthermore, we know from Einstein's theory of relativity that the energy released from converting a mass M into energy is given by $E = Mc^2$. In this problem, we assume that all stars burn almost the same fraction of their mass during their active life.

a) If the fraction of the mass converted into energy is $\alpha \ll 1$, first find an expression for the lifetime of a star in terms of its mass, luminosity, α and physical constants.

b) Using the luminosity-mass equation, find the lifetime of a star as a function of its mass.

c) If we know that the lifetime of our Sun is about 10 billion years, what is the lifetime of a star with a mass $M = 5M_S$?

Question 1	a	b	c	d	e	f
Question 2	a	b	c	d	e	f
Question 3	a	b	c	d	e	f
Question 4	a	b	c	d	e	f
Question 5	a	b	c	d	e	f
Question 6	a	b	c	d	e	f
Question 7	a	b	c	d	e	f
Question 8	a	b	c	d	e	f
Question 9	a	b	c	d	e	f
Question 10	a	b	c	d	e	f
Question 11	a	b	c	d	e	f
Question 12	a	b	c	d	e	f
Question 13	a	b	c	d	e	f
Question 14	a	b	c	d	e	f
Question 15	a	b	c	d	e	f
Question 16	a	b	c	d	e	f
Question 17	a	b	c	d	e	f
Question 18	a	b	c	d	e	f
Question 19	a	b	c	d	e	f
Question 20	a	b	c	d	e	f
Question 21	a	b	c	d	e	f
Question 22	a	b	c	d	e	f
Question 23	a	b	c	d	e	f
Question 24	a	b	c	d	e	f
Question 25	a	b	c	d	e	f

CAP High School Prize Exam

April 2013

Solutions

1 Part A: Multiple Choice

Question 1	**a**	b	c	d	e	f
Question 2	a	b	**c**	d	e	f
Question 3	**a**	b	c	d	e	f
Question 4	**a**	b	c	d	e	f
Question 5	**a**	b	c	d	e	f
Question 6	a	**b**	c	d	e	f
Question 7	a	**b**	c	d	e	f
Question 8	**a**	b	c	d	e	f
Question 9	a	b	**c**	d	e	f
Question 10	a	b	**c**	d	e	f
Question 11	a	b	c	**d**	e	f
Question 12	a	b	c	**d**	e	f
Question 13	a	**b**	c	d	e	f
Question 14	a	b	c	**d**	e	f
Question 15	a	**b**	c	d	e	f
Question 16	a	b	**c**	d	e	f
Question 17	a	**b**	c	d	e	f
Question 18	a	**b**	c	d	e	f
Question 19	**a**	b	c	d	e	f
Question 20	a	b	**c**	d	e	f
Question 21	a	b	c	d	**e**	f
Question 22	a	b	**c**	d	e	f
Question 23	a	**b**	c	d	e	f
Question 24	**a**	b	c	d	e	f
Question 25	a	b	c	**d**	e	f

2 Part B: Problems

Problem 1

The internal resistance of the capacitor is small compared with the dynamic resistance of the diode at the operating current, so it can be ignored.

$$Q(t)/C + R(t)I + V_{LED} = 0$$

$I = I_0$ is to be kept constant at 800 mA, so

$$Q(t) = Q_0 - I_0 t$$

... where Q_0 is the initial charge.

$$Q_0 = V_0 C$$
$$(Q_0 - I_0 t)/C - R(t)I_0 - V_{LED} = 0$$
$$V_0 - I_0 t/C - R(t)I_0 - V_{LED} = 0$$
$$V_0/I_0 - t/C - R(t) - V_{LED}/I_0 = 0$$
$$R(t) = -t/C - V_{LED}/I_0 + V_0/I_0$$

Therefore R_0 has to be

$$R_0 = (V_0 - V_{LED})/I_0 = (5 - 2.7)/0.5 = 2.9\ \Omega$$

$$R(t) = -t/C + R_0 = R_0 - t/C$$

$R(t)$ cannot be negative, so $t_{max} = R_0 C = 3000 \times 2.9 = 8600$ s, that is 2.4 hours.

Problem 2

First, we must check the possibility for the spring to push the two blocks away in presence of friction. The force exerted by the spring on each block is in magnitude

$$F = kx = 3.85\ \text{N/m} \times 0.08\ \text{m} = 0.308\ \text{N}$$

This value must be compared to the static friction force exerted on each block.

The lighter block to the left experiences the force of static friction

$$f_l = \mu_s n = \mu_s m_l g = 0.120 \times 0.250\ \text{kg} \times 9.80\ \text{m/s}^2 = 0.294\ \text{N}.$$

This block will start moving when pushed by the spring.

Similarly, the block to the right experiences the force of static friction

$$f_r = \mu_s n = \mu_s m_r g = 0.120 \times 0.500\ \text{kg} \times 9.80\ \text{m/s}^2 = 0.588\ \text{N}.$$

Static friction for this block is greater than the spring force, so it will stay motionless.

The lighter block will gain speed as long as the spring force is larger than the kinetic friction force: that is, until the spring compression becomes

$$x_f = 0.100 \times 0.250\ \text{kg} \times 9.80\ \text{m/s}^2 = x_f \times 3.85\ \text{N/m} \Rightarrow x_f = 0.0636\ \text{m}$$

So, the energy of the lighter block as it moves to the maximum speed point can be found from

$$K_i + U_i - f_k d = K_f + U_f$$

$$0 + \frac{(3.85 \times 0.08^2)\ \text{J}}{2} - (0.100 \times 0.250 \times 9.80)\ \text{N} \times (0.08 - 0.0636)\ \text{m} = \frac{0.250\ \text{kg} \times {v_{max}}^2}{2} + \frac{3.85\ \text{N/m} \times 0.0636^2}{2}$$
$$\Rightarrow v_{max} = 0.0642\ \text{m/s}$$

Problem 3

a) The life of the star is the time it takes to use up all of its energy resources. By the definition of luminosity, after a time T the total energy emitted is LT. Using Einstein's theory of relativity, the total energy of the star, if it has mass M, is Mc^2, but only a fraction α of it is used. Therefore the total energy resources of the star are αMc^2. Setting the two expressions for the total energy emitted as equal:

$$LT = \alpha Mc^2 \Rightarrow T = \frac{\alpha Mc^2}{L}$$

b) Substituting the luminosity of the star from the equation, as a function of mass:

$$T = \frac{\alpha Mc^2}{L}$$

$$L = L_s(\frac{M}{M_s})^{3.5}$$

$$\Rightarrow T = \frac{\alpha Mc^2}{L_s} \times (\frac{M_s}{M})^{3.5} = \frac{\alpha M_s c^2}{L_s} \times (\frac{M_s}{M})^{2.5}$$

... but using the formula from part (a), the expression $\frac{\alpha M_s c^2}{L_s}$ is just the lifetime of the Sun, therefore

$$T = T_s(\frac{M_s}{M})^{2.5}$$

c) Using the previous equation:

$$T = (10 \times 10^9 \text{ years})(M_s/5M_s)^{2.5} = (10^{10} \text{ years}) \times 1/5^{2.5} = 1.79 \times 10^8 \text{ years}$$

CAP High School Prize Exam

April 8th, 2014
9:00 - 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS

Family Name: ______________________ Given Name: ______________________

Home Address: ____

______________________ Postal Code: ______________________

Telephone: (   ) ______________________ Email: ______________________

School: ______________________ Grade: ______________________

Physics Teacher: ____

Date of Birth: ______________________ Sex: Male ☐ Female ☐

Citizenship: ____

Would you prefer further correspondence in French or English? ______________________

If you are not a Canadian citizen, what is your Immigration Status? ______________________

For how many years have you studied in a Canadian school? ______________________

Sponsored by:

Canadian Association of Physicists,
Canadian Physics Olympiad,

The University of British Columbia,
Department of Physics and Astronomy.

Canadian Association of Physicists 2014 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on students' performance on sections A and B of the exam. Performance on the questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. Part A consists of twenty-five multiple-choice questions. The questions in part B span a range of difficulties, and may require graphing. Be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway.

Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions ***on the answer sheet*** provided; most importantly, write your solutions to the three long problems on ***three separate*** sheets as they will be marked by people in different parts of Canada. Good luck.

Notice: Full marks will be given to a student who provides any full correct solution to the long problems. Partial marks will be given for partial solutions. There are no penalties for incorrect answers. The questions are not of equal difficulty. Remember we are challenging the best physics students in Canada; it is possible that even the best papers may not achieve an overall score of 80%. This is meant to be tough!

Data

Speed of light $c = 3.00 \times 10^8\,\mathrm{m/s}$
Gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$
Acceleration due to gravity $g = 9.80\,\mathrm{m/s^2}$
Normal atmospheric pressure $P_0 = 1.01 \times 10^5\,\mathrm{Pa}$
Density of fresh water $\rho = 1.00 \times 10^3\,\mathrm{kg/m^3}$
Specific heat of water $C_w = 4186\,\mathrm{J/(kg \cdot K)}$
Specific heat of ice $C_i = 2050\,\mathrm{J/(kg \cdot K)}$
Latent heat of water $L_w = 2260\,\mathrm{kJ/kg}$
Latent heat of ice $L_i = 334\,\mathrm{kJ/kg}$
Density of ice $\rho_i = 916\,\mathrm{kg/m^3}$
Fundamental charge $e = 1.60 \times 10^{-19}\,\mathrm{C}$
Mass of electron $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of proton $m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$
Planck's constant $h = 6.63 \times 10^{-34}\,\mathrm{Js}$
Coulomb's constant $1/(4\pi\epsilon_0) = 8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2}$
Boltzmann's constant $k = 1.38 \times 10^{-23}\,\mathrm{J/K}$
A.U. Astronomical Unit $= 1.49598 \times 10^{11}\,\mathrm{m}$: The approximate distance from the Sun to the Earth.
Radius of the Earth $R_E = 6.371 \times 10^6\,\mathrm{m}$
Radius of the Sun $R_S = 6.96 \times 10^8\,\mathrm{m}$
Stefan's constant $\sigma = 5.6704 \times 10^{-8}\,\mathrm{W/(m^2 \cdot K^4)}$

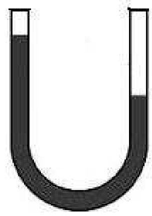
Part A: Multiple Choice

Each multiple choice question is worth 1 point.

Question 1
Consider the electric power dissipation due to resistance in a circuit. Which of the following changes leave the dissipated electric power unchanged?

a) Doubling the voltage and reducing the current by a factor of two.
b) Doubling the voltage and increasing the resistance by a factor of four.
c) Doubling the current and reducing the resistance by a factor of four.
d) None of the above.
e) Both (b) and (c) are correct.
f) (a),(b) and (c) are correct.

Question 2
A U-shaped glass tube closed at both ends and positioned vertically is partly filled with water. At a certain time the levels of water are different in the two arms of the tube due to a difference in air pressure above each arm. If there is no temperature change, what will happen to the water level on both sides?



a) The water levels will stay stationary.
b) The water levels will equalize rapidly — within seconds.
c) The water levels will slowly equalize — within days.
d) The difference between the levels in each arm will increase.

Question 3
How does the magnitude of the gravitational force with which the Moon attracts the Earth compare to the magnitude of the gravitational force with which the Earth attracts the Moon?

a) They are equal.
b) The first is greater.
c) The first is smaller.

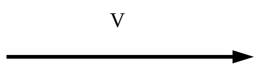
Question 4
The friction force acting on a bicycle is 20 N. What power does a cyclist need in order to travel at 18 km/h?

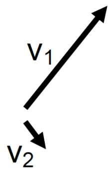
a) 50 W
b) 100 W
c) 1800 W
d) 3600 W

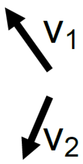
Question 5
What is the average horizontal force acting on a ball when it elastically bounces off a wall, assuming the collision time is 0.1 s and the momentum of the ball before the bounce was $2\,\mathrm{kg \cdot m/s}$, perpendicular to the wall?

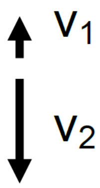
a) 0.2 N
b) 0.4 N
c) 40 N
d) 20 N

Question 6
A small explosion occurs in a model airplane which rips it into two pieces. The model was flying with velocity v just before the explosion. Which combination of v_1 and v_2 are possible velocities of the two pieces after the explosion?

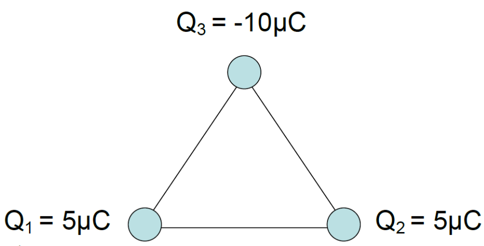


a) 

b) 

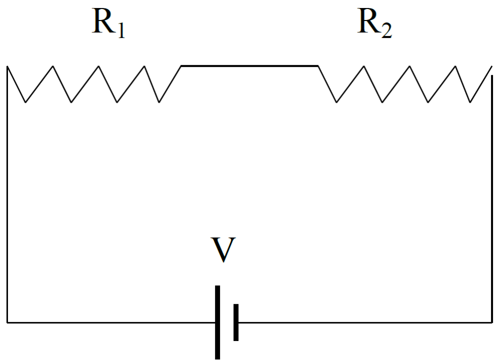
c) 

Question 7
Three ping-pong balls are electrically charged and are arranged in the plane of the page in an equilateral triangle as shown below. What is the direction of the force acting on the ping-pong ball charged with $Q_3 = -10\,\mu\mathrm{C}$?



a) Towards the top of the page.
b) Towards the bottom of the page.
c) Towards the left.
d) Towards the right.
e) Another direction.

Question 8
In the circuit shown below, the resistance R_1 is increased. What happens to the magnitude of the potential difference across R_1?



a) It increases.
b) It decreases.
c) It remains the same.

Question 9
Two identical loudspeakers, placed close to each other, are supplied with the same sinusoidal voltage. One can imagine a pattern around the loudspeakers with alternating areas of increased and decreased sound intensity. Which of the actions below will **not** change the positions of these areas?

a) Moving one of the speakers.
b) Changing the amplitude of the voltage.
c) Changing the frequency.
d) Replacing the air in the room with a gas of a different density.

Question 10
Two artificial satellites, named Argo and Navis, have circular orbits of radii R and $2R$, respectively, about the same planet. The orbital speed of Argo is v. What is the orbital speed of Navis?

a) $v/2$
b) $v/\sqrt{2}$
c) v
d) $v\sqrt{2}$
e) $2v$

Question 11
When someone drags their fingernails across a chalkboard, a terrible high-pitched sound is produced due to small bumps in the chalkboard. Assume these bumps are uniformly spaced by 0.5 mm. Audiologists have determined that humans find sounds in the range of $2 \sim 4\,\mathrm{kHz}$ to be very annoying. An evil teacher wants to produce the longest duration continuous sound in this range by dragging her fingernails across the chalkboard. At what speed should she drag her nails to accomplish this?

a) 0.28 m/s
b) 0.56 m/s
c) 1.00 m/s
d) 2.00 m/s

Question 12

The Hall Effect occurs when both a current and magnetic field are present and perpendicular to each other in a solid. The result is the generation of an electric field and a corresponding potential difference (the Hall voltage) across the width of the solid. Suppose a two-dimensional rectangular material carries a current of 0.5 A in the positive x direction and is penetrated by a magnetic field of 1.4 mT in the negative z direction. The number of mobile charges per unit area of the material is $0.2\,\mu\text{C/m}^2$. What is the magnitude of the Hall voltage, and the direction of the generated electric field? (Assume a right-handed coordinate system)

a) $70\,\mu$V, negative y direction
b) $70\,\mu$V, positive y direction
c) 3.5 kV, negative y direction
d) 3.5 kV, positive y direction

Question 13

^{214}Po atoms have a mass of 3.55×10^{-22} g and decay into ^{210}Pb with a half-life of $160\,\mu$s. A detector encompassing 1g of ^{214}Po counts the number of ^{210}Pb daughters produced. An experimentalist rigs an oscillator so that the frequency of electromagnetic radiation it emits matches the frequency of ^{210}Pb counts measured by detector. After 8 ms, what type of electromagnetic radiation is produced by the oscillator?

a) Radar
b) Red light
c) Ultra-violet
d) X-rays

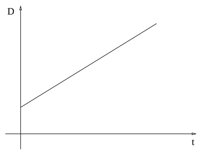
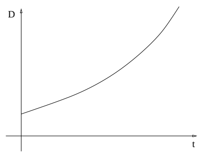
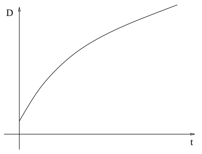
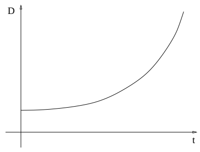
Question 14

To submerge a block of wood which is less dense than water, one needs to exert a force downward which does a positive amount of work on the block. Which of the following is correct for this situation to occur?

a) The work done by the external force is stored as potential energy in the block.
b) The block is moving downward, therefore its potential energy is decreasing, thus the work done by the external force is all converted to heat, due to friction.
c) The work done by external force is all stored as kinetic energy in the block.
d) The potential energy of the water is increased while the potential energy of the block is decreased.
e) The total energy of the block is conserved.
f) The total energy of the block and water is conserved.

Question 15

In 1929, Edwin Hubble discovered that the universe is expanding. He observed that galaxies far away from us are moving away at a speed that is proportional to their distance from us (you can assume the constant of proportionality is time-independent). For a galaxy that obeys Hubble's law, which of the following can be the graph of distance (from Earth) versus time? For each plot, $t = 0$ corresponds to the present.

a) 
b) 
c) 
d) 

Question 16

Which of the following is closest to the thickness of a piece of paper?

a) 10^{-3} m
b) 10^{-4} m
c) 10^{-5} m
d) 10^{-6} m
e) 10^{-7} m

Question 17

The paper-folding theorem states that in order to fold something in half, it's length must be at least π times its thickness. How many times can you fold a standard sheet of printing paper ($0.216\,\text{m} \times 0.279\,\text{m}$) if you always fold from the middle of the longer edge?

a) 5
b) 6
c) 7
d) 12

Question 18

A circuit contains nothing but a battery of voltage V wired to three resistors of resistance R. Which of the following **cannot** be the power dissipated in the circuit (assuming negligible resistance for the wires)?

a) $P = V^2/(3R)$
b) $P = 3V^2/R$
c) $P = 3V^2/(2R)$
d) $P = 2V^2/(3R)$
e) All of the above are possible

Question 19

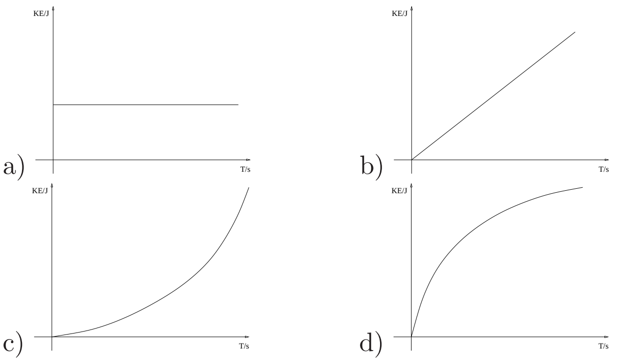
In a binary star system consisting of two stars of equal mass, where is the gravitational potential equal to zero? Assume that for a single star in empty space, the potential is zero at infinity.

a) Exactly halfway between the stars.
b) Along a line bisecting the line connecting the stars and perpendicular to the plane of the stars' orbit.
c) Infinitely far from the stars.
d) At any point on a plane bisecting the line connecting the stars and perpendicular to the plane of the stars' orbit.

Question 20
In Young's double-slit experiment, both slits are illuminated by a laser beam and the interference pattern is observed on a screen. If the viewing screen is moving farther from the slits, what will happen to the interference pattern?

a)The pattern gets brighter.
b)The pattern gets brighter and the fringes move closer to each other and to the central fringe.
c)The pattern gets brighter and the fringes move farther from the central fringe and from each other.
d)The pattern gets less bright and fringes appear farther apart.
e)There is no change in the pattern.
f)The pattern becomes unfocused.

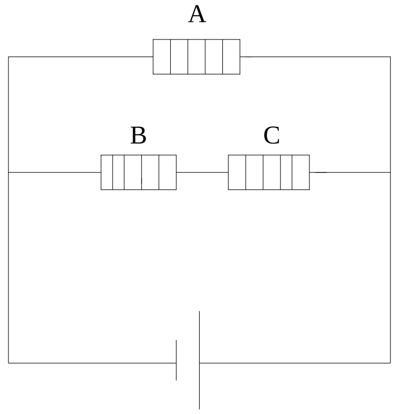
Question 21
A ball is dropped to the Earth from a height of 2 m. Neglecting the air resistance, which one of the following graphs represents the kinetic energy of the ball vs. time?



Question 22
What will happen to the magnitude of the optical power of a lens when it is placed in water ($n = 1.33$) compared to its power in the air ($n = 1$)?

a)It will increase.
b)It will decrease.
c)It will stay the same.
d)It will depend on whether the lens is converging or diverging.

Question 23
The three electric heaters in the following circuit all have the same resistance. Given that the total heat emitted by a heater is proportional to the power dissipated, the total heat produced by B and C together, compared with the heat produced in A, is:



a) A quarter as much.
b) Half as much.
c) The same.
d) Twice as much.
e) Four times as much.

Question 24
A box sits on a horizontal surface that exerts a normal force N on the box. You apply a horizontal force to it and it does not move. If you had applied a force twice as large, it still would not have moved. Let μ_s be the coefficient of static friction of the surface. While you are applying your initial force, which of the following is true of the force of friction acting on the box?

a) $F_f = 0$
b) $F_f < \mu_s N$
c) $F_f \leq \mu_s N$
d) $F_f = \mu_s N$
e) $F_f \geq \mu_s N$
f) $F_f > \mu_s N$

Question 25
Which of the following is closer to the dimensions of a solar cell panel that can produce enough energy for a family in Vancouver in summer time? The following information might be useful:
The price of residential electricity in British Columbia is approximately 6.90 cents per kWh. A typical household's monthly electricity bill is $40 in the summer. The power per area from sunlight that reaches the city of Vancouver is about $0.5\,\mathrm{kW/m^2}$ (averaged over 24 hours) during the period of June-September.
The efficiency of a typical solar cell is about 20%.

a) $30\,\mathrm{cm} \times 30\,\mathrm{cm}$
b) $3\,\mathrm{m} \times 3\,\mathrm{m}$
c) $30\,\mathrm{m} \times 30\,\mathrm{m}$
d) $300\,\mathrm{m} \times 300\,\mathrm{m}$
e) The size of the panel must be much bigger than any of these numbers, since it is always cloudy in Vancouver and the city does not get enough sunshine!

Part B: Problems

Problem 1

When an objects breaks, many of the molecular bonds get detached, but in this process, some of the energy that caused the break turns into heat and sound wave energy.

In this problem we want to understand what percentage of the energy that causes a break is actually used to break the molecular bondings. As a simple model we can think of glass as a cubical structure, which means each SiO_2 molecule occupies a cube of side length a, and each cubic molecule site has 1 bonding with each of its neighbouring cubes. The energy required to break this bond is called bonding energy, denoted by E_b.

Furthermore, in this problem we are only interested in the **order of magnitude** of the values we obtain. The following image shows a piece of glass of dimensions 25 mm × 75 mm × 1 mm that is fallen down from a height 150 cm and broken into pieces.

a) Using the image, estimate the total length of cracks and thereby the total number of broken bonds.

b) Using the latent heat of vaporization for glass, estimate the bonding energy of glass.

c) What percentage of the energy in this collision is used to break the bonds?

Numerical Values:

- Density of glass: $\rho_g = 2\,\mathrm{g/cm^3}$
- Molecular mass of SiO_2: $M_{SiO_2} = 60\,\mathrm{g/mol}$
- Latent heat of Vaporization for glass: $L_g = 10\,\mathrm{kJ/g}$

Problem 2

The main span of the Lion Gate Bridge has a length of 473 m. On each end there are the expansion joints like the one on the photo below. They allow the span to expand horizontally without warping the steel frame of the span. One day the temperature in Vancouver changed from −4 to +15 degrees between 6 in the morning and 2 in the afternoon.

a) What was the average speed of the "tooth" in one of the expansion joints?

b) At 6 in the morning a piece of tire rubber fell into a crack in front of one of the "fingers", filling it completely. The rubber was 10 cm long and had a cross section of 4 cm^2. What was the force acting on the rubber along its length at 2 PM?

c) What will be the vertical force needed to pull this piece of rubber vertically to remove it?

Clearly state all the assumptions that you made while solving this problem. Data: The elastic constant of such a piece of rubber compressed along its length is 28 N/m. The coefficient of friction between the rubber and steel is 0.35. The linear Expansion coefficient of steel $13 \times 10^{-6}\,\mathrm{K^{-1}}$. The linear Expansion coefficient of rubber $77 \times 10^{-6}\,\mathrm{K^{-1}}$.

Problem 3

A planet without atmosphere in orbit around a star radiates away an amount of energy equal to the amount of energy it receives from the star. Therefore, the average surface temperature of the planet is constant over decades. Imagine an Earth-sized planet without atmosphere orbiting around a Sun-sized star (with the same radiation power as the sun) at a distance of 1 AU (AU is the Astronomical Unit, equal to the Earth-Sun distance).

a) What portion of the energy radiated by the star is captured by the planet? (Assume that planet absorbs all the energy it receives)

b) According to Stefan-Boltzmann's law, the power per area emitted from the surface of an object at temperature T is $I = \sigma T^4$ for which σ is Stefan's constant. Assuming the power output of the star is the same as the Sun (3.85×10^{26} W), find the surface temperature of the planet. Compare this result to the average surface temperature of the Earth (288 K). How much difference does the atmosphere make in the surface temperature of Earth? Would Earth be warm enough to be habitable for humans if it did not have an atmosphere?

c) Neptune is the outer-most planet in our solar system at distance 30.4 AU. If we assume that Neptune absorbs all the energy it receives from the Sun, what is the average uniform surface temperature of this planet?

Question 1	a	b	c	d	e	f
Question 2	a	b	c	d	e	f
Question 3	a	b	c	d	e	f
Question 4	a	b	c	d	e	f
Question 5	a	b	c	d	e	f
Question 6	a	b	c	d	e	f
Question 7	a	b	c	d	e	f
Question 8	a	b	c	d	e	f
Question 9	a	b	c	d	e	f
Question 10	a	b	c	d	e	f
Question 11	a	b	c	d	e	f
Question 12	a	b	c	d	e	f
Question 13	a	b	c	d	e	f
Question 14	a	b	c	d	e	f
Question 15	a	b	c	d	e	f
Question 16	a	b	c	d	e	f
Question 17	a	b	c	d	e	f
Question 18	a	b	c	d	e	f
Question 19	a	b	c	d	e	f
Question 20	a	b	c	d	e	f
Question 21	a	b	c	d	e	f
Question 22	a	b	c	d	e	f
Question 23	a	b	c	d	e	f
Question 24	a	b	c	d	e	f
Question 25	a	b	c	d	e	f

1 Multiple Choice

Question 1	a	b	c	d	e	**f**
Question 2	a	b	**c**	d	e	f
Question 3	**a**	b	c	d	e	f
Question 4	a	**b**	c	d	e	f
Question 5	a	b	**c**	d	e	f
Question 6	**a**	b	c	d	e	f
Question 7	a	**b**	c	d	e	f
Question 8	**a**	b	c	d	e	f
Question 9	a	**b**	c	d	e	f
Question 10	a	**b**	c	d	e	f
Question 11	a	b	**c**	d	e	f
Question 12	a	b	**c**	d	e	f
Question 13	**a**	b	c	d	e	f
Question 14	a	b	c	**d**	e	f
Question 15	a	**b**	c	d	e	f
Question 16	a	**b**	c	d	e	f
Question 17	a	b	**c**	d	e	f
Question 18	a	b	c	d	**e**	f
Question 19	a	b	**c**	d	e	f
Question 20	a	b	c	**d**	e	f
Question 21	a	b	**c**	d	e	f
Question 22	a	**b**	c	d	e	f
Question 23	a	**b**	c	d	e	f
Question 24	a	**b**	c	d	e	f
Question 25	a	**b**	c	d	e	f

2 Written Response

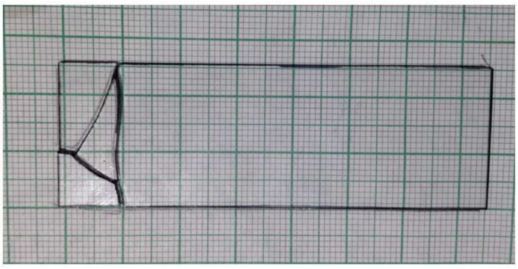
Problem 1

When an objects breaks, many of the molecular bonds get detached, but in this process, some of the energy that caused the break turns into heat and sound wave energy.

In this problem we want to understand what percentage of the energy that causes a break is actually used to break the molecular bondings. As a simple model we can think of glass as a cubical structure, which means each SiO_2 molecule occupies a cube of side length a, and each cubic molecule site has 1 bonding with each of its neighbouring cubes. The energy required to break this bond is called bonding energy, denoted by E_b.

Furthermore, in this problem we are only interested in the **order of magnitude** of the values we obtain.

The following image shows a piece of glass of dimensions 25 mm $\times$75 mm $\times$1 mm that is fallen down from a height 150 cm and broken into pieces.



a) Using the image, estimate the total length of cracks and thereby the total number of broken bonds.

b) Using the latent heat of vaporization for glass, estimate the bonding energy of glass.

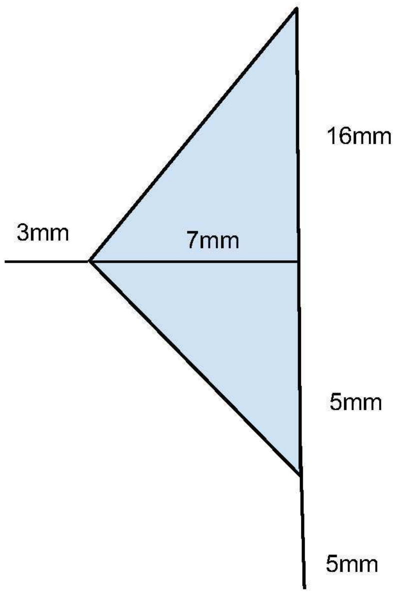
c) What percentage of the energy in this collision is used to break the bonds?

Numerical Values:

- Density of glass: $\rho_g = 2$ g/cm^3
- Molar mass of SiO_2: $M_{SiO_2} = 60$ g/mol
- Latent heat of Vaporization for glass: $L_g = 10$ kJ/g

Solution

a) From the figure, the glass cracked into 4 pieces. The middle piece is approximately triangular and should be divided into two triangles as in the figure below. The hypotenuses of these two triangles are determined from Pythagoras' theorem to be 17.5mm and 8.6mm for the top and bottom sides respectively.



Adding up all the side lengths gives $3 + 17.5 + 8.6 + 16 + 5 + 5 = 55.1$mm. Acceptable answers can differ from this by several millimetres due to the irregularity of the cracks.

Now, the volume occupied by a single molecule is

$$V = \frac{(60\text{g/mol})}{(2\text{g/cm}^3)(6.022 \times 10^{23}\text{molecules/mol})} = 4.98 \times 10^{-23}\text{cm}^3/\text{molecule}. \quad (1)$$

This is the volume of a cube with side length $a = V^{1/3} = 3.7 \times 10^{-8}cm= 3.7 \times 10^{-7}$mm. Each face of the cube has an area of 1.37×10^{-13}mm^2, with one bond protruding from each face.

The glass is 1mm thick, so the total surface area of the cracks is 55.1mm×1mm= 55.1mm^2. Thus the total number of broken bonds is

$$N = \frac{55.1\text{mm}^2}{1.35 \times 10^{-15}\text{mm}^2/\text{bond}} = 4.02 \times 10^{14}. \tag{2}$$

b) The latent heat of Vaporization is the energy density required to vaporize the glass. That is, to break all the bonds of the glass. There are 6 bonds attached to each molecule, but these are shared with other molecules, so 6 bonds per molecule would be over-counting. Consider a cube containing $n \times n \times n$ molecules. Then there are $(n-1) \times (n-1) \times (n-1) \approx n^3$ bonds oriented along each direction. There are 3 directions along which the bonds can lie (x, y, z), so there are a total of $3n^3$ bonds, or 3 bonds per molecule.

Now, the amount of energy per molecule required to vaporize the glass is:

$$(10\text{kJ/g})(60\text{g/mol})\left(\frac{1\text{mol}}{6.022 \times 10^{23}\text{molecules}}\right) = 9.96 \times 10^{-22}\text{kJ/molecule}. \tag{3}$$

Since there are 3 bonds per molecule, we get a bonding energy of

$$(9.96 \times 10^{-22}\text{kJ/molecule})\left(\frac{1\text{molecule}}{3\text{bonds}}\right) = 3.32 \times 10^{-22}\text{kJ/bond} = 3.32 \times 10^{-19}\text{J/bond}. \tag{4}$$

c) The total energy in the system is just the gravitational potential energy of the glass before it is dropped.

$$E_{\text{total}} = \rho V g h = (0.002\text{kg/cm}^3)(1.875\text{cm}^3)(9.81\text{m/s}^2)(1.5\text{m}) = 5.52 \times 10^{-2}\text{J}, \tag{5}$$

where the volume is determined from the dimensions of the glass $V = (2.5\text{cm})(7.5\text{cm})(0.1\text{cm}) = 1.875\text{cm}^3$. Meanwhile, the energy used to break the glass was

$$(3.32 \times 10^{-19}\text{J/bond})(4.04 \times 10^{14}\text{bonds}) = 1.34 \times 10^{-4}\text{J}. \tag{6}$$

So the percentage of the total energy that was used to break the bonds was

$$\frac{1.34 \times 10^{-4}\text{J}}{5.52 \times 10^{-2}\text{J}} \times 100\% = 0.24\%. \tag{7}$$

The rest of the energy is transferred to thermal energy in the glass and floor.

Problem 2

The main span of the Lion Gate Bridge has a length of 473m. On each end there are the expansion joints like the one on the photo below. They allow the span to expand horizontally without warping the steel frame of the span. One day the temperature in Vancouver changed from –4 to +15 degrees between 6 in the morning and 2 in the afternoon.

a) What was the average speed of the "tooth" in one of the expansion joints?

b) At 6 in the morning a piece of tire rubber fell into a crack in front of one of the "fingers", filling it completely. The rubber was 10 cm long and had a cross section of 4 cm^2. What was the force acting on the rubber along its length at 2 PM?

c) What will be the vertical force needed to pull this piece of rubber vertically to remove it?

Clearly state all the assumptions that you made while solving this problem.

Data:
The elastic constant of such a piece of rubber compressed along its length is 28N/m.
The coefficient of friction between the rubber and steel is 0.35.
The linear Expansion coefficient of steel is $13 \times 10^{-6}\ K^{-1}$.
The linear Expansion coefficient of rubber is $77 \times 10^{-6}\ K^{-1}$.

a) In 8 hours the span increases its length by:
$\Delta L = L0\alpha\Delta T = 473\ m \bullet 13 \times 10^{-6}\ K^{-1} \bullet (15-(-4))K = 0.12\ m$
It expands uniformly in both directions so the tooth moves by 0.06 m
The speed is
$0.06/(8 \bullet 3600) = 2 \bullet 10^{-6}\ m/s = 2\ \mu m/s$

b) The expansion of cm of rubber is negligible so we can assume that it was compressed by 0.06m so force acting on the rubber from the sides is
$F = k\Delta x = 28N/m \bullet 0.06 = 1.6\ N$

c) It acts on both sides of the rubber so total normal force is twice that = 3.2 N
The force of friction is about $3.2 \bullet 0.35 = 1.1$ N
To lift the rubber one has to apply this force plus weight of rubber. We did not specify the density of rubber (which is $1.1 \bullet 10^3\ kg/m^3$) so one could ignore it or say that this density was similar to water and calculate the weight to be about 0.15 N.
So the force needed to pull the rubber was about 1.2 N

Assumptions:
Uniform expansion of the span of the bridge in both directions due only to the expansion of the steel.
Rubber compressed uniformly without "buckling".
Thermal expansion of the rubber was negligible compared to its compression by the tooth.

Solution to Problem 3.

(a) The portion of energy captured by the planet of radius *r* that is (upon the problem description) a black body and orbiting the star at the distance *d* from its center is given by

$$\gamma = \frac{\Delta E_p}{\Delta E_s} = \frac{\pi r^2}{4\pi d^2} = \left(\frac{r}{2d}\right)^2 = 4.534 \text{ x } 10^{-10}$$

(b) Since the planet is a black body, the power from the star, absorbed by the planet, is equal to the power, emitted by the planet in all directions. Or

$$(\Delta E_p / \Delta t) = I_p (4\pi r^2) = \gamma (\Delta E_s / \Delta t)$$

where $(\Delta E_s / \Delta t) = 3.85 \text{ x } 10^{26}$ W.
The average temperature T_p on the planet surface can be found from $I_p = \sigma T_p^4$,

$$T_p = \left[\frac{\gamma(\Delta E_s / \Delta t)}{4\pi r^2 \sigma}\right]^{1/4} = \left[\frac{\Delta E_s / \Delta t}{16\pi d^2 \sigma}\right]^{1/4} = 279 \text{ K.} \quad (1)$$

This corresponds to + 6°C.
The real average Earth surface temperature is 288 K = 15°C. If the Earth had no atmosphere, its average surface temperature of 279 K could theoretically be warm enough to produce life and not freeze or burn the living beings in case it were somehow evenly distributed over the surface. However, the Moon surface is known of being much hotter than 100°C on the side facing our Sun and much cooler than 0°C on the dark side. Very like temperature distribution can be expected for any planet without an atmosphere in the Solar system, including Earth. Water as the main solvent boiling on the planet side, exposed to Sun, would kill proteins, and life would be impossible.

(c) The only chance to calculate the average surface temperature of Neptune is given by assumption that Neptune is also a black body without an atmosphere (which is really not so). According to the Stefan-Boltzmann's law and following the formula (1) for a different distance d_N between Sun and Neptune, the average temperature on the surface is given by

$$T_N = \left[\frac{\Delta E_s / \Delta t}{16\pi d_N^2 \sigma}\right]^{1/4} = T_p \sqrt{\frac{d}{d_N}} = 50.6 \text{ K.}$$